Noise Trading and the Effects of Monetary Policy Shocks on Nominal and Real Exchange Rates

by

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**Abstract**

A number of empirical studies have reported the result that exchange rates show a delayed overshooting in response to monetary policy shocks. This result is puzzling. Economic theory suggests that the overshooting should occur immediately after the shock, not with a delay. This paper uses a ‘new open economy macroeconomics’ model with pricing-to-market to analyze whether the assumption of noise trading in the foreign exchange market helps to resolve the delayed overshooting puzzle. The implications of noise trading for the effects of monetary policy shocks on the nominal and on the real exchange rate are analyzed.

**Keywords:** Monetary Policy; Noise trading; Exchange rate overshooting

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1. Introduction

Influential empirical research in international macroeconomics has established the ‘puzzle’ that exchange rates often show a delayed overshooting in response to monetary policy shocks (Lastrapes 1992, Clarida and Gali 1994, Eichenbaum and Evans 1995; see, however, Faust and Rogers 1999). The delayed overshooting of the exchange rate describes a situation in which a monetary expansion triggers an immediate depreciation of the exchange rate that then, however, gradually continues for some months or quarters before the exchange rate starts appreciating to its long-run post-shock level. This hump-shaped pattern of exchange rate dynamics in response to a monetary policy shock is in sharp contrast to the predictions of the classical sticky-price model pioneered by Dornbusch (1976). This workhorse model of international macroeconomics implies that the exchange rate overshooting triggered by an unanticipated expansionary permanent monetary policy shock should occur immediately after the shock, not with a delay.

In this paper, I analyze how the delayed overshooting of the exchange rate can be modeled within the context of a dynamic optimizing ‘new open economy macro’ (NOEM) model of the type recently developed by Obstfeld and Rogoff (1995). As compared to the Dornbusch (1976) model often used in previous theoretical research on exchange rate overshooting (see, e.g., Levin 1994, Kempa and Nelles 1999), a major advantage of using a NOEM model to reassess the effects of monetary policy shocks on exchange rate dynamics is that NOEM models are full-fledged micro-founded general equilibrium models. NOEM models, therefore, allow taking into account the intertemporal budget constraints and the dynamic optimization of the private sector when analyzing the effects of monetary policy shocks in open economies. Within a NOEM model, I explore the implications of an explanation for the delayed overshooting puzzle that is based on the notion that some agents who trade in the foreign exchange market use simple ‘rules of thumb’ when forecasting exchange rates and, thus, fail to form rational exchange rate expectations. In the context of the Dornbusch (1976) model,
Levin (1998) has spelled out the implications of relaxing the assumption of rational expectations for the overshooting phenomenon.

In my model, those agents who use simple ‘rules of thumb’ form their forecasts of exchange rates by comparing the actual exchange rate with some kind of medium-run target exchange rate. As time evolves, they update their computations of the medium-run target exchange rate. Thus, the agents who use a simple ‘rule of thumb’ to forecast exchange rates learn over time about the nature of exchange rate movements and update their forecasts correspondingly. This idea that the gradual accumulation of information in an environment in which agents need some time to learn about the way their environment changes has attracted attention in the recent literature on the delayed overshooting puzzle.

In a recent contribution to this literature, Gourinchas and Tornell (2002) have shown that delayed overshooting can arise in a model of exchange rate determination in which agents learn about the persistence of monetary policy (interest rate) shocks and interest rates react only gradually to the shock. In addition, they allow for the possibility that agents misperceive the second moments of the monetary policy process. Anderson and Beier (2000) have made a further recent contribution to this literature. Anderson and Beier have shown how learning about the persistence of monetary policy shocks may generate delayed overshooting of the nominal exchange rate in a variant of the optimizing general equilibrium NOEM model developed by Obstfeld and Rogoff (1995). Moreover, they have demonstrated that learning may increase the persistence of the effects of nominal shocks and, in addition, has important implications for the dynamics of the current account and, thus, for the long-run effects of monetary policy shocks.

The research I present in this paper can be viewed as an extension of the work of Anderson and Beier (2000). My analysis closely resembles their analysis insofar as I build a learning mechanism into a dynamic optimizing two-country NOEM model. However, there are some significant differences between their research strategy and the research strategy I
adopt in this paper. These differences regard both the macroeconomic model I use to
determine exchange rate dynamics and the learning mechanism I built into this model to
generate delayed exchange rate overshooting.

To model the impact of monetary policy shocks on exchange rate dynamics, I employ an
optimizing dynamic NOEM model featuring sticky goods prices and pricing-to-market (PTM)
behavior of firms. NOEM models built on the assumption of PTM have recently attracted
much attention in the international macro literature (Betts and Devereux 1996, 2000,
Kollmann 2001, Senay 1998). PTM implies that firms can set different prices for their good
across segmented national goods markets. Combining PTM with the assumption of sticky
prices implies that the prices of goods are sticky in the currency of the country that imports
the goods. In line with the available empirical evidence (see, e.g., Engel and Rogers 1996,
Knetter, 1993), assuming sticky prices and PTM limits the extent of pass-through of exchange
rate fluctuations onto goods prices and cushions the expenditure switching effect of exchange
rate changes. Furthermore, PTM rests on the assumption that national goods markets are
segmented, implying that monetary policy shocks in general lead to deviations of the real
exchange rate from purchasing power parity (PPP). This has the advantage that, as in the
traditional Dornbusch (1976) model, the implications of ‘rules of thumb’ forecasting for the
dynamics of the real exchange rate can be studied. Because Anderson and Beier (2000) use in
their analysis a close variant of the model pioneered by Obstfeld and Rogoff (1995), a model
in which purchasing power parity always holds, they cannot study how monetary policy
shocks affect the dynamics of the real exchange rate.

