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US Phillips-Curve Dynamics:
Mark-Up Cyclicality, Effective Hours
and Regime-Dependency*

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Abstract
This paper re-examines the validity of the Phillips-Curve framework using US data. We make
three main innovations. First, we introduce into the well-known Calvo price staggering framework,
a regime-dependent price-changing signal. This means that a state-dependent linearization is no
longer required to derive the Phillips relationship and thus that questions of regime dependency
can be addressed. Second, we engage on a careful modeling of long-run supply in the economy,
which permits more data-coherent measures of output gaps and real marginal costs indicators
consistent with underlying, frictionless supply. Finally, we include two types of labor adjustment
costs reflecting the intensive and extensive participation decisions. As regards the latter, we
introduce the concept of "effective" working hours into the production technology which generates
an overtime function directly into the mark-up equation. This, it turns, out has first-order
implications for the cyclicality and econometric fit of the mark-up implied by the Phillips-curve
representation.

Keywords: Phillips Curve, Mark-up Cyclicality, Effective Hours, Factor-Augmenting Technical
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JEL:
Word Count:
1. Introduction

New Keynesian Phillips Curves (NKPCs) have become increasingly popular for analyzing inflation dynamics and accounting for real and nominal economic interactions. The specification - predicated on rational expectations, monopolistic competition and price staggering - models current inflation, $\pi_t$, as a function of its future expectation and some fundamental driving variable:

$$\pi_t = \beta E_t[\pi_{t+1}] + \lambda x_t,$$

where $\beta$ is a discount factor ($\beta < 1$); $\lambda (\lambda > 0)$, the slope of the Phillips curve, is a composite parameter arising from the optimization environment; and $x$ is the assumed fundament (e.g., real marginal costs) implying $\pi_t = \lambda \sum_{j=0}^{\infty} \delta^j x_{t+j}$. The literature on the performance and plausibility of NKPCs is by now substantial (e.g., Roberts, 1995; Galí and Gertler, 1999; Rudd and Whelan, 2007; McAdam and Willman, 2004). In this paper, however, we estimate NKPCs on aggregate US data correcting for what we believe are three fundamental (and arguably neglected) point of issue.

First, we consider the case where the coefficients of the NKPC rather than being constant are a function of economic conditions (i.e., state dependent rather than time dependent). Second, relates to modeling the intensive employment margin; in so doing, we introduce the concept of “effective labor hours”. An innovative aspect of our framework is that the former margin turns out to have a key spillover onto firms’ optimal pricing decisions. Finally, we estimate the underlying supply side allowing the elasticity of substitution to deviate from unity and for technical progress to deviate from Harrod-Neutrality. Accordingly, we avoid the common practise of using labor income share as the default representation of real marginal costs.

Taken together, these three innovations may shed light on a key question arising from the Phillips curve literature: namely the cyclicality of the difference between factor cost prices and real unit labor costs.

2 A Time-Varying Calvo-Price Setting Signal

Let us start with the most general Calvo-NKPC framework, namely one which incorporates “intrinsic” inflation persistence via “rule-of-thumb” price re-setters (although, note, our framework follows equally well assuming no rule-of-thumb price setters).1 In our set up the Calvo price reset signal is presumed to be state dependent, a corollary of which is that at the beginning of each period firms receive a time-varying signal regarding price setting in the following three-valued manner:

1 Hereafter, for simplicity and consistent with many other studies, we use the term NKPC (rather than hybrid NKPC, or H-NKPC) to denote a Phillips curve with intrinsic persistent (i.e., a backward-looking component).
(2) With a probability \((1-\theta_t)\omega\) firm \(j\) is allowed to change its price following a backward-looking pricing rule, as in Gali and Gertler (1999), \(P_t^j = P_t^b = (P_{t-1}^j / P_{t-2}^j)P_{t-1}^*,\) where \(P_{t-1}^*\) is the average price level selected by firms able to change price at time \(t-1,\) and where \(\omega \in [0,1]\) represents the fraction of firms able to reset prices but who do so in this rule-of-thumb manner.

(3) With a probability \((1-\theta_t)(1-\omega)\) firm \(j\) receives the signal that allows it to reset its price on the profit-maximization level, \(P_t^j = P_t^f.\)

As we shall see, the advantage of the three-valued signal is that in the beginning of each period, before the outcome of the signal is known, each firm faces exactly the same optimization problem, (i.e. in an ex-ante sense, all firms are profit-maximizers). By contrast, in the conventional approach there is a fixed, ex-ante classification of firms into profit-maximizers or non profit-maximizers and thus uncertainty concerns only whether the firm is or is not allowed to change its price. If allowed, the firm knows with ex-ante certainty whether it belongs to the rule-of-thumb or profit-maximizing group and hence ex-ante there are two behaviorally different groups of firms: profit maximizers and rule-of-thumb price setters.

