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Center for Comparative and International Studies (CIS)

Working Paper No 48, 2009

Published by the Center for Comparative and International Studies (ETH Zurich and University of Zurich)

Who gains from apparentments under D'Hondt?

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Acknowledgements

I am in dept to Sebastian Maier, Friedrich Pukelsheim, Peter Leutgäb, Hanspeter Kriesi, and Alex Fischer, who provided very insightful comments on earlier versions of this paper.

Manuscript

Who gains from apparentments under D'Hondt?

Apparentments – or coalitions of several electoral lists – are a widely neglected aspect of the study of proportional electoral systems. This paper proposes a formal model that explains the benefits political parties derive from apparentments, based on their alliance strategies and relative size. In doing so, it reveals that apparentments are most beneficial for highly fractionalised political blocs. However, it also emerges that large parties stand to gain much more from apparentments than small parties do. Because of this, small parties are likely to join in apparentments with other small parties, excluding large parties where possible. These arguments are tested empirically, using a new dataset from the Swiss national parliamentary elections covering a period from 1995 to 2007.

Keywords: Electoral systems; apparentments; mechanical effect; PR; D'Hondt.

Apparentments, a neglected feature of electoral systems

Seat allocation rules in proportional representation (PR) systems have been subject to widespread political debate, and one particularly under-analysed subject in this area is list apparentments. Theoretical work and studies based on simulation models has shown that the *division allocation rule with rounding down* (the most frequently applied PR allocation rule, referred to as the *D'Hondt* or *Jefferson* method) strongly favours large parties over small parties (see, among many others, Pennisi, 1998; Elklit, 2007; Schuster, Pukelsheim, Drton, and Draper, 2003). Other common seat allocation rules, such as *Hamilton's* or *Hare/Niemayer's largest remainder method*, or the *Sainte-Laguë/Webster divisor method with standard rounding* are unbiased with regards to party size. Several countries apply the D'Hondt allocation rule in combination with the possibility of list apparentments—a course of action aimed at reducing bias in favour of large parties. Here, we define apparentments as the formal linking of several electoral lists in list PR systems, where the initial seat allocation treats these apparentments jointly as a single unit.[1] In a secondary step, the mandates won by a coalition are allocated to each of the electoral lists in the coalition.

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[1] See also Lijphart (1994, p. 134, 2000) or Taagepera (2007, p. 41) for a similar definition. Other studies have used a broader definition, which also includes electoral alliances in open-list PR, in which the mandates are allocated to the candidates with most votes within alliances, regardless of whether this might lead to an overrepresentation of their own party (cf. Suojanen, 2007; Kreuzer and Pettai, 2003, p. 84). Lijphart (1994, p. 135) implicitly applies a wide definition of apparentments, including forms that I would define as alliances or joint electoral lists of several parties.
The election of candidates on joint lists involves the same coordination problem as the Single Non-Transferable Vote. A party needs to present as many candidates as it expects to win seats, and it needs to guarantee that the votes are approximately equally distributed on all candidates. Parties that fulfil this strategic task better can win more seats in the seat allocation process within an alliance (Christensen, 1996). Since it is easier to estimate the number of mandates that small parties would win in an alliance, and since this number would often be one (after rounding), they can avoid the problem of equal vote allocation on several candidates. Thus, it is likely that

In this study, I propose a formal model that predicts the partisan effects of list apparentments. My dependent variable is the benefit that political parties might expect from an apparentment. And my calculations are based on a party's alliance strategy and size, without knowing the vote distribution that will result from an election in detail.

My model shows that apparentments are most beneficial for political blocs that are highly fractionalised. However, within the apparentments, the largest parties profit most. For this reason, it would seem that small parties prefer joining apparentments with other small parties, excluding large parties wherever possible. These results are widely confirmed by empirical tests that I have carried out using a new database, taken from the Swiss national parliamentary elections in the period from 1995 to 2007. For these tests, I aggregated the electoral results and list apparentments by cantons and parties, and calculated the net effect of list apparentments on the seat allocation.