I show that the joint modeling of the delayed overshooting of the nominal and of the real
exchange has the advantage that it provides additional insights into the question under which
condition learning in the foreign exchange market implies a delayed overshooting of the
nominal and of the real exchange rate. I show that the analysis of learning for the joint
delayed overshooting of the nominal and of the real exchange rate is important because in my
PTM-NOEM model delayed nominal exchange rate overshooting does not automatically imply delayed real exchange rate overshooting. In fact, in my model situations can arise in which the nominal exchange rate exhibits delayed overshooting but the real exchange rate does not. This result is counterfactual because, in the real world, nominal and real exchange rates tend to be highly correlated with correlation coefficients typically around 0.9 or higher.

In consequence, a delayed overshooting of the nominal exchange rate should be mirrored in a delayed overshooting of the real exchange rate. It follows that, if learning is at the root of the delayed overshooting ‘puzzle’, it should give rise to delayed overshooting of the nominal and of the real exchange rate. I find that a simultaneous delayed overshooting of the nominal and of the real exchange rate arises if the parameters that describe the learning mechanism imply that agents update their forecasts relatively quickly and their expectations are relatively sensitive to exchange rate movements. Because a number of empirical studies has reported evidence on these parameters for different sample periods and different currencies, this result can be used in future empirical research to conduct further systematic research on the link between learning, ‘rule of thumb’ forecasting, and the delayed overshooting ‘puzzle’.

The learning mechanism I build into my PTM-NOEM model to generate delayed exchange rate overshooting is different from the learning mechanisms advocated by Anderson and Beier (2001) and Gourinchas and Tornell (2002). In both studies, agents seek to learn about the nature of monetary policy shocks. In my model, in contrast, agents seek to learn directly about exchange rate dynamics. Further, in both studies agents rely on relatively complex forecasting and learning techniques. In the model used by Anderson and Beier (2002), agents also form rational expectations. In my model, in contrast, some agents use a simple ‘rule of thumb’ to form exchange rate expectations. In the context of my model, this constitutes a deviation from the assumption of rational expectations. Specifically, to study the implications of learning in the formation of exchange rate expectations for the effects of monetary policy shocks, I follow DaSilva (2001) and Devereux and Engel (2002) and extend
an otherwise standard PTM-NOEM model by allowing for the type of small deviations from rational expectations discussed in the large and rapidly growing ‘noise trader’ literature.

The main difference between the study of Devereux and Engel (2002) and my study is that they seek to explain are concerned with the so-called exchange rate ‘disconnect puzzle’ which describes the stylized fact that exchange rates are extremely volatile and that, at the same time, exchange rate volatility does not affect the volatility of macroeconomic ‘fundamentals’. In contrast, I focus on the delayed overshooting ‘puzzle’. This difference in the focus of the analysis requires that I analyze the consequences of a form of noise trading that is different from the form of noise trading studied by Devereux and Engel. In the study of Devereux and Engel, noise traders have conditionally biased exchange rate expectations. This assumption mainly serves to amplify the volatility of the exchange rate. In contrast, in my PTM-NOEM model, noise traders employ, as in DaSilva (2001), a ‘rule of thumb’ provided by technical chart analysis to forecast exchange rate movements. This implies that noise trading has important implications in a dynamic context. The empirical relevance of the form of deviations from rational expectations I assume in my analysis has been documented in numerous empirical studies of the exchange rate forecasting techniques used by foreign exchange traders. A by no means exhaustive list of significant contributions to this literature includes the studies of Allen and Taylor (1990), Frankel and Froot (1987a, 1987b), and Cheung and Chinn (2001).

The results of my study are in line with the results derived in different theoretical settings in the literature on the implications of learning for delayed exchange rate overshooting (i.e., I find that these results are robust). In addition, my results offer some interesting new insights. I find that, in the context of a stochastic, dynamic, optimizing PTM-NOEM model, noise trading in foreign exchange markets helps explaining the delayed overshooting of the nominal and of the real exchange rate in response to an unanticipated persistent expansionary
monetary policy shock. Moreover, I find that the form of noise trading I analyze in this paper helps explaining the delayed overshooting ‘puzzle’ if noise traders update their exchange rate expectations quickly and/or if noise traders’ exchange rate expectations are relatively sensitive to deviations of the current exchange rate from its medium-run target value. In line with the results reported by Anderson and Beier (2000), I find that noise trading has implications for the dynamics of the current account and leads to systematic deviations from the condition of uncovered interest rate parity (UIP). Further, noise trading helps explaining the delayed overshooting puzzle if monetary policy shocks are relatively persistent or even permanent as in the classic Dornbusch (1976) study. The link between noise trading and the delayed overshooting of nominal exchange rates weakens of monetary policy shocks are not very persistent.

I organize the remainder of the paper as follows. In Section 2, I lay out the theoretical model I use to study the link between noise trading, monetary policy shocks, and exchange rate dynamics. In Section 3, I use impulse response functions to analyze how the dynamics of the nominal and of the real exchange rate and of other key macroeconomic variables in the aftermath of a monetary policy shock depend upon the behavior of noise traders and the persistence of monetary policy shocks. In Section 4, I offer some concluding remarks.

2. The Model

To analyze how noise trading changes the way monetary policy shocks affect exchange rate dynamics, I use an in many respects standard PTM-NOEM model. This guarantees that I use a consensus model and that the results I derive do not hinge upon uncommon and arbitrary assumptions regarding households’ preferences and firms’ price setting. Betts and Devereux (1996, 2000) have done the pioneering work on PTM in NOEM-style general equilibrium models, and the model I use closely resembles their model.
The structure of the model is as follows: The world is made up of two countries, Home and Foreign. The two countries are of equal size. Each country is inhabited by infinitely-lived identical households who maximize their expected lifetime utility. In addition, each country is populated by a continuum of profit maximizing firms. The households own the respective domestic firms. The firms sell differentiated products in a monopolistically competitive goods market. Firms have monopoly power and are, therefore, price setters. Because there are no possibilities to arbitrage away price differentials between segmented national goods markets, firms can set different prices in their respective home country and abroad. All firms set prices in the local currency used by their buyers. Thus, there is complete PTM. As in the PTM-NOEM models developed by Senay (2001), Betts and Devereux (2001), and Kollmann (2001), the price setting of firms is governed by a staggered-contracts mechanism as in Calvo (1983). The only production factor used by firms is labor. Firms hire labor in a perfectly competitive labor market. There is no migration of labor across countries.