Given this, the aggregate price level, \(p_t,\) (lower case denoting logs) can then be taken as the simple weighted sum of the reset and lagged price:\(^2\)

\[
p_t = (1-\theta_t)p_t^* + \theta_t p_{t-1}
\]

where

\[
p_t^* = (1-\omega)p_t^f + \omega p_t^b
\]  

(2)

Inserting (2) into (1), and subtracting \(p_{t-1}\) from both sides and rearranging, yields,

\[
[\theta_t + (1-\theta_t)\omega]p_t = (1-\theta_t)(1-\omega)p_t^f + \omega(p_t^* - p_{t-2})
\]

where \(p_t = \Delta p_t\) denotes inflation. Furthermore, using (1) to solve for \(p_{t-1}^*\) and inserting into (3), we derive,

\[
\left(\frac{\theta_t}{1-\theta_t} + \omega\right)p_t = \frac{\omega}{1-\theta_t}p_{t-1} + (1-\omega)(p_t^f - p_t)
\]

\[
(4)
\]

---

\(^2\) See Appendix A of McAdam and Willman (2007b) shows the formal derivation.
To proceed, we require a derivation of the profit-maximizing price, \( P_f \) as well as operationalizing of the time-varying Calvo signal, \( \theta_i \), to which we now turn.

3 Regime Dependency of the Signal

Although still exogenous to firms, we assume that the Calvo signal maps to the fundamentals of firms’ overall price-setting environment, namely inflation and market structure. Specifically, in a high-inflation environment price changes are presumed more frequent than otherwise. Similarly, price changes in highly-competitive markets are presumed to be more frequent than in less competitive ones. In terms of the no reset relationship we have \( \theta_i = g(\pi, \epsilon) \), where \( -\epsilon \leq -1 \) is the price elasticity of demand, this would imply \( g_\pi g_\epsilon < 0 \). The following, simple functional form captures these ideas:

\[
\theta_i = \theta \cdot \left( \frac{P_i}{P_{i-1}} \right)^{1-\epsilon} = \theta \cdot (1 + \pi_i)^{\epsilon}
\]  

where \( \theta \in [0, 1] \) and \( \theta_i \in [0, \theta] \) \( \forall \pi_i \geq 0 \). Thus it can be easily demonstrated that the higher is inflation and the more competitive is the economy, the more likely is a probability to reset prices, i.e., \( 1 - \theta_i \) tends to 1. With positive inflation, parameter \( \theta \) in (5), note, sets the upper bound to the time-varying price-fixing probability, \( \theta_i \), which materializes either under zero inflation or pure monopoly.

Moreover, a puzzling feature of estimated NKPCs is their apparent tendency to over-estimate price stickiness, given by \( 1/(1-\theta) \). One aspect of this puzzle may be that in time-dependent models, firms change prices only on a periodic basis. Accordingly, time-dependent pricing rules might lead to stickier prices than state-dependent ones for a continuum of shocks. In addition, since this duration is typically estimated from diffuse inflation histories (recall Chart 1) which, importantly, deviate markedly from zero inflation, some bias might be expected. Our framework, however, might shed light on this puzzle since here \( \theta \) provides an upper limit for average contract duration which, as already stated, precisely materializes only under zero inflation (i.e., the maintained hypothesis of the standard NKPC). Whilst in our framework, price stickiness and duration are time-varying, and can be recursed from (5) for a given \( \epsilon \).

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3 For instance, the ECB’s ‘Inflation Persistence Network’ found that sectors with a higher inflation rate and higher inflation variability, typically exhibit more frequent price changes (e.g., Altissimo, Ehrmann and Smets, 2006, see also Cecchetti, 1986).

4 Again, this is consistent with the evidence, e.g., Levy et al. (1997).

5 Whilst equation (5) may be justified as a quasi demand schedule for price resetting, it remains, like that of the original Calvo and Taylor (1980) frameworks, not predicated on any explicit micro-foundations. Our signal is essentially a way of incorporating the acknowledged real world feature of price staggering in a tractable manner, albeit in a way that goes beyond Calvo (1983). As Calvo comments, “Like cash-in-advance, price stickiness models fill a vacuum in general equilibrium theory without which one cannot even begin to address some basic policy issues in monetary economics. Unfortunately, the micro foundations are still weak.”. Macroeconomic Dynamics, 9, 2005, 123-145.

6 Smets and Wouters (2003), for example, estimate price durations in the euro area at around 2.5 years, which contrasts with comparable micro evidence which suggests around 1 year (e.g., see the summary paper of Altissimo, Ehrmann and Smets, 2006).
4 Factor Demands and the Profit-Maximizing Price Level

In the following sections we derive the first-order intertemporal maximization conditions of the firm accounting for Calvo-pricing signals. In addition, we account for adjustment costs associated with labor in a framework where wage contracts are fashioned in terms of normal working hours with a pre-set overtime premium.

4.1 Effective Labor Hours and Pricing

Typically, around two-thirds of the variation in total hired hours originates from employment; the rest from changes in hours per worker, e.g., Kyland (1995), Hart (2004). The relatively small proportion of the variation of paid hours per worker can be explained by the fact that typically labor contracts are framed in terms of normal working hours. Therefore it is difficult for firms to reduce hired hours per worker below the norm and problematic to increase them above that norm without increasing marginal costs. Under these conditions it may be optimal for firms to allow the intensity at which the hired labor is utilized, to vary in response to shocks. Hired hours may therefore underestimate the true variation of the utilized labor input over the business cycle leaving effective hours as the correct measure of labor input in the production function. An empirical difficulty with the effective hours is that they are not directly observable; although in our framework we demonstrate that they can be expressed in terms of observables.

Hence, as before, assume that output is defined by production function, \( F(K, H, L) \), where, in particular, \( H \) is 'effective' labor hours (where \( H = N \cdot h \); \( N \) is the number of employees, \( h \) 'effective working hours' per employee). To illustrate the idea of “effective” labor hours, assume an employee is paid for, say, 8 “normal” hours, even though there may be periods when she works considerable below that, say, to only 5 hours work with “full” intensity. From the production-function standpoint, the logically correct measure of the labor input is 5 hours (i.e., the effective labor input) which implies that for effective labor hours the identity \( H = N \cdot h \) must hold.