In this fashion, I have worked to flesh out one of the least studied aspect of electoral systems.[2] As noted before, the scholarly understanding of these apparentments, and of their partisan effects, has been rather rudimentary. In particular, there is substantial confusion about the *effect* of apparentments. Some scholars see them as predominantly advantageous to small parties:

> "Because even PR systems tend to favour the larger parties to some extent, the possibility of *apparentement* makes it possible for small parties to counteract to some extent the disadvantages of their small size." (Lijphart, 1994, p. 134)[3]

Others dispute this line of reasoning, claiming that apparentments benefit large parties most. Still others suggest that apparentments' outcome is random—following the logic of a lottery rather than any reasonable rule (Pukelsheim and Leutgäb, 2009).[4]

Apparentments are also believed to induce 'vote pooling' among parties, and to spur the formation of inter-party alliances (Lijphart, 1994, p. 150).

In many situations, politicians and election experts surmise the effect of apparentments based on recent local experiences. Systematic analytical studies, or sound theoretical explanations, have remained rare. The lack of academic knowledge in this field is even more astonishing, given that list apparentments remain highly disputed politically.[5]

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small parties can win more seats within an alliance in an open-list system than they would be entitled to proportionally, at the expense of larger alliance partners.

[2] The rare studies known to the author that investigate the functioning of list apparentments date from many decades ago (von Bortkiewicz, 1919; Schmid, 1961), and more importantly, they hardly offer generally applicable conclusions about the partisan effect of list apparentments. In a recent study, Pukelsheim and Leutgäb (2009) argue that the effect of list apparentments is, to a wide extent, devoid of logic.

[3] A similar statement can be found in Bentz (2003, p. 8).

[4] See Elklit (2007). Similarly, the Swiss newspaper „Der Bund" writes that in the Swiss PR system, large parties profit from apparentments, particularly the Social Democrats, which in many countries are allied with a smaller Green party (Rudolf Burger, Der Bund, 15. Oktober 2007, Seite 2, „SP-Verluste, Gewinne der Grünen")

[5] For instance, in the period of 1997 to 2008, five out of 17 Swiss cantons have abolished list apparentments. Since, the parliament of at least one other canton (Zug) has decided to abolish the instrument, arguing that list apparentments lead to a flux of electoral lists, and that it increases the complexity of the voting process (Kanton Zug, Protokoll des Kantonsrates, 28 May 2009 and 25 June 2009). Others criticise apparentments as

This paper measures the real effect that apparentments have on seat allocation under the D'Hondt system, using a theoretical and empirical perspective, in order to allow for the prediction of apparentments-derived partisan benefits. The model established in this paper suggests that the frequent believe that apparentments correct the disproportionalities of the D'Hondt rule can not be maintained.

The first section of this paper defines apparentments and distinguishes them from conceptually different forms of party alliances. Section 2 sketches out my theoretical model. And Section 3 tests it, using data from Swiss national parliamentary elections.

Definition and use of apparentments

Several electoral systems allow parties to ally themselves in the certain ways, and this allows them to win more seats than if they were to run alone.

> "Apparentement means the formal linking of party lists that is allowed in some list systems of proportional representation, and the term also denotes the set of parties that are thus linked. ... The party lists that belong to an apparentement appear separately on the ballot, and each voter normally votes for one list only, but in the initial allocation of seats, all of the votes cast for the lists in the apparentement are counted as having been cast for the apparentement. The next step is the proportional distribution of the seats won by the apparentement to the individual party lists that belong to it." (Lijphart, 2000, p. 16, italics in the original)

While other types of electoral alliances require parties to run with a joint list, in list apparentments each parties keeps its own list and they are only treated as groups for the purpose of seat allocation. Votes are cast for single parties and not for the full alliance, and the seat allocation of parties within the apparentment occurs proportionally in relation to each parties' votes.[6] While apparentments are often used among political partners (notably the Swiss national elections) (XXX, 2009), they can also be viewed as solely technical alliances of parties with no political affinities (Andeweg, 2005, p. 504). This would hardly be imaginable for other types of electoral alliances, where the allied parties do not appear separately on the ballot, and where votes can not be cast for each party separately.

intransparent: voters do not know who is profiting from their votes, and their effects are at random (among many others, Kanton Aargau, Botschaft des Regierungsrates an den Grossen Rat vom 27. September 2006, 06.220, p. 22). For a response to these critiques, see Bochsler (2009).

[6] In the case of joint lists, votes can only be cast for the alliance of parties as a whole, and not for single lists within the alliance. The procedure relies on the parties to define how seats are allocated between members of the alliance (in certain systems it is defined through the order of candidates on the list). In open-list PR, voters can choose which candidate (and, thus, for which party) they cast their vote. However, in open-list PR, seats are allocated within alliances based on the votes each candidate receives, and are not proportional to parties' vote shares.