The special feature of my PTM-NOEM model is the structure of international financial markets. Three features of international financial markets are worth mentioning. First, the only internationally traded assets are nominal one-period bonds. Second, while Foreign households hold only Foreign currency denominated bonds, Home households hold both Home and Foreign currency denominated bonds. This is merely a modeling device that alleviates the analysis. Third, Home households can directly trade in Home currency denominated bonds but all of their trading in Foreign currency denominated bonds is, as in Devereux and Engel (2002), carried out by specialized foreign exchange dealers.

Foreign exchange dealers act in a competitive market and seek to maximize the discounted expected returns of buying and selling Foreign currency denominated bonds. Because the discounted expected returns of trading in Foreign currency denominated bonds are sensitive to exchange rate changes, foreign exchange dealers have to form exchange rate expectations. To form exchange rate expectations, they conduct a questionnaire among
specialized foreign exchange rate forecasters. Some of these foreign exchange rate forecasters are ‘noise traders’ who use a simple ‘rule of thumb’ to compute exchange rate forecasts.

2.1 Households Preferences and Budget Constraints

Home and Foreign households have identical preferences and maximize the present value of their expected lifetime utility. In the case of a Home household the expected lifetime utility is given by

\[ U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left( \log(C_t) + \chi \left( \frac{M_t}{P_t} \right)^{-\varepsilon} / (1 - \varepsilon) - \kappa N_t^2 / 2 \right), \] (1)

where the Greek letters are such that \( \kappa > 1, \chi > 0, \varepsilon > 0, 0 < \beta < 1 \) and \( E_t \) are expectations conditional on time \( t \) information. As regards the arguments entering into the utility function, \( C_t \) denotes a real consumption index, \( N_t \) denotes hours worked, and \( M_t / P_t \) denotes the end-of-period real money holdings of the household, where \( M_t \) is the supply of Home central bank money (there is not currency substitution) and \( P_t \) is the aggregate Home consumer price index (CPI). The real consumption index, \( C_t \), is defined over a continuum of differentiated, perishable Home and Foreign consumption goods of total measure unity:

\[ C_t = \left[ \int_0^1 c_t(z)^{(0-1)/\theta} \, dz \right]^{(\theta-1)/\theta}, \] (2)

where \( \theta > 1 \) denotes the elasticity of substitution and \( c(z) \) denotes consumption of good \( z \) at time \( t \). The Home consumer price index, \( P_t \), is defined as the minimum expenditure required to buy one unit of the aggregate consumption index, \( C_t \):

\[ P_t = \left[ \int_0^{1/2} p_t(z)^{1-\theta} \, dz + \int_{1/2}^1 q_t(z)^{1-\theta} \, dz \right]^{1/(1-\theta)}, \] (3)
where $p_t(z)$ denotes the Home currency price of a Home-produced good and $q_t(z)$ denotes the Home currency price of a Foreign PTM good. The Foreign price index is given by a similar formula.

In addition to real balances, Home and Foreign households hold nominal bonds. Foreign households only hold Foreign currency denominated bonds. Home households can take positions in both Home and Foreign currency denominated bonds. However, they cannot trade directly in Foreign currency denominated bonds. All trading in Foreign currency denominated bonds is carried out by specialized foreign exchange dealers which act in the interest of Home households. The net payment (denominated in Home currency units) Home households receive for their investment account from the foreign exchange dealer is denoted by $\Pi_t^f$. Depending on the ‘good luck’ or ‘bad luck’ of the foreign exchange dealer, this net payment may be negative or positive. Households take the net payment they receive from the foreign exchange dealer as given. The net payment is defined as

$$\Pi_t^f = S_t^*B_{h,d}^{*,t+1} - d_t^*S_tB_{h,d}^*,$$

(4)

where $S_t$ denotes the nominal exchange rate defined as the amount of Home currency units required to buy one unit of the Foreign currency, $B_{h,d}^*$ denotes foreign exchange dealers’ holdings of Foreign currency denominated bonds paying one unit of foreign currency in period $t+1$, and $d_t^*$ denotes the period $t$ price of Foreign currency denominated bonds. The first component of $\Pi_t^f$ denotes the amount Households receive in period $t$ from the foreign exchange dealer. The second component of $\Pi_t^f$ denotes the new payments Households make in period $t$ to the foreign exchange dealer.

Home households receive interest income for holding Home bonds, profit income for the ownership of Home firms, labor income for hours worked, and transfers from the government.
In addition, they receive funds from or, in bad times, are required to pay funds in to their foreign exchange dealer. Given the various components of their income, Home households determine their optimal consumption, decide on their optimal Home bond holdings, and determine the optimal holding of domestic outside money. Their period budget constraint can be written as

\[ P_iC_t + d_iB_t + M_t = B_{t-1} + M_{t-1} + w_iN_t + \Pi_t + \Pi_{t^f} + P_Tt, \]  

where \( B_i \) stands for the quantity of Home currency denominated nominal bonds paying out one unit of Home currency in period \( t+1 \), \( d_i \) stands for the period \( t \) price of these bonds, \( T \) stands for real lump-sum transfers (denominated in terms of the consumption aggregate, \( C \)), \( w_i \) stands for the nominal wage rate, and \( \Pi_t \) stands for the nominal profit income the household receives from domestic firms.

Using similar arguments, it can be shown that the period budget constraint for a Foreign household obeys the following difference equation:

\[ P_i^*C_t^* + d_i^*B_{t^f}^* + M_t^* = B_{t^f,t-1}^* + M_{t-1}^* + w_i^*N_t^* + \Pi_t^* + P_T^*t, \]

where \( B_{t^f}^* \) gives the Foreign households’ holdings of Foreign currency denominated bonds and an asterisk denotes a Foreign variable.