Further, in the spirit of indivisible labor (e.g., Kinoshita, 1987, Trejo, 1991, Rogerson, 1998) assume that contracts are drawn up in terms of fixed (or normal) working hours per employee, normalized to unity: \( \bar{h} = 1 \). In general, effective hours in excess of normal hours attract a premium. Conversely, employers have limited possibilities to decrease paid hours when effective hours fall below normal ones. Hence, total wage costs, recalling equation, can be presented as a convex function of the deviation of effective hours from normal hours.\(^7\) Using a variant of the

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\(^7\) Whilst, the overtime pay schedule of a single worker takes a kinked form, this is not so at a firm level, if there are simultaneously employees working at less than full intensity and those working overtime at full intensity (see the discussion in Bils, 1987).
“fixed-wage” model of Trejo (1991) for overtime pay, the following function gives a local approximation of this relation in the neighborhood of effective hours equaling normal hours:

\[
W_i[H_i + A_t(N_i, H_i)] = W_i \left[ h_t N_i + \frac{a_h}{2} \left( \frac{h_t N_i - \bar{h} N_i}{\bar{h} N_i} \right)^2 \right]; \quad a_h \geq 0
\]  

(6)

where \( W_i \) is the real straight-time wage rate which each firm takes as given. Conditional on the contracted straight-time wage rate and the overtime wage premium function, effective hours are completely demand determined. Firms can also freely (but not costlessly) determine the allocation of total effective hours into effective hours per employee and the number of employees.

Setting the number of employees, \( N \), to 100 and \( W=1 \), the linear schedule, Figure 2, illustrates the dependency of total wage costs if deviations of effective hours from normal hours attract no premium, i.e. \( a_h = 0 \). Convex curvature in wage costs results for \( a_h > 0 \), and the greater the curvature, the greater is the incentive to adjust total effective hours, \( H \), by changing the number of employees. Indeed, if changing the number of employees is costless, all adjustment is done via this margin and, independently from the size of \( a_h \), effective hours \( H \) equals \( N \) for all periods. However, in reality, changes in the number of employees are associated with non-trivial costs.

Figure 2: Wage Costs and Effective Hours

![Figure 2: Wage Costs and Effective Hours](image)

\( h(\bar{h} = 1), \) Effective Labor Hours per Employee

4.2. The Theoretical Model

Assume that each firm solves its profit-maximization problem in the beginning of the period with full information on all current-period variables - except for the price-setting category to which it belongs ex-post. Regarding the Calvo signal itself, the prior probability distribution is known, i.e. \( E_i \theta_i = \theta_i \) and, hence, the \( j^{th} \) firm’s expected price level is,

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\( ^8 \) Trejo’s (1991) focus was in overtime hours and, therefore, he did not distinguish between effective and paid hours. Hence, our formulation is compatible with his when \( h_t \geq \bar{h} \) and effective and paid hours are equal. However, our formulation also accounts for the possibility that effective hours are below normal (i.e. paid) hours.
Thus, although at the firm level, we find, 

\[ E_t p^i_t = \theta_t p^i_{t-1} + (1 - \theta_t) \left( (1 - \omega) p^i_t + \omega p^*_t \right) \]  

(7)

\[ E_t p^i_t \neq p^i_t, \quad E_t p_t = p_t \]

continues to hold at the aggregate level (this can be inferred from taking expectations of (1) and (2)). Now the profit-maximization problem is identical for each firm independently of the ex-post outcome of the Calvo signal. In general form, therefore, each \( j \)th firm maximizes,

\[
\begin{align*}
\max E_j \sum_{i=0}^{\infty} R_{j,i+i} \left\{ p^i_{t+i} Y^j_{t+i} - \frac{W_{t+i}}{P_{t+i}} \left[ h^j_{t+i} N^j_{t+i} + \frac{\alpha_s N^j_{t+i}}{2} (h^j_{t+i} - 1)^2 + A_N \left( N^j_{t+i}, N^j_{t+i+1} \right) \right] - \frac{Q_{t+i}}{P_{t+i}} K^j_{t+i} \right\} \\
\end{align*}
\]

(8)

\[
Y^j_t = F(K^j_t, h^j_t N^j_t) 
\]

(8a)

\[
Y^j_t = Y_t \left( \frac{P^j_t}{P_t} \right)^{-\varepsilon} = P_t Y_t \left( \frac{P^j_t}{P_t} \right)^{-\varepsilon} = P_t Z_t \left( P^j_t \right) 
\]

(8b)

where \( W \) and \( Q \) denote nominal wages and the user-cost-of-capital respectively, \( Z \) is a convenient factoring term, \( F \) represents some generalized production technology (e.g. Klump, McAdam and Willman, 2007), and \( R_{j,i+i} = (1 + r_i) \prod_{j=0}^{i} \left( 1 + r_{t+j} \right)^{i} \) with \( r_i \) being the risk-free, real interest rate.