Apparentments are currently used in Belgium, Denmark, Germany, Israel, the Netherlands, and in Switzerland.[7] Previously, they had been used in the French Fourth Republic, and, until the 1940s, in Norway and in Sweden. Certain legislatures also allow apparentments *within* an apparentment, or "*sub-apparentments",* with several electoral lists of the same or different parties. Sub-apparentments themselves are part of a larger apparentment. In the analytical part of this paper, I will argue that they make a crucial difference in how apparentments are used and who benefits from them (see also the appendix). In sum, apparentments do not necessarily stop at the second level of sub-apparentments: in one empirical case, from the 1958 elections in the Swiss canton of Bern, a *third degree* list apparentment was noted, thus constituting a sub-sub-apparentment.[8]

Apart from correcting for disproportionalities, apparentments might help small parties to gain parliamentary seats collaboratively, in cases where they were not strong enough separately (Gallagher and Mitchell, 2005, p. 589; XXX, 2009). This function might prevail if apparentments were to be used under allocation methods, which do not provide any genuine advantage to larger parties in seat allocation, as the D'Hondt method does.[9] The theoretical model presented in this paper and its empirical application focus solely on the D'Hondt method, which is frequently applied in PR systems.

A theoretical model of the partisan effect of apparentments

This article focuses on the mechanical impact of list apparentments, which – according to Duverger (1951) – is the direct effect of the seat allocation rule in electoral systems.[10] This section develops a theoretical model that predicts the gains and losses that result when political parties form list apparentments. While such benefits can be measured ex-post, an ex-ante estimation is usually not feasible, since even minor changes in the distribution of votes might affect parties' seat allocation, and the overall effect of apparentments. Without knowing the exact number of votes per party in advance, an exact prediction of the effect of apparentments is impossible.

Like all proportionality rules, D'Hondt allocates a number of seats to each party in proportion to the number of votes won. Usually this results in fractions and a rounding method is needed. The D'Hondt system is a divisor method with rounding down. Votes are divided by a divider, and the

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[7] The information for these countries was found in (Lijphart, 1994, p. 134); for Belgium in Coffé (2006, p. 104); Germany in (Meyer, 2007, p. 442-443); Israel in Rahat & Hazan (2005, p. 338); and the Netherlands in Andeweg (2005, p. 496). Interestingly, in the Netherlands, the LR-Hare method is used for the secondary seat allocation within apparentments.

[8] In the 1958 cantonal legislative elections in the Swiss canton of Bern, three centre-right parties (the BGB, the Christian Socials, and the Radicals) formed an apparentment, within which the BGB and the Christian Socials formed a sub-apparentment. Since the BGB party was running with two distinct regional lists (North and South) within the constituency Bern-countryside both lists (BGB North and BGB South) were allied in a sub-sub-apparentment (Schmid, 1961, p. 202).

[9] As I discuss elsewhere, list apparentments perform political functions under other PR formulas too. They can help to avoid wasted votes for small parties, by directing them to an allied partner (XXX, 2009).

[10] The model does not capture possible strategic reactions of political parties to list apparentments, so that I neglect what Duverger would have called a 'psychological effect'

resulting seat numbers, which are not integers, are rounded down to the next integer, as is illustrated in the following section.

The mechanical effect of list apparentments

Using an imaginary example, I illustrate the seat allocation mechanism of the D'Hondt method, showing how seats are allocated under list apparentments. I posit four parties, A, B, C, and D, competing in a district with nine seats (table 1). Before showing the effect of list apparentments, I calculate the seat allocation without any apparentments.