The first-order conditions for a Home household can be written as (the usual transversality condition applies):

\[ d_i = \beta E_i \left( \frac{C_p}{C_{t+1}} \right), \]

\[ \kappa N_tC_t = \frac{w_i}{P_t}, \]
\[
\frac{M_{t}}{P_{t}} = \left( \frac{\gamma C_t}{1 - d_t} \right)^{1/\varepsilon}.
\] (9)

Following the same steps that led to equations (7) - (9), analogous first-order conditions can be derived for the Foreign household.

### 2.3 Foreign Exchange Dealers

As in Devereux and Engel (2002), Foreign exchange dealers act in a competitive market and maximize the discounted expected returns of investing in Foreign bonds:

\[
\max E_{t}^{fx} \left( d_{t} S_{t+1} B_{t+1}^* - d_{t}^* S_{t} B_{t}^* \right),
\] (10)

where \( E_{t}^{fx} \) denotes foreign exchange dealers conditional expectations and \( d_{t} \) is the state contingent value of domestic currency delivered in period \( t + 1 \). This equation is a zero-expected-profit condition that can be used to derive the following form of the no-arbitrage condition of uncovered interest rate parity (UIP):

\[
d_{t}^* = E_{t}^{fx} \left[ d_{t+1} S_{t+1} / S_{t} \right].
\] (11)

Equations (10) and (11) show that the optimal investment strategy the foreign exchange dealers choose depends upon exchange rate expectations. To form exchange rate expectations, foreign exchange dealers conduct a questionnaire among foreign exchange rate forecasters. There are two groups of foreign exchange rate forecasters. The first group consists of foreign exchange rate forecasters who apply a simple ‘rule of thumb’ to forecast exchange rate fluctuations. In the following, I use the term ‘noise traders’ to refer to this group of exchange rate forecasters. The second group of exchange rate forecasters forms model-consistent rational expectations to forecast exchange rate fluctuations. The exchange rate expectations of this second group of forecasters are given by:
\[ E'_t (\tilde{S}_{t+1} - \tilde{S}_t) = E_t (\tilde{S}_{t+1} - \tilde{S}_t). \]  

(12)

The term \( E'_t \) denotes the conditional expectations operator applying in the case of exchange rate forecasters forming rational expectations and a variable with a hat denotes percentage deviation from the steady state.

As regards the noise traders, I assume that this group of exchange rate forecasters derives exchange rate forecasts from the following simple ‘rule of thumb’:

\[ E^n_t (\tilde{S}_{t+1} - \tilde{S}_t) = -\phi (\tilde{S}_t - \tilde{S}^T_t). \]  

(13)

The parameter \( \phi \) denotes the elasticity of noise traders’ expectations with respect to the deviation of the actual exchange rate change from a medium-run exchange rate target, \( \tilde{S}^T_t \). The term \( E^n_t \) denotes the conditional expectations operator applying in the case of noise traders. Noise traders calculate the medium-run exchange rate target, \( \tilde{S}^T_t \), as the geometrically-weighted moving average of the exchange rates observed during the last \( ma > 0 \) periods:

\[ \tilde{S}^T_t = \sum_{j=1}^{ma} \left( (ma + 1 - j) / J \right) \tilde{S}_{t-j}, \quad \text{with} \quad J = \sum_{j=1}^{ma} j. \]  

(14)

Equations (13) and (14) show that noise traders use a simple technical trading rule to forecast exchange rate fluctuations. Variants of the simple technical trading rule given in equations (13) and (14) have often been studied in the theoretical noise trader literature. Because such rules have been estimated in the empirical noise trader literature, it is straightforward to calibrate these rules. Of course, more sophisticated forecasting rules for the expectations formation of noise traders could be analyzed. However, the simple forecasting rule given in equations (13) and (14) suffices to derive the main results of this paper. Moreover, using more complex exchange rate forecasting rules to describe the behavior of noise traders would
conflict with the idea that noise traders use simple ‘rules of thumb’ to forecast exchange rate changes.

Guided by the results reported in the empirical noise trader literature, I assume that the elasticity, $\phi$, of noise traders’ expectations with respect to the deviation of the actual exchange rate change from its medium-run exchange rate target is positive. This assumption implies that noise traders form regressive exchange rate expectations. The empirical evidence reported by Frankel and Froot (1987a, 1987b, 1990) and others suggests that exchange rate forecasters form regressive rather than extrapolative exchange rate expectations at forecasting horizons of several months. These forecasting horizons are the relevant ones for business cycle analyses.

Finally, to close the model, I invoke the assumption that foreign exchange dealers form their exchange rate expectations upon computing the weighted arithmetic average of the exchange rate forecasts of noise traders and rational traders

$$E_t^f (\hat{S}_{t+1} - \hat{S}_t) = gE_t^n (\hat{S}_{t+1} - \hat{S}_t) + (1 - g)E_t^r (\hat{S}_{t+1} - \hat{S}_t), \quad \text{with} \quad g \in [0,1]. \quad (15)$$

See, for instance, DeGrauwe and Dewachter (1993) and DaSilva (2001) for similar approaches to aggregate the exchange rate expectations of heterogeneous agents.

### 2.4 Pricing-to-Market

Firms maximize expected profits. To maximize expected profits, firms hire labor in a competitive labor market in order to produce a differentiated good. The production function of firms is given by $y_i(z) = N_i(z)$. The nominal profits of a Home firm producing good $z$ consist of profits from sales at Home and of sales in the Foreign economy and can be expressed as

$$\Pi_i(z) = y_i^D(z)[p_i(z) - w_i] + y_i^F(z)[S_i p_i^*(z) - w_i], \quad \text{where} \quad p_i^*(z) \text{ denotes the Foreign PTM price of a Home product and } y_i^D(z) \text{ and } y_i^F(z) \text{ denote the demand for the}$$
product of the firm at Home and abroad. Because prices are sticky, output is demand determined in the short run. The demand functions are given by:

\[ y_t^D(z) = (p_t(z)/P_t)^{-\theta} C_t, \quad (16) \]
\[ y_t^F(z) = (p_t^*(z)/P_t^*)^{-\theta} C_t^*. \quad (17) \]