After applying expectation rule (7), we can separate \( P^j_t Y^j_t \) and \( Y^j_t \) conditional on \( P^j_{t+i} = P^j_t \), on one hand, and conditional on all other possible prices, i.e. \( P^j_{t+i} \neq P^j_t \), on the other:

\[
E_i P^j_{t+i} Y^j_{t+i} = (1 - \theta_i) (1 - \omega) \Theta_{t,i+i} P^j_t Z_t E_j \left[ P^j_{t+i} | P^j_{t+i} = P^j_t \right] + E_i \Omega_{t,i+i} \left[ P^j_{t+i} | P^j_{t+i} = P^j_t, P^j_{t+i} \neq P^j_t \right] 
\]

(9a)

\[
E_i Y^j_{t+i} = (1 - \theta_i) (1 - \omega) \Theta_{t,i+i} E_j \left[ P^j_{t+i} | P^j_{t+i} = P^j_t \right] - E_i \left[ \frac{P^j_{t+i}}{P^j_t} \Omega_{t,i+i} \left[ P^j_{t+i} | P^j_{t+i} = P^j_t, P^j_{t+i} \neq P^j_t \right] \right] 
\]

(9b)

where \( \Theta_{t,i+i} = \frac{1}{\theta_i} \prod_{j=0}^{i} E_i \Theta_{t+j} = \Theta^t E_i \left( \frac{P^j_{t+i}}{P_t} \right)^{-\varepsilon} \) and \( \Omega_{t+i} \) is the probability-weighted sales corresponding to all possible sales price \( P^j_{t+i} \neq P^j_t \) deflated by the aggregate price level. The profit-maximization problem of (8) can now be presented as,
\[
\begin{align*}
\text{Max} \ E_\epsilon \sum_{j=0}^{\infty} & R_{t+i} \left[ (1 - \theta_j) (1 - \phi_j) \right] E_{t+i} P_{t+i} Z_{i+t} \left[ P_{i+t} = P_i^f \right] + \Omega_{t+i} \left[ P_{i+t} Z_{i+t} \left( P_{i+t} \neq P_i^f \right) \right] - \\
& \frac{W_{t+i}}{P_{i+t}} \left[ h_j(i) - \frac{\alpha_{N_j}}{2} \left( h_j(i) - 1 \right)^2 + \frac{\alpha_{N_j}}{2} \left( h_j(i) - 1 \right) \right] a_{N_j} - \frac{\alpha_{N_j}}{2} a_{N_j} (N_j(i) - N_{j-i}) - \frac{\alpha_{N_j}}{2} a_{N_j} (N_j(i) - N_{j-i}) + \\
& \lambda_{t+i} \left[ \frac{P_{i+t}}{P_i^f} \Omega_{t+i} \left[ P_{i+t} \left( P_{i+t} \neq P_i^f \right) \right] \right]
\end{align*}
\]

(10)

where the terms containing \( \Omega_{t+i}(\cdot) \) are independent from maximized variables. Now, the first-order conditions with respect to \( P_i^f \), \( h_i^j, N_i^j \) and \( K_i^j \) yield, respectively,

\[
(1 - \epsilon) E_\epsilon \sum_{j=0}^{\infty} R_{t+i} \theta_j E_{t+i} P_{t+i} Z_{i+t} \left[ P_{i+t} = P_i^f \right] + \epsilon E_\epsilon \sum_{j=0}^{\infty} R_{t+i} \theta_j E_{t+i} \lambda_{t+i} Z_{i+t} \left[ P_{i+t} = P_i^f \right] = 0
\]

(11)

\[
\frac{W_{t+i}}{P_{i+t}} \left( 1 + \frac{\alpha_{N_j}}{2} \left( h_j(i) - 1 \right)^2 \right) \lambda_{t+i} N_j(i) = \frac{\partial F}{\partial H_i}
\]

(12)

\[
\lambda_{i+1} \frac{\partial F}{\partial K_i} \equiv \frac{Q_i}{P_i}
\]

(13)

\[
\text{The demand function defined by (8b) implies that}
\]

\[
Z_{i+1} \left( P_{i+1} \left| P_i = P_i^f \right. \right) = \left[ \frac{Y_{i+1}}{P_i} \right]^{\epsilon - 1} Z_i \left( P_i \left| P_i = P_i^f \right. \right)
\]

Using this relation and the definition of \( \Theta_{t+i} \), conditions (11) and (12) can be transformed into:

\[
P_i^f = \frac{\epsilon}{\epsilon - 1} \sum_{j=0}^{\infty} \theta_j E_{t+i} R_{t+j} \left[ \frac{Y_{i+j}}{Y_i} P_{i+j} \lambda_{i+j} \right]
\]

(11')

\[
\lambda_{i+1} = \frac{W_{i+1} \left[ 1 + \frac{\alpha_{N_j}}{2} \left( h_j(i) - 1 \right) \right]}{P_{i+j} F_h}
\]