xxx Include table 1 about here. xxx

The most intuitive way to allocate h seats to parties $[1, ..., j, ..., N]$ would be to multiply their vote shares p_j with h. Hence, in a nine-seat district ($h=9$), party C with 23.5% of the votes would be entitled some 2.1 seats, or one seat for every 11.1% of the votes. However, the resulting number of seats-by-party is not an integer number. The D'Hondt rule rounds the number of seats by party off to the next integer. If for each party its seats are rounded off by a fraction of a seat, then the sum of all seats allocated to parties will be lower than the number of available seats h. The more parties there are, the more fractions of seats will be rounded off, so that with an increasing number of parties, the gap between the number of seats h and the sum of the allocated seats increases. For this reason, there is a correcting divisor υ introduced. Hence, the vote share p_j is multiplied by the number of seats in the electoral district h and with this divisor υ. If υ=1, then an exactly proportional seat allocation would result, but due to the rounding off rule, this would only in rare exceptions allocate all h seats in the district. Therefore, the divisor υ needs to be slightly more than 1; for small districts and a highly fractionalised distribution of votes, it might even become larger than 1.5. The divisor υ is established newly for each seat allocation performed under the D'Hondt rule.[11] In the present example, the divisor $\upsilon=1.3$ decides the distribution of the nine seats in the district. As a result, party A obtains five mandates, B and C two mandates each, and D obtains zero. The divisor v is established through iterations. For each distribution of votes, there is a range of divisors that is suitable, in order to allocate exactly h mandates to a party. In my example, each number in the range 1.292 to 1.307 would be possible. All divisors in this range lead to the same seat allocation.

In the second example, the three smaller parties (B, C, and D) consolidate their lists in an apparentment. In the first-step of seat allocation, mandates are allocated to A and to the B-C-D apparentment, before mandates are allocated to the parties within the apparentment. For the first step, the same procedure is applied as before, but with only two units, so that the resulting number of seats is rounded off only twice – once for A, and once for B-C-D. Due to the lower

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[11] There are several ways how the D'Hondt seat allocation can be calculated, which all lead to identical results. For a similar calculation, although with a divisor that is defined as *1/hv*, and operates on the basis of votes, instead of vote shares, see Balinski and Young (2001, p. 19).

fractionalisation of the vote, the divisor might be slightly lower. Indeed, $v=1.1$ results, and stands as a viable divisor. The calculation shows that party A loses one of its existing five seats to the allied parties. For the secondary seat allocation within the apparentment, $h'=5$ seats are available for distribution, and they are allocated according to the D'Hondt rule among the remaining three parties (B, C, and D). Their overall vote share adds up only to *57%*. Now, the divisor v'=1.31 emerges, allocating two seats each to parties B and C, and one seat to party D.

Estimation of the gains through apparentments

This example illustrates how apparentment work, and it also shows that, once the exact vote distribution is known, we can establish the exact effect of list apparentments on the seat allocation. Instead of a case-wise ex-post calculation, based on the vote distribution, this papers aims at establishing general rules about the partisan effect of list apparentments under the D'Hondt rule, which helps to understand the functioning of list apparentments. A function that describes the partisan effect of list apparentments can also be of practical use, enabling political party strategists to anticipate the expected effect of a list apparentment. Certainly, the latter can also be done, relying on a supposed vote distribution, and a calculation of the resulting seat allocation, using the D'Hondt rule. However, results of such an estimation would not be robust: due to the discrete character of the seat allocation, the direct calculation of gains through list apparentments using the D'Hondt seat allocation formula becomes highly unsteady.[12] Even tiny changes in the vote distribution can affect the seat allocation substantially, so that at the absence of the (almost) exact numbers of votes, any prediction relying directly on the D'Hondt rule might be erroneous.

Instead my general model of the partisan effect of list apparentments will rely on theoretical considerations about the functioning of apparentments, and lead to robust results, which are based only on the *approximate vote share* of the allied share (taken, for instance, from the results of previous elections or from survey data), not requiring exact voting results. While my approximation will not calculate the exact effect of apparentments in single cases with exactly known vote shares, it establishes general rules that help to evaluate the average expected effect of apparentments. Rather than estimating the average effect solely on empirical grounds, I first provide a solely theoretically derived formal model (Taagepera, 2008), whose empirical accuracy will be tested below. Such models are rare in social sciences, but more common in sciences. As will be shown, in the present case, I can make fairly accurate predictions, which go far beyond directional hypotheses of the effect of single variables, or their interaction effects. On the other hand, my model will be less parsimonious and produce multicollinearity. This is certainly a challenge for standard empirical

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[12] For instance, in the example in table 1, in the iteration with no apparentment, an increase of votes for party D of a mere 0.1% would allow this party to win a seat at the expense of party A. After such a tiny shift of the votes, an apparentment would not affect the seat allocation any more. Furthermore, if party B were to win 0.126% of party D's votes, it would win an additional seat – instead of party D – in the seat allocation within the apparentment.

testing methodology used in social sciences, however, it is out-weighted by the advantage of having a theoretical well-founded model, which leads to an exact prediction.