Because each firm has monopoly power on the Home and Foreign market for its differentiated good, it treats the prices, \( p_t(z) \) and \( p_t^*(z) \), it charges for its product as choice variables. As in Senay (1998), Betts and Devereux (2001), and Kollmann (2001), firms’ price setting is subject to a discrete time version of the sluggish price-setting mechanism developed by Calvo (1983). According to the Calvo-style price-setting mechanism, when maximizing its expected profits each firm has to take into account that there is a positive probability \( 0<\gamma<1 \) that it cannot revise the prices \( p_s(z) \) and \( p_s^*(z) \) it set in period \( s<t \) in period \( t \). In consequence, firms set the current Home and Foreign price of their product, \( p_t(z) \) and \( p_t^*(z) \), so as to maximize the expected present value, \( V_t(z) \), of current and future profits, where period \( s \), \( s>t \), profits are weighted by the probability that the current period prices, \( p_t(z) \) and \( p_t^*(z) \), will still be in force in period \( s \). Firms maximize

\[ \max_{p_t(z), p_t^*(z)} V_t(z) = E_t \sum_{s=t}^{\infty} \gamma^{s-t} R_{t,s} \Pi_s(z), \quad (18) \]

where \( R_{t,s} \equiv \Pi_j^{t-s} d_j \) is the market nominal discount factor. The first-order conditions for this maximization problem are:

\[ p_t(z) = \left( \frac{E_t \sum_{s=t}^{\infty} \gamma^{s-t} R_{t,s} C_s (1/P_s)^{-\delta} w_s}{E_t \sum_{s=t}^{\infty} \gamma^{s-t} R_{t,s} C_s (1/P_s)^{-\delta}} \right) \]

\[ \left( \frac{0}{0-1} \right). \quad (19) \]
\[ p^*_t(z) = \left( \frac{0}{0 - 1} \right) \frac{E \sum_{s=t}^{\infty} \gamma^{s-t} R_{t,s} C_s^*(1/P_s^*)^{-\theta} w_s}{E \sum_{s=t}^{\infty} \gamma^{s-t} R_{t,s} C_s^*(1/P_s^*)^{-\theta} S_s}. \] (20)

Similar expressions can be derived for the profit-maximizing prices, \( q_t(z) \) and \( q^*_t(z) \), set by Foreign firms.

### 2.5 Government

The Home and Foreign governments finance real transfers by seignorage. The period-budget constraint for the Home government can, thus, be written as

\[ T_t = (M_t - M_{t-1})/P_t. \] (21)

The Home money supply is governed by a simple AR(1) process:

\[ \dot{M}_t = \alpha \dot{M}_{t-1} + \varepsilon_{M,t}, \] (22)

where \( \varepsilon_{M,t} \) denotes a serially uncorrelated stochastic disturbance term and the parameter \( \alpha \in [0,1] \) captures the persistence of monetary policy shocks. The Foreign government sector is structured in a completely analogous way.

### 2.6 Definition of Equilibrium, Model Solution, and Model Calibration

Following the NOEM literature, I focus on a symmetric monopolistic competition equilibrium in each country. This equilibrium is defined as an allocation for output, consumption, the exchange rate, prices, interest rates, wage rates, and bond holdings such that the following six conditions are satisfied: (i) the labor market in each country clears, (ii) the first-order conditions for households intertemporal utility maximization are satisfied, (iii) the consolidated budget constraint for each country is satisfied, (iv) the bond markets are in
equilibrium and the expected profits of foreign exchange dealers are zero, (v) PTM firms set
prices according to equations (19) and (20), and, (vi) the Home and Foreign money supply are
given by equation (22) and its foreign counterpart.

To solve for the vector of endogenous variables, I solve the model numerically. The
solution algorithm consists of three steps. In a first step, I follow the NOEM literature and
log-linearize the model around a symmetric flexible-price steady state in which Home
households foreign asset position is zero. In a second step, I calibrate the model. The
 calibration of the model is given in Table 1. All parameter values are within the range of
empirical estimates. The parameter values I use are as in Betts and Devereux (2001). Other
authors have used similar parameter values (see, e.g., by Senay 1998). Following the model
calibrations carried out in these studies guarantees that the results I report in this paper can be
compared with those reported in the literature. As regards the parameters describing how
noise traders form their exchange rate forecasts, I begin my analysis by assuming $\phi = 0.5$ and
$ma = 2$. The numerical value for the parameter $\phi$ lies within the range of empirical estimates
(see, e.g., Frankel and Froot 1987a, Cavaglia et al. 1993). Because the model is calibrated to
quarterly data, the numerical value for the parameter $ma$ implies that noise traders use a
geometric moving average of length two quarters (i.e., six months) to compute the medium-
run exchange rate target value. In Section 3.2.4 below I analyze how the simulation results
change if the parameters $\phi$ and $ma$ are varied. In a third step, I simulate the model
numerically in order to explore how the presence of noise traders in the foreign exchange
market changes the impact of monetary policy shocks on the dynamics of the nominal and of
the real exchange rate and of other key macroeconomic variables. I use Klein’s (2000)
algorithm in order to find the solution of the system. This solution determines the paths of the
endogenous variables of the model in terms of the predetermined and exogenous state
variables of the model. For details, see Klein (2000).
3. Noise Trading and the Propagation of Monetary Policy Shocks

This section comes in three parts. In the first part, I analyze the implications of noise trading in the foreign exchange market for the propagation of a permanent Home monetary policy shock. I show that noise trading can result in the type of delayed overshooting of the exchange rate documented in the empirical literature. I show that noise trading implies deviations from UIP and I also analyze the implications of noise trading for the impact of a permanent Home monetary policy shock on the current account. Moreover, I discuss the implications of noise trading for the real exchange rate. In the second part, I analyze how variations in the parameters that describe how noise traders form their exchange rate forecasts affect these results. In the third part, I analyze the implications of noise trading for the propagation of a temporary Home monetary policy shock.