(12')

where \( MC \) is the real marginal cost of labor and \( F_h \) represents the marginal product of effective hours \( H_i^j \). There are in (11') and (12') two features worth noting. Firstly, from (11') we see

\[8\]
that the optimal price now has been reduced to depend on the constant $\theta$ instead of the time-varying $\theta_i$. Secondly, real marginal costs depend also on the deviation of effective hours from normal, which is the case if $a_n > 0$ and the adjustment of the number of employees is costly. From the point of view of empirical application this might be problematic, because effective hours are unobservable. However, in the present framework that is not the case if the production function is known and on the bases of (8a) the unobservable $h_i^j$ can be expressed in terms of observable variable as follows:

$$h_i^j = \frac{F^{-1}(y_i^j, K_i^j)}{N_i^j}$$

Furthermore, by assuming that households have access to a complete set of contingent claims, and that identical consumers maximize their intertemporal utility, $\sum_i \beta^i U(C_{t+i})$, it holds for the discount rate $R_{t,t+i}$:

$$\frac{1}{R_{t,t+i}} \beta^i E \left[ \frac{U_{t+i}(C_{t+i})}{U_t(C_t)} \right] = 1$$

where $C$ denotes consumption, and $\beta$ is the discount factor. For the standard case of logarithmic utility and under the assumption that market growth equals consumption growth, i.e. $Y_{t+i} \over Y_t = C_{t+i} / C_t$, equations (11') (12') and (16) imply,

$$p_t^i = \frac{\mu}{\epsilon - 1} \sum_{j=0}^{\infty} (\beta \theta)^j P_{t+i} MC_{t+i} = (1 + \mu) (1 - \beta \theta) \sum_{j=0}^{\infty} (\beta \theta)^j E_{i} MCN_{t+i}$$

where $\mu = \frac{1}{\epsilon - 1}$ and $MCN$ are the mark-up and nominal marginal cost of labor, respectively.

The logarithmic approximation of (17) can be written as,

$$p_t^i = (1 - \beta \theta) \sum_{j=0}^{\infty} (\beta \theta)^j E_{i}(mcn_{t+i} + \mu)$$

Equation (17') resembles a conventional profit-maximizing, price-setting rule, but it is worth noting that it has been derived to hold for the log levels of the left- and right-hand side variables, i.e. not only for the corresponding log differences of the variables from their presumed steady state
values, as is usually the case - and where, consequently, the presumed fixed mark-up drops out.
In fact (17') corresponds to the specification implied by Rotemberg’s (1987) (two-stage optimization) framework but, unlike in that paper, we have explicitly derived it here in the profit-maximization environment. Furthermore, the derivation of (14') crucially requires that the reset probability is state-dependent, as defined in equation (5). Finally, the definition of marginal costs contains also the deviation of effective hours from normal hours.

5 The State-Dependent New Keynesian Phillips Curve

Finally, to derive our State-Dependent NKPC we start by inserting equation (17') into (4) to obtain:

\[
\left( \frac{\theta_t}{1-\theta_t} + \omega \right) \pi_t = \frac{\omega}{1-\theta_t} \pi_{t-1} + \left( 1 - \omega \right) \left( 1 - \beta \theta \right) \left( \sum_{i=0}^{\infty} (\beta \theta)^i E_i \left( mcn_{t+i} + \mu \right) \right) - p_t
\]

(18)

We next shift (18) forward by one period and take expectations at the beginning of period t.

\[
E_t \left[ \left( \frac{\theta_{t+1}}{1-\theta_{t+1}} + \omega \right) \pi_{t+1} \right] = \frac{\omega}{1-\theta_t} \pi_t + \left( 1 - \omega \right) \left( 1 - \beta \theta \right) \left( \sum_{i=0}^{\infty} (\beta \theta)^i E_i \left( mcn_{t+i} + \mu \right) \right) - E_t p_{t+1}
\]

(19)

We see that the expectation of the left-hand side of (19) is a non-linear function of the next-period inflation. Therefore, to derive a closed-form solution, we linearize this left-hand-side term around current-period inflation:

\[
E_t \left[ \left( \frac{\theta_{t+1}}{1-\theta_{t+1}} + \omega \right) \pi_{t+1} \right] \approx \left( \frac{\theta_t}{1-\theta_t} + \omega \right) E_t \pi_{t+1} + \frac{\theta_t (1-\omega)}{(1 + \pi_t)(1-\theta_t)^2} \left( E_t \pi_{t+1} - \pi_t \right) \pi_t
\]

(20)

Now, using the approximation (20) in (19), multiplying both sides of (19) by \( \beta \theta \) and subtracting it from (18) we end up, after some manipulations, with our reformulated State-Dependent NKPC (SD-NKPC):

\[
\pi_t = \gamma_t \pi_{t-1} + \gamma_t f E_t \pi_{t+1} + \lambda_t (mcn_t + \mu - p_t) - \xi_t (E_t \pi_{t+1} - \pi_t) \pi_t
\]

(21)

where,
\[ \gamma^b_i = \omega \frac{(1-\theta_i)}{(1-\theta_{i-1})} \phi^{-1}_i, \quad \gamma^f_i = \beta \theta \phi^{-1}_i, \]

\[ \lambda_i = (1-\omega)(1-\theta_i)(1-\beta \theta) \phi^{-1}_i, \quad \xi_i = \frac{\theta_i}{(1+\pi_i)(1-\theta_i)} \beta \theta \phi^{-1}_i, \]

\[ \phi_i = \theta_i + \omega [1-\theta_i(1-\beta)] \]

where, as before, \( \theta_i = \theta \cdot (1+\pi_i)^{-\varepsilon} \).

We see that although equation (21) notionally resembles the conventional hybrid NKPC, it differs in three key respects.