First, I estimate the number of seats (s_j) that a party (j) with an approximate vote share of p_j might win, ignorant of the exact distribution of votes across parties.[13] Because the D'Hondt rule rounds the number of seats off to the next integer, the resulting function is characterised by discontinuities, and its results depend on the votes of all other parties. The function can, however, be approximated by a short formula, incorporating vote share, district size, and the number of competing parties, based on the property of the D'Hondt rule to reduce the result by half a seat. I assume that the remaining fractions that are rounded off in every seat allocation are randomly distributed between 0 and 1.[14] If this is the case, then their mean is 0.5, so that a rounding rule would on average reduce the number of seats of each electoral list by 0.5. On average, for an electoral district with h seats, the average expected number of seats for party j can be estimated in this fashion, using the divisor υ, which still needs to be established:

$$s_j = p_j \cdot \upsilon \cdot h - \frac{1}{2} \qquad [1]$$

The divisor υ is slightly larger than 1. If seats were to be allocated according to formula 1, but with $\upsilon=1$, then every party would gain half a seat *less* than under exact proportionality. Overall, not all the seats in the district would be allocated. For N parties, assuming that for each party the number of seats is rounded off by 0.5, $h - N/2$ seats would be allocated. However, the seats allocated to all parties $1, ..., j, ..., N$ need to add up to h, with the vote shares of the same parties adding up to 1 (unless if there is a legal electoral threshold). Both conditions allow us to establish an estimate of the divisor υ. (See also Gfeller (1890) and Happacher and Pukelsheim (1996, p. 374).)[15]

$$\sum_{j=1}^{N} s_j = h \qquad [2]$$

$$\sum_{j=1}^{N} p_j = 1 \qquad [3]$$

$$\sum_{j=1}^{N} \left(p_j \cdot \upsilon \cdot h - \frac{1}{2} \right) = h \qquad [1 \text{ in } 2]$$

$$\upsilon \cdot h \cdot \sum_{j=1}^{N} p_j - \frac{N}{2} = h$$

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[13] For their estimation of gains resulting from apparentments, Pukelsheim and Leutgäb (2009) employ a different method, which relies on the expected ordinal strength of a political party, in terms of votes received. Hence, the expected effect of apparentments on a party can *only* be established once the vote shares of other parties are known. My method, which relies solely on the expected vote share of a political party, does not require any information about the vote share of other parties; the only required information is the overall number of parties and of parties within apparentments.

[14] If many micro-parties compete, which win votes that correspond to a very small share of one seat, this assumption might be violated.

[15] Real values of the divisor will deviate, due to the rounding process.

Since $\sum_{j=1}^{N} p_j = 1$, this equation can be simplified: $\upsilon \cdot h \cdot 1 - \frac{N}{2} = h$, or $\upsilon = 1 + \frac{N}{2 \cdot h}$ [4]

If this is included in formula 1, we can now estimate the number of seats s_j for a party j:

$$s_j = p_j \cdot \left(1 + \frac{N}{2 \cdot h}\right) \cdot h - \frac{1}{2} = p_j \cdot h + \frac{p_j \cdot N}{2} - \frac{1}{2} \quad [4\ in\ 1]$$

This formula [4 in 1] allows us to compare the number of seats that a party j is expected to win when running alone *(s_j)* with the number of seats it seems likely to win as part of an apparentment *($\hat{s}_j$)*. The difference between both estimates constitutes the relative benefit g_j of party j. I symbolise the number of parties participating in the apparentment with party j as n_j, and the overall vote share of all parties within the apparentment amounts as p_n.

If party j were to run alone, the number of seats it would win (s_j) can be estimated according to formula [4 in 1].

$$s_j = p_j \cdot h + \frac{p_j \cdot N}{2} - \frac{1}{2}$$

However, if the same party with the same vote share were to run as part of an apparentment, the number of seats must first be calculated for the alliance as a whole; thereafter, these seats are allocated to parties within the apparentment. Since the allied parties count as a single unit in the primary seat allocation, the overall number of units is reduced from N to $N\!-\!n_j\!+\!1$ (for one apparentment with n_j parties), or to $N\!-\!n_T\!+\!a_T$ (for a_T apparentments with overall n_T parties). I follow with the estimation for one apparentment with n_j parties, and a generalisation for a_T apparentments can easily be adopted later.