3.1 The Effects of a Permanent Monetary Policy Shock

Figure 1 depicts the impact of a unit permanent Home monetary policy shock on key Home variables. To compute the impulse responses depicted in the figure, I hold – in line with Dornbusch’s (1976) analysis – the Foreign money stock constant. In order to assess the implications of noise trading for the propagation of the monetary policy shock, I plot for each variable depicted in Figure 1 two impulse response functions: one impulse response function depicts a situation in which noise traders play no role for exchange rate forecasting (solid line) and one impulse response function depicts a situation in which foreign exchange rate dealers take noise traders’ forecasts into account when forming exchange rate expectations (dashed line).

I start with the analysis of the impulse response functions applying if there are no noise traders in the foreign exchange market. Figure 1 shows that the Home monetary policy shock
results in a temporary decline of the Home nominal interest rate. In consequence, the nominal international interest rate differential becomes negative. In addition, because PPP does not hold in the short run under PTM, real interest rates can differ across countries in the aftermath of the monetary policy shock. The Home real interest rate, which is linked to the nominal interest rate by the Fisher parity condition, declines in the short run. This has the implication that optimizing households increase their consumption spending in the aftermath of the monetary policy shock.

A further effect of the absolute and relative decline of the Home nominal interest rate is that the nominal exchange rate must depreciate in order to restore the condition of UIP. The magnitude of the required depreciation of the nominal exchange rate depends upon the influence of noise traders on the exchange rate forecasts of foreign exchange dealers. If there are no noise traders active in the foreign exchange market, the depreciation of the nominal exchange rate comes to a stop only when the rational appreciation expectations of the foreign exchange dealers are large enough to cover the negative international nominal interest rate differential. As a result, the nominal exchange rate overshoots its long-run post-shock steady-state value. As in the model championed by Dornbusch (1976), the exchange rate overshooting can be observed in the immediate aftermath of the monetary policy shock. Thus, there is no delayed overshooting. As shown by Betts and Devereux (2000), exchange rate overshooting will occur in the model so long as the elasticity of money demand with respect to real balances is smaller than one.

The depreciation of the nominal exchange rate translates into a corresponding depreciation of the CPI-based real exchange rate. A further effect of the monetary policy shock is that Home firms expand production in the short run in order to meet the increased demand for their products. This is profitable because prices are above marginal costs.
What are the implications of noise trading for the macro-dynamic effects of a permanent monetary policy shock? The impulse responses depicted in Figure 1 highlight three core results. The first core result is that noise trading allows generating a delayed exchange rate overshooting in response to a permanent monetary policy shock. The second core result is that a negative international nominal interest rate differential can arise alongside with a depreciation of the nominal exchange rate. The third core result is that noise trading in the foreign exchange market has important implications for the response of the current account to monetary policy shocks.

### 3.1.1 Noise Trading and Delayed Exchange Rate Overshooting

The first core result highlighted by Figure 1 is that noise trading can give rise to a delayed overshooting of the nominal exchange in response to a permanent monetary policy shock. Why does the nominal exchange rate exhibit a delayed overshooting rather than an immediate overshooting in the aftermath of the permanent monetary policy shock? The key to the answer to this question is the UIP condition. This condition can be written as

\[ \hat{d}_t^n - \hat{d}_t^n - gE_t^n(\Delta \hat{S}_{t+1}) = (1 - g)E_t^n(\Delta \hat{S}_{t+1}), \]  

(23)

where the Greek letter \( \Delta \) denotes the first-difference operator. The key feature of equation (23) is that there is an additional term, \( E_t^n(\Delta \hat{S}_{t+1}) \), in the UIP condition if noise traders influence the exchange rate forecasts of foreign exchange dealers. This term captures the fact that the depreciation of the nominal exchange rate triggered by the Home monetary policy shock induces noise traders to compare the actually realized nominal exchange rate with their medium-run target exchange rate. Because noise traders update their forecast for the medium-run exchange-rate target only gradually, the actual exchange rate lies above its target rate in the immediate aftermath of the monetary policy shock. Hence, noise traders expect a
reversion of the exchange rate trend in the medium run. In consequence, they form appreciation expectations: $E_t^n(\Delta \hat{S}_{t+1}) < 0$.

As witnessed by equation (23), the appreciation expectations of noise traders serve to mitigate the effects of the negative international nominal interest rate differential on rational forecasters’ exchange rate expectations. It follows that if the impact of noise traders on the formation of exchange rate expectations is strong enough (i.e., if the parameter $g$ is large enough) the appreciation expectations of the noise traders will dominate the effect of the negative international interest rate differential. In consequence, rational foreign exchange forecasters take into account the strong impact of noise traders and form corresponding depreciation expectations. Because these expectations are rational, it follows that the only viable solution to the model in this case is one that implies that the exchange rate continues depreciating for some periods following the monetary policy shock. The result is the delayed overshooting of the exchange rate shown in Figure 1. Because noise traders continuously update their expectations regarding the medium-run target exchange rate, the delayed overshooting phases out gradually. In the long run, the medium-run target exchange rate of noise traders is identical with the post-shock steady-state exchange rate.

### 3.1.2 Noise Trading and Deviations from UIP

The second core result highlighted by Figure 1 is that the international nominal interest rate differential is negative in the aftermath of the permanent monetary policy shock and, at the same time, that the nominal exchange rate starts depreciating. To shed further light on this result, I plot in Figure 1 the deviation from the condition of UIP in the aftermath of a monetary policy shock. I define the deviation from UIP as the residual that obtains if the international nominal interest rate differential is subtracted from the exchange rate forecasts of rational traders. In other words, I define the deviations from UIP in terms of the exchange rate.
rate forecasts of noise traders. Hence, deviations from the condition of UIP can arise if the influence of noise traders on the formation of exchange rate expectations is erroneously neglected.

As can be seen in Figure 1, if noise traders have no influence on the exchange rate forecasts of foreign exchange dealers, the deviation from UIP is identically zero. However, once foreign exchange dealers take into account the exchange rate forecasts of noise traders when forming exchange rate expectations, the deviation from UIP is in general non-zero. In particular, note that a positive deviation from UIP (i.e., noise traders from appreciation expectations) arises in the first periods following the permanent monetary policy. These are the periods during which the delayed overshooting occurs. In consequence, the model implies that in the aftermath of a permanent expansionary Home money supply shock a depreciation of the nominal exchange rate, a negative international nominal interest rate differential, and a positive deviation from UIP can be observed. Eichenbaum and Evans (1995) have described this phenomenon in detail in their empirical study.