First, and most obviously, all "coefficients" are time-varying, resulting from the dependency of \( \theta_i \) on inflation. Moreover, it can be shown that the SD-NKPC parameters are increasing in inflation \( \gamma^b_i, \gamma^f_i, \lambda_i, \phi_i \) except for that capturing inflation volatility, \( \xi_i \). Our formulation thus turns out therefore to be consistent with Ball, Mankiw and Romer’s (1988) well-known finding of a positive correlation between the slope of the Phillips curve and inflation (or trend inflation). Finally, conditional on estimates of the state-dependent NKPC, the time-varying Calvo re-set signal and time-varying duration would then be solved recursively by inserting the estimate of \( \theta_i \), into \( \theta_i = \theta \cdot (1+\pi_i)^{-\varepsilon} \) and \( (1-\theta_i)^{-1} \), respectively. Note, equation (21) can be expressed in terms of \( \theta_i \), \( \theta_i \), or, as here, both. For estimation purposes, though, we would replace \( \theta_i \) by \( \theta(1+\pi_i)^{-\varepsilon} \). The time-varying Calvo signal and its duration over time would then be solved by inserting the estimate of \( \theta_i \), into (5) and \( (1-\theta_i)^{-1} \) respectively.

A natural consequence of this framework is that question of monetary and inflation regime can now be examined directly since our SD-NKPC is not dependent on a linearization of the reset price around any given value of inflation. Whereas the standard NKPC is valid locally only around a zero-inflation regime, or, at best, a low and stable one.

Second, our NKPC specification differs in being derived directly from the log-levels of the underlying variables, without relying on deviations from a zero (or indeed non-zero) inflation steady state.

Finally, looking at the last right-hand term in (18), the equation contains second-order inflation terms. The importance of this is strengthened the higher is inflation, on one hand, and the more volatile it is, on the other. However, for steady inflation, i.e., \( \pi_{t+1} = \pi_t \), this term disappears independently from the value of that steady rate. Also, it is straightforward to see that the time-varying coefficients asymptotically approach those of the hybrid NKPC, when inflation converges to zero, i.e. the linearization point of conventional equation. Likewise the second-order inflation term vanishes when \( \pi_t \to 0 \). Hence, equation (21) indicates that in an inflation regime at

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9 The aggregate elasticity of demand, \( \varepsilon \), or equivalently the mark-up, \( \mu \), which enters \( \xi \), meanwhile would require either prior estimation or calibration.

10 Although, note that \( \xi_t \neq 0 \forall \pi_t \).
or close to zero with relatively small inflation variability, the conventional (hybrid) NKPC should give a good approximation to (21).

6. The Long-Run Supply-Side System

Before being applicable for empirical estimation the real marginal cost must be operationalized. Deviating from a general practice we do not rely the a priory assumption of the Cobb-Douglas production function but allow both the elasticity of substitution between capital and labor to differ from unity as well as technical progress from Harrod neutrality. To increase the robustness of our results we estimate the underlying production technology by applying the system approach, i.e. we estimate the first order conditions of maximization as a system. Our estimation task is alleviated by the fact that for identifying the technology parameters we need to estimate only the steady state form of the profit maximization system determined by (8a) and (11)-(14), i.e. we assume that adjustment costs, $\Delta N = 0$; thus, (10) reduces to $H_t = N_t$, (i.e., $h_t = h = 1$ and, hence, normal working hours $N_t$ equals 'effective' hours $H_t$). Now the supply-side system can be re-expressed as:

\[
\frac{W_t N_t}{P_t Y_t} = \frac{N_t}{(1 + \mu)Y_t} \partial F \partial N_t
\]

(21)

\[
\frac{Q_t}{W_t} = \frac{\partial F/\partial K_t}{\partial F/\partial N_t}
\]

(22)

\[
Y_t = F(K_t, N_t, \ell)
\]

(23)

where $q_t = r_t + \delta$ is the user cost of capital.

6.1 “Normalized” CES Production with Time-Varying Factor Augmenting Technical Progress.

6.1.1 Normalization of Production functions

In estimating system (21-23), our technology assumption is the "normalized" CES production function allowing for time-varying factor-augmenting technical progress. The importance of explicitly normalizing CES functions was discovered by de La Grandville (1989), further explored by Klump and de La Grandville (2000), and Klump and Preißler (2000), and first implemented empirically by Klump, McAdam and Willman (2007).

Normalization starts from the observation that a family of CES functions whose members are distinguished only by different substitution elasticities need a common benchmark point. Since the elasticity of substitution is defined as a point elasticity, one needs to fix benchmark values for the level of production, factor inputs and marginal rate of substitution, or equivalently for per-capita production, capital intensity and factor income shares. Normalization is crucial when dealing with CES functions: (a) It is necessary for identifying in an economically meaningful way the constants
of integration which appear in the solution to the differential equation from which the CES
production function is derived. (b) it is necessary for securing the neo-classical property of a
strictly positive relationship between the elasticity of substitution and the level of output, (c) it is
(implicitly or explicitly) employed in all empirical studies of CES functions, (d) it is convenient
when biases in technical progress are to be empirically determined (as in this paper).
The normalized production function is given by

\[
\frac{Y}{Y_0} = \left\{ \left[ \frac{N}{N_0} \cdot e^{\pi g_i(t)} \right]^\sigma - \pi \left[ \frac{K}{K_0} \cdot e^{\pi g_i(t)} \right]^\sigma \right\} \right.^{\frac{1}{\sigma}}
\] (24)

where \( \pi_0 \) is the capital share evaluated at the normalization point (subscript 0) and \( g_i(t) \) define
the (indexed) level of technical progress from factor \( i \) and where \( \sigma \in [0, \infty) \) is the elasticity of
substitution.