For the primary seat allocation, I estimate the number of seats ($\hat{s}_n$) that will be allocated to the whole apparentment.

$$\tilde{s}_n = p_n \cdot h + \frac{p_n \cdot (N - n_j + 1)}{2} - \frac{1}{2} \quad [5]$$

Thereafter, the seats ($\tilde{s}_n$) are allocated within the apparentment to the participating parties, according to the D'Hondt formula. The number of seats that party j obtains depends on the overall number of seats ($\tilde{s}_n$) allocated to the apparentment (similar to h in the main formula), on the number of parties within the apparentment (n_j, similar to N in the main formula), and on the vote share of party j of all votes of the apparentment (p_j/p_n, similar to p_j in the main formula).

$$\tilde{s}_j = \frac{p_j}{p_n} \cdot \tilde{s}_n + \frac{p_j \cdot n_j}{p_n \cdot 2} - \frac{1}{2} \quad [6]$$

Since I am interested in gauging how party j benefits from the apparentment, I calculate this variable using the formulas for s_j and $\tilde{s}_j$.

$$g_j = \tilde{s}_j - s_j = \left[\frac{p_j}{p_n} \cdot \tilde{s}_n + \frac{p_j \cdot n_j}{p_n \cdot 2} - \frac{1}{2}\right] - \left(p_j \cdot h + \frac{p_j \cdot N}{2} - \frac{1}{2}\right) \quad [1,4,6]$$

$$g_j = \left[\frac{p_j}{p_n} \cdot \left(p_n \cdot h + \frac{p_n \cdot (N - n_j + 1)}{2} - \frac{1}{2}\right) + \frac{p_j \cdot n_j}{p_n \cdot 2} - \frac{1}{2}\right] - \left(p_j \cdot h + \frac{p_j \cdot N}{2} - \frac{1}{2}\right) \text{[with 5]}$$

$$g_j = p_j \cdot h + \frac{p_j \cdot (N - n_j + 1)}{2} - \frac{p_j}{p_n} \cdot \frac{1}{2} + \frac{p_j \cdot n_j}{p_n \cdot 2} - \frac{1}{2} - p_j \cdot h - \frac{p_j \cdot N}{2} + \frac{1}{2}$$

$$g_j = \frac{p_j \cdot (N - n_j + 1)}{2} - \frac{p_j \cdot N}{2} + \frac{p_j \cdot n_j}{p_n \cdot 2} - \frac{p_j}{p_n \cdot 2}$$

$$g_j = \frac{p_j \cdot (n_j - 1)}{p_n \cdot 2} - \frac{p_j \cdot (n_j - 1)}{2} = \frac{p_j \cdot (n_j - 1) \cdot (1 - p_n)}{p_n \cdot 2} \quad [7a]$$

A further generalisation regarding several apparentments, as well as the estimation of the effect of apparentments on parties that are running alone, can be achieved through substitution of the term $N – n_j + 1$ in formula 5 with $N – n_T + a_T$. The result is very similar to formula 7a:[16]

$$g_j = \frac{p_j \cdot (n_j - 1)}{p_n \cdot 2} - \frac{p_j \cdot (n_T - a_T)}{2} \quad [7b]$$

Formula 7b allows a direct estimation of the expected losses of a party m, which is running alone, setting $n_j = 1$ and $p_n = p_m$.

$$g_m = -\frac{p_m \cdot (n_T - a_T)}{2} \quad [7c]$$

What can we learn for political practice?

My model suggest that when there are apparentments, non-affiliated parties might lose seats, with formula 7c estimating the number of lost seats. The larger the number of affiliated parties (n_T), the larger the losses of non-affiliated parties (p_m) will be (since the overall number of parties within an apparentment (n_T) is always larger than the overall number of apparentments (a_T, $n_T > a_T$), if there is at least one apparentment.

In formula 7a/7b the first summand of the term g_j describes which parties are expected to win these seats As long as the first summand outnumbers the (negative value of the) second summand,

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16 See Pukelsheim and Leutgäb (2009) for a similar formula, relying on the ordinal rank order of parties, in terms of votes received. Peter Leutgäb (personal communication, 29 April 2009) shows that both formulas converge, under certain distributional assumptions.

the party is expected to benefit from apparentments. Every additional apparentment in which a party takes part raises the expected benefit for that party.[17] This is in line with previous work which proved that merging two lists under the D'Hondt rules never results in a loss of a seat (Balinski and Young, 2001, p. 149-151).