3.1.3 Noise Trading, Delayed Overshooting, and the Current Account

The third core result highlighted by Figure 1 is that, as in the NOEM model used by Anderson and Beier (2000), noise trading in the foreign exchange market has important implications for the impact of monetary policy shocks on the dynamics of the Foreign bond holdings of Home households and, thus, on the current account. To start the analysis of this result, I go one step back and assume that there are no noise traders in the foreign exchange market. In such a scenario, the model is simply a dynamic version of the PTM-NOEM model developed by Betts and Devereux (1996, 2000). From their analysis, it is known that monetary policy shocks do not lead to changes in the current account and, thus, leave the bond holdings of households unaffected. The log-linear version of the difference equation governing the dynamics of the Foreign bond holdings of Home households is given by
\[ \beta \hat{B}_{h,t} = (\hat{x}_t + \hat{p}_t(z) - \hat{p}_t)/2 + (\hat{v}_t + \hat{S}_t + \hat{p}_t^*(z) - \hat{p}_t)/2 + \hat{B}_{h,t-1} - \hat{C}_t, \]  

(24)

where \( \hat{x}_t \) (\( \hat{v}_t \)) denotes output sold by Home PTM firms at home (abroad). If monetary policy shocks do not result in a change in \( \hat{B}_{h,t} \), it must hold that in the immediate aftermath of the shock, before firms begin adjusting prices, the following equation applies: \( \hat{y}_t + \hat{S}_t - \hat{C}_t = 0. \) Hence, the impulse response functions depicted in Figure 1 show that the current account effect of the monetary policy shock is zero if \( g = 0. \)

What changes if noise traders enter the scene? The analysis of Figure 1 shows that the effect of allowing for noise trading on output and consumption is relatively small. In consequence, \( \hat{y}_t \) and \( \hat{C}_t \) do not change substantially. However, noise traders exert a substantial effect on the nominal exchange rate. In consequence, it must be the case that \( \hat{y}_t + \hat{S}_t - \hat{C}_t \neq 0, \) implying that monetary policy shocks now trigger an adjustment of the stock of Foreign bond holdings of Home households. In other words, monetary policy shocks affect the current account. This implies that money is no longer neutral in the long run. In consequence, what has begun as a purely nominal phenomenon (noise trading and monetary policy shocks) eventually results in changes in the allocation of real resources.

Of course, the interpretation of this result should not be stretched too far because, as pointed out by Obstfeld and Rogoff (1995), the non-neutrality of monetary policy in the long-run is proportional to the effect of the interest income on Foreign bond holdings on the households optimizing behavior. For the assumed calibration of the model, this effect is fairly small, implying that money is ‘almost’ neutral in the long run.

### 3.2 Sensitivity Analyses

To analyze how changes in the parameter that govern the way in which noise traders form their exchange rate forecasts affect the results reported in Section 3.1, I summarize in Figure 2
the results of some sensitivity analyses. In the first row of this figure, I plot impulse response functions for a model in which \( ma = 2 \) and the elasticity of noise traders’ exchange rate forecast with respect to the deviation of the current exchange rate from its medium-run target value is equal to \( \phi = 0.5 \) (solid lines) and \( \phi = 0.2 \) (dashed lines). In the second row of the figure, I fix the elasticity of noise traders’ exchange rate forecast with respect to the deviation of the current exchange rate from its medium-run target value at \( \phi = 0.5 \) and increase the number of lagged exchange rate realizations used by noise traders to compute the medium-run exchange rate target from \( ma = 2 \) (solid lines) to \( ma = 4 \) (dashed lines).

--- Insert Figure 2 about here. ---

The figure shows that decreasing the elasticity of noise traders’ exchange rate forecasts with respect to the deviation of the current exchange rate from its medium-run target value implies that the nominal exchange rate tends to reach its maximum deviation from the steady state relatively rapidly. In consequence, the delayed overshooting of the nominal exchange rate tends to be transformed more and more into an immediate overshooting as the numerical value assigned to the parameter \( \phi \) decreases.

Interestingly, the impulse response functions graphed in the first row of Figure 2 show a situation in which there is still a delayed overshooting of the nominal exchange rate but no longer a delay in the overshooting of the real exchange rate. While the nominal exchange rate gradually depreciates for some periods after the monetary policy shock, the real exchange starts appreciating in the immediate aftermath of the shock. The main factor responsible for this effect is that in the situation depicted in the figure the delayed overshooting of the nominal exchange rate is overshadowed by the dynamics of the CPIs. In fact, the Home CPI starts to rise relatively sharply in the aftermath of the monetary policy shock and converges thereafter rapidly towards its new steady-state value. In addition, because the Home monetary policy shock has positive temporary spillover effects on the Foreign economy (not depicted in
Figure 1), the CPI in the Foreign economy increases in the short-run. But this effect tends to be smaller than in the Home country. The result is that the real exchange rate starts appreciating immediately following its impact devaluation.

This result shows that the simultaneous modeling of the nominal and of the real exchange rate in a PTM-NOEM model provides important information on which numerical values for the parameters describing how noise traders form their exchange rate forecasts should be used in the simulations of the model in order to generate a delayed overshooting of the nominal and of the real exchange rate.

The second row of Figure 2 graphs a situation in which noise traders use a relatively long history of the exchange rate trajectory to compute their medium-run exchange rate target. In the depicted scenario, noise trading causes an undershooting of the nominal exchange rate. The reason is that if the number of lagged exchange rates used by noise traders to compute the medium-run target exchange rate is relatively large, this target exchange rate is updated rather slowly. Thus, noise traders learn relatively slowly about the nature of exchange rate changes. This implies that any depreciation of the nominal exchange rate caused by the monetary policy shock gives rise to relatively strong appreciation expectations of noise traders. In consequence, for the exchange rate to reach its post-shock steady-state value, rational exchange rate forecasters have to form relatively strong depreciation expectations. The result is that the contemporaneous effect of the monetary policy shock on the nominal exchange rate tends to be relatively small in order to leave enough room for a further gradual depreciation of the nominal exchange rate.