We suggest normalization points should be calculated from sample averages (denoted by a
bar). However, due to the non-linearity of the CES functional form, sample averages (arithmetic
or geometric) need not exactly coincide with the implied fixed point of the underlying CES
function. That would be the case only if the functional form is log-linear i.e. Cobb Douglas with
constant technical growth. Therefore, we capture and measure the possible emergence of such a
problem by introducing an additional parameter, \( \zeta \), which should be close to unity. This allows us
to express the fixed point in terms of the geometric sample averages of output and inputs,

\[ Y_0 = \zeta \cdot \bar{Y}, K_0 = \bar{K}, N_0 = \bar{N}, \]

and the arithmetic sample averages of capital income share and
time: \( \pi_0 = \bar{\pi}, t_0 = \bar{t} \). Distribution parameter \( \bar{\pi} \) can be calculated directly from the data or it can
be estimated jointly with the other parameters of the model. We apply the former approach.

6.1.2 Flexible Modeling of Technical Progress

Recall that neo-classical growth theory suggests that, for an economy to possess a steady state
with positive growth and constant factor income shares, the elasticity of substitution must be
unitary (i.e., Cobb Douglas) or technical change must exhibit labor-augmentation (i.e., Harrod
Neutrality).

Under Cobb Douglas, the direction of technical change is irrelevant for income distribution since it
is not possible to determine any biases in technical change. In contrast, pronounced trends in
factor-income distribution witnessed in many industrialized countries support the more general CES
function and make biases of technical progress a central issue. For CES, though, a steady state
with constant factor income shares is only possible if technical progress is purely labor
augmenting. Acemoglu (2003) was able to derive this same result in a model with endogenous

---

11 However, if the normalization point is implicitly defined, then all baseline values are equal to one, and the baseline factor
income shares become equal to one half. This may be an interesting theoretical case but is of limited empirical relevance.
12 The reader is directed to McAdam and Willman (2007a) for a more extensive discussion of normalization.
13 If factors are perfect substitutes in production \( \sigma = \infty \) (i.e., linear production function) and if non-substitutable \( \sigma = 0 \)
(Leontief); \( \sigma = 1 \) signifies Cobb-Douglas.
innovative activities but demonstrated that, over significant periods of transition, capital-augmenting progress can be expected resulting from endogenous changes in the direction of innovations. Indeed, abstracting from theory, it would be surprising if the decline in the price (and rise in usage) of goods such as computers and semi-conductors since the 1970s had not induced some capital-augmenting technical change.

Earlier work on CES functions, moreover, tended to assume constant technical growth. However, following recent debates about biases in technical progress over time, it is not obvious that growth rates should always be constant; accordingly, we follow an agnostic approach and model technical progress drawing on a well-known flexible, functional form (Box and Cox, 1964):

$$g_i(t; t_0, \gamma_i, \lambda_i) = \frac{\gamma_i t_0}{\lambda_i} \left[ \frac{t}{t_0} \right]^{\lambda_i} - 1, \quad t > 0 \quad (25)$$

Technical progress, \(g_i(t)\), is, thus, a function of time, \(t\) (around its normalization point) and a curvature parameter, \(\lambda_i\) with a growth rate of \(\gamma_i\) at the point of normalization. When \(\lambda_i = 1\) (=0 [<-0]), technical progress displays linear (log-linear) [hyperbolic] dynamics:

$$g_i(t) \Rightarrow \begin{cases} \lim_{t \to \infty} g_i(t) = \infty & \text{if } \lambda_i \geq 0 \\ \lim_{t \to \infty} g_i(t) = -\frac{\gamma_i t_0}{\lambda_i} & \text{if } \lambda_i < 0 \end{cases} \quad \frac{\partial g_i(t)}{\partial t} = \gamma_i \left( \frac{t}{t_0} \right)^{\lambda_i - 1}$$

Thus, if \(\lambda_i \geq 0\), the steady-state level of technical progress accruing from factor \(i\) tends to infinity but is bounded otherwise. If \(\lambda_i = 1\) the growth of technical progress is constant (i.e., the “textbook” case) but asymptotes to zero for any \(\lambda_i < 1\). This flexible (Box-Cox) modeling of technical progress allows the data to decide on the presence and dynamics of factor-augmenting technical change rather than being imposed a priori.15

6.3 Estimates of the Supply Side

Combining our previous sections, we can write down our final estimated supply side system:

$$\log \left( \frac{w_i N_i}{w_i N_i + q_i K_i} \right) = \log (1 - \pi) - \frac{1 - \sigma}{\sigma} \left[ \log \left( \frac{Y_i}{N_i} \right) + \log \left( \frac{q_i K_i}{K_i} \right) \right]$$

$$\log \left( \frac{q_i K_i}{w_i N_i} \right) = \log \left( \frac{1 - \pi}{1 - \pi} \right) - \frac{1 - \sigma}{\sigma} \left[ \log \left( \frac{N_i}{N_i} \right) \right]$$

$$\log \left( \frac{w_i N_i}{w_i N_i + q_i K_i} \right) = \log (1 - \pi) - \frac{1 - \sigma}{\sigma} \left[ \log \left( \frac{Y_i}{N_i} \right) + \log \left( \frac{q_i K_i}{K_i} \right) \right]$$

$$\log \left( \frac{q_i K_i}{w_i N_i} \right) = \log \left( \frac{1 - \pi}{1 - \pi} \right) - \frac{1 - \sigma}{\sigma} \left[ \log \left( \frac{N_i}{N_i} \right) \right]$$

14 Note we scaled the Box-Cox specification by \(t_0\) to interpret \(\gamma_X\) and \(\gamma_K\) directly as the rates of labor- and capital-augmenting technical change at the fix point.

15 Assuming a specific, albeit flexible, function form for technical progress has the added advantage of circumventing problems related to Diamond et al.’s (1978) non-identification theorem.
\[
\log \left( \frac{y_T}{N_T/N} \right) = \log(\kappa) + \frac{\gamma}{\lambda_N} \left( \frac{y}{\kappa} \right)^{-\frac{\lambda}{\gamma}} - \frac{\sigma}{1 - \sigma} \log \left[ \frac{\gamma}{\lambda_N} \left( \frac{y}{\kappa} \right)^{-\frac{\lambda}{\gamma}} \right] + \left( \frac{\kappa}{N_T/N} \right)^{-\frac{\lambda}{\gamma}} + (1 - \pi)
\]

where \( \bar{\pi} = \frac{\bar{q}K}{wN + \bar{q}K} \) is the capital share evaluated at the fixed point (sample mean). Thus we can see that under Cobb-Douglas, equations (27) and (28) collapse to equating factor income shares to their normalization value plus a stochastic component.

Table 1 gives the estimation of the above supply side system for the unconstrained (CES) and imposed Cobb-Douglas case. In both cases, we can see that the aggregate mark-up is identified at around 11%-12%.

7. NKPC Empirical Results:

Tables 2 and 3 give the results of estimating the NKPC under a number of guises: under time-dependent and state dependent pricing; where real marginal costs are determined by Cobb-Douglas or CES supply side; and, finally, where the deviation of effective hours from normal hours are incorporated. (We also, for completeness, show free and constrained estimates of the discount factor and examples where real marginalized costs are expressed in deviation from their sample mean). Overall, most results appear economically plausible – parameters are generally significant at the 1% level and measures of price stickiness, given by \((1-\theta)^{-1}\) in time-dependent case, are in many cases comparable to that of typical micro evidence, i.e., 4-7 quarters.

Further, in most cases – CES or Cobb-Douglas – we see that our derivation of the overtime premia has statistically significant spillover effects onto optimal pricing. Our estimates of the share of backward-looking price setters is a little more mixed: overall, we are almost as likely to find the share above as below 0.5. However for our preferred method, namely state-dependent forms, we tend to find a relatively high share of backward-looking price setters, around 0.7.

8. Conclusions

In this paper, we sought to make three principal contributions to the field of Phillips-Curve modeling, in the New-Keynesian tradition. These three contributions can we hope make a constructive input into understanding inflation dynamics and real and nominal interactions in the economy, but also commenting on the cyclicality of the mark-up over time (which is a key concern for understanding economic fluctuations and informing stabilization policy).

Acknowledgements

We thank Bob Hart, Rainer Klump, Richard Rogerson, Robert Rowthorn and Bob Solow for helpful comments. The opinions expressed are not necessarily those of the ECB. McAdam is also an honorary Reader in macroeconomics at the University of Kent and a CEPR and EABCN affiliate.
Data Appendix

[To be completed]
Table 1: Alternative Supply-Side Estimations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Cobb-Douglas</th>
<th>CES</th>
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</thead>
<tbody>
<tr>
<td>$\zeta$</td>
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<td>1.0342</td>
</tr>
<tr>
<td></td>
<td>(0.0021)</td>
<td>(0.0022)</td>
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<td>$\gamma_N$</td>
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<td>$\lambda_{N1}$</td>
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<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0005)</td>
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<td>$\lambda_K$</td>
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<td></td>
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<td>$\sigma$</td>
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<td></td>
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<td>$\epsilon$</td>
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<td>Log Det.</td>
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Table 2: Alternative Estimates of the New Keynesian Phillips Curve (Cobb-Douglas Supply)

<table>
<thead>
<tr>
<th>Parameter</th>
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<th></th>
<th>State-Dependent</th>
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<th></th>
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</thead>
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<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.8458 (0.0209)</td>
<td>0.8558 (0.0217)</td>
<td>0.7853 (0.0563)</td>
<td>0.7814 (0.0353)</td>
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<td>$\beta$</td>
<td>1.0017 (0.0144)</td>
<td>0.9090 (−)</td>
<td>0.9090 (−)</td>
<td>0.9090 (−)</td>
<td>0.9090 (−)</td>
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<tr>
<td>$\omega$</td>
<td>0.4089 (0.0772)</td>
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<td>0.5132 (0.1357)</td>
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<tr>
<td>$a_h$</td>
<td>–</td>
<td>–</td>
<td>0.4400 (0.1933)</td>
<td>0.5076 (0.5847)</td>
<td>0.4593 (0.2477)</td>
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<tr>
<td>J-test</td>
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<td>0.7506</td>
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Table 3: Alternative Estimates of the New Keynesian Phillips Curve (CES Supply)

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<th></th>
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<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
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<td>0.9900 (−)</td>
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<td>$\omega$</td>
<td>0.4748 (0.1205)</td>
<td>0.3008 (0.0790)</td>
<td>0.2902 (0.1732)</td>
<td>0.7187 (0.0530)</td>
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<tr>
<td>$a_h$</td>
<td>0.6724 (0.1724)</td>
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<td>–</td>
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<td>J-test</td>
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<td>7.8059</td>
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References