Within an apparentment, the gains and losses are proportional to p_j. Thus, allied parties benefit from an apparentment's gains proportionally to their respective vote shares. This factor is crucial to how apparentments are leveraged politically. Often, it is suggested that apparentments can remedy the distorting effect of the D'Hondt formula. My model suggests, however, that while apparentments can help alliances of small parties avoid the disadvantages of running in a fractionalised fashion, the largest parties in an alliance benefit the most. Because of this, it is reasonable to believe that apparentments might help to correct for disproportionalities in cases where several small parties can run jointly. Parties that aim to maximise their number of seats in parliament should always favour small parties as coalition partners over large parties (see appendix).

Formula 7a allows us to estimate the effects of enlarging an existing apparentment. For the apparentment as a whole, and for the newly included parties, the inclusion of additional parties proves to be either beneficial or to have no effect. However, the same cannot be said for pre-existing parties. Indeed, if an apparentment of smaller parties is joined by one or several large parties, these new partners will likely gain the most from the apparentment. Subsequently, it is in smaller parties' best interest to favour small alliances with other small parties. For instance, if two parties with an overall vote share of 20% agreed on an apparentment, they would benefit from the inclusion of a third party only if the new party's vote share did not exceed 13.3% (or if two additional parties did not exceed 30% of the vote share). In general, larger parties are expected to marginalize the gains reaped by smaller parties. However, as the appendix illustrates, sub-apparentments may be a viable solution to this problem, since they allow small parties to ally within larger apparentments without being marginalized so intensely. This sheds light on why sub-apparentments are formed mostly by small political parties, within apparentments of parties of different sizes.

Empirical investigation of the Swiss National Council elections, 1995-2007

This section tests the model on empirical data collected from Swiss national parliamentary elections in the period 1995-2007. In these elections, apparentments and sub-apparentments[18] were

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[17] An additional apparentment increases (n_j-1) to the same extent as (n_T-a_T). As long as the apparentment does not involve all parties (which means that $0<p_n<1$), and since $n_j > 1$, this will increase the first summand more than the second one.

[18] Sub-apparentments are restricted to lists with a similar name, or a common element in the denomination. Often, sub-apparentments are agreed upon within the same party—for instance, between the main party list and a youth list, or for separated lists for different regions or sexes. However, as parties are not legal entities under Swiss legislation, that the law states that the name of election lists that are related in a sub-apparentments

widespread (occurring in 194 to 195 of the 200 seats) and there was wide variance in district magnitude and in the number of competing party lists. The laws of Switzerland state that each of the 26 Swiss cantons forms its own electoral district, ranging from 1 to 34 seats. In multi-seat districts (1995 and 1999: in 21 out of 26 cantons; 2003 and 2007: in 20 out of 26 cantons), PR with the D'Hondt divisor method is applied.

My novel database contains all the political parties' electoral results by district for the four elections (votes and seats) and all inter-party apparentments and sub-apparentments.[19] Overall, these results contain 730 party lists, covering all electoral districts. 67% of the parties were part of an apparentment. On average, each party was linked to the lists of two other parties.

To complete my two-step analysis, I first re-calculated the fictitious seat allocation that would have resulted in the absence of *any* inter-party apparentments or sub-apparentments. This allowed me to establish the gains and losses of seats brought about by apparentments, for each political party. This was based on the voting results in all cantons, using the software BAZI, which calculates different seat allocation for a given distribution of votes (Maier and Pukelsheim, 2007).[20] In my second step, which I will explain in detail later on, I systematically analyzed these gains and losses.[21]

Overall, my initial seat-allocation calculation showed that 632 parties were not affected by apparentments in any way; 49 parties won one seat each; and those 49 seats were in turn lost by 49 other parties. In each of the four elections, there were 10 to 14 of these zero-sum seat transfers, brought about by apparentments. Table 2 shows the net gains and losses by parties and by election year. The main beneficiaries of list apparentments were medium and small parties, and parties of the centre-left: the Green party (GPS) gained 8 seats through apparentments, the Christian Social Party (CSP) 4 seats, and 2 seats each the Green Alternatives (FGA), the 'Landesring der Unabhängigen' (LdU), and the large centre-left party, the Social Democrats (SP). While a few small centre-right parties (EDU, FPS, Lega) profited from apparentments in a small way, the medium and large centre-right parties lost substantially—none more so than the Swiss People's Party (SVP) which lost 11 seats, and the Radicals (FDP), who lost 10 seats.

xxx Include table 2 about here. xxx

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needs to have a common element. Occasionally, different parties agree in sub-apparentments, introducing a label for their alliance into the list name.

19 I define each political organisation or group that runs within its own electoral list in elections as a political party—apart from cases where the same political organisation was present with several electoral lists, which are counted as single parties.

20 The analysis for the 2007 elections is available in Bochsler (2007), previous elections are taken from the Federal Office of Statistics of Switzerland (personal correspondence); Seitz (2002: 66); Seitz/Schneider (2007: 71).

21 The two-step analysis relies on the assumption apparentment lists do not assert a psychological effect on the electorate—an assumption that cannot be tested within the framework of the chosen research design, since the counterfactuals are estimated in a simulation model. The design of the analysis is built, instead, to show the mechanical effect of list apparentments, and is well suited for this purpose.

In the second step of my analysis, I explain these gains and losses systematically through a review of the structure of the apparentments. We must remember, however, that there is a strong random element. As my calculations have shown (table 1), an apparentment's prospective benefit is linked strongly to the margin upon which a party has won a mandate, or the margin by which a party may have lost a mandate. Marginal transfers of just a few votes might lead to different outcomes. This random element aside, however, I expect to find generally regular tendencies. The net change experienced by each political party depends greatly upon the apparentments of which the party is part, on other apparentments in the same canton and year, as well as upon possible sub-apparentments.

Dependent variable

My dependent variable g_j represents the *net benefit derived from a specific list apparentment*. This variable compares a party's gains (or losses) in seats to the above-mentioned hypothetical situation (in which no inter-party apparentments are allowed or accorded). For the 730 cases in my database, the dependent variable g_j varies between –1 and +1, meaning that no party experienced an overall gain or loss of more than one mandate. The mean is theoretically fixed at 0, since every net gain is reflected in a loss by another party. Standard deviation is 0.37.

Explanatory variables

The variables that might contribute to the overall effect of apparentments have already been established and arranged as a formula, as described above. The expected average net benefit (g_j) of a party (j) is a function of the strength of party (j) itself, in conjunction with the number of parties participating in an apparentment minus one (n_j-1), the strength of all parties within the apparentment (pj / p_n), and in relation to all parties competing in elections (p_j). The party might lose seats, in relation to its vote share p_j, if there are other apparentments (a_T) of other parties (n_T) (see table 3).

$$g_j = \frac{p_j \cdot (n_j - 1)}{p_n \cdot 2} - \frac{p_j \cdot (n_T - a_T)}{2}$$

From this formula, I derive a model that is testable in multivariate linear regression models.

$$g_j = \alpha + \beta_1 \cdot n_j \cdot p_j / p_n + \beta_2 \cdot p_j / p_n + \beta_3 \cdot n_T \cdot p_j + \beta_4 \cdot a_T \cdot p_j + \beta_x \cdot \text{controls} + \varepsilon$$

xxx Include table 3 about here. xxx

The effect of sub-apparentments is expected to be similar to the effect of apparentments. While in the case of apparentments, the vote share (p_j) is measured in relation to the overall number of votes that were cast in the electoral district, in the case of sub-apparentments, the formula refers to the apparentment in which the sub-apparentment is nested—and *not* to the electoral district. The overall number of parties that participate in an apparentment is sn_j; the vote share is related to sp_n, the overall vote share of the apparentment, of which the sub-apparentment is part. Furthermore, while n_T and a_T refer to the overall number of apparentments, this is not the case for sub-apparentments; in the case of sub-apparentments, sa_T identifies the number of inter-party sub-apparentments within the same apparentment, and sn_T counts the number of parties that participate in these sub-apparentments.

$$sg_j = \frac{p_j \cdot (sn_j - 1)}{sp_n \cdot 2} - \frac{p_j \cdot (sn_T - sa_T)}{p_n \cdot 2}$$

Overall effect:

$g_j + sg_j$ + controls, with

$sg_j = \alpha + \