To sum up, delayed overshooting of the nominal and of the real exchange rate in response to a monetary policy shock tends to occur whenever noise traders update their expectations regarding the medium-run exchange rate target relatively quickly. Moreover, delayed overshooting becomes more likely if the elasticity of the forecasts of noise traders with
respect to deviations of the current exchange rate from this medium-run target value is sufficiently large.

3.3 The Persistence of Monetary Policy Shocks and Delayed Overshooting

So far, I have analyzed how the presence of noise traders in the foreign exchange market shapes the dynamic effects of key macroeconomic variables in response to a permanent Home monetary policy shock. This is the kind of analysis carried out by Dornbusch (1976) and by the researchers who have used variants of his model to study the robustness of the overshooting phenomenon. I now study the implications of noise trading for the dynamic effects of a temporary Home monetary policy shock. To this end, I compare in Figure 3 the implications of noise trading in the foreign exchange market for the response of the nominal exchange rate to a permanent and to a temporary unit Home money supply shock.

--- Insert Figure 3 about here. ---

In the case of a temporary monetary policy shock, both noise traders and rational exchange rate forecasters expect that the contemporaneous nominal depreciation of the Home currency will be reversed in the long run. In consequence, both groups of foreign exchange rate forecasters form appreciation expectations. Hence, there is no delay in the overshooting of the nominal exchange rate. Thus, as in the models developed by Gourinchas and Tornell (2002) and Anderson and Beier (2000), the persistence of monetary policy shocks deserves special attention in analyses of the delayed overshooting ‘puzzle’. As expected, the magnitude of the actual depreciation of the nominal exchange rate in the aftermath of the temporary monetary policy shock becomes stronger the stronger is the influence of noise traders on the exchange rate expectations of foreign exchange dealers. Also note that even though there is no delayed overshooting the presence of ‘rule of thumb’ forecasters implies that, in line with empirical evidence (Faust and Rogers 1999), there are still deviations from UIP.
4. Conclusion

The analysis in this paper demonstrated that adding noise trading to an otherwise standard PTM-NOEM model offers a number of interesting insights into the impact of monetary policy shocks on nominal and on real exchange rate dynamics. Specifically, I showed that noise trading helps to model a delayed overshooting of the exchange rate in response to a monetary policy shock. Moreover, noise trading is important for the dynamics of the current account and may be one of the modeling devices that could help generating persistent deviations from UIP. Further, the magnitude of the parameters that capture how noise traders form their exchange rate forecasts is important. Specifically, noise trading tends to help generating delayed overshooting of the nominal and of the real exchange rate if (a) noise traders update their medium-run exchange rate target relatively quickly, and (b) the elasticity of noise traders’ forecasts with respect to the deviation of the exchange rate from this medium-run target rate is sufficiently large.

The latter results are interesting because they imply that the delayed overshooting ‘puzzle’ can be linked to the parameters that describe how noise traders compute their exchange rate forecasts. From the research on exchange rate expectations, empirical evidence on the magnitude of these parameters is available for different sample periods and for a cross-section of currencies. Given this evidence, it would be interesting to use the results I reported in this paper to study empirically whether the delayed exchange rate overshooting ‘puzzle’ can be linked in a systematic way to the structural parameters that capture how foreign exchange traders form their exchange rate expectations.
References


Table 1 — The calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Subjective discount factor</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>8.2</td>
<td>Inverse of the elasticity of utility with respect to real balances</td>
</tr>
<tr>
<td>$\theta$</td>
<td>10.0</td>
<td>Elasticity of substitution between differentiated products</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.75</td>
<td>Probability of adjusting prices</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.5</td>
<td>Elasticity of noise traders’ expectations with respect to exchange rate changes</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1 (0.5)</td>
<td>Autoregressive parameter of the money supply process in the case of permanent (temporary) money supply shocks</td>
</tr>
<tr>
<td>$ma$</td>
<td>2</td>
<td>Number of lags used by noise traders to compute the moving average of the exchange rate</td>
</tr>
</tbody>
</table>

*Note:* The numerical values for the parameter of households’ utility function and for firms’ price setting are taken from Betts and Devereux (2001). The parameter $\phi$ lies within the range of the empirical estimates of, e.g., Frankel and Froot (1987a) and Cavaglia et al. (1993). The numerical value of the parameter $ma$ implies that noise traders use two lags of quarterly exchange rate data to compute the medium-run exchange rate target value.
Figure 1: Persistent monetary policy shocks, noise trading, and delayed overshooting

Note: The figure plots the responses of key Home variables to a unit permanent Home monetary policy shock. Dashed lines obtain when setting $g = 0.5$ and solid lines obtain when setting $g = 0.0$. The variables are measured in terms of percentage deviations from the pre-shock steady state. The international interest rate differential is measured in terms of percentage point deviations from the steady state.
Figure 2: Noise Trading and the Dynamics of the Nominal and of the Real Exchange Rate

Note: The first row of the figure gives the impulse response functions for the nominal and for the real exchange rate obtaining in a model with $ma = 2$. Solid lines denote the case of $\phi = 0.5$ and dashed lines denote the case of $\phi = 0.2$. The second row of the figure gives the impulse response functions for the nominal and for the real exchange rate obtaining in a model with $\phi = 0.5$ and $ma = 2$ (solid lines) and $ma = 4$ (dashed lines). The nominal and the real exchange rates are measured in terms of percentage deviations from the pre-shock steady state.
Figure 3: The persistence of monetary policy shocks and the dynamics of the nominal exchange rate

\textit{Note}: The figure depicts the response of the nominal exchange rate to a permanent (upper panel) and temporary (lower panel) money supply shock. The nominal exchange rate is measured in terms of percentage deviations from the pre-shock steady state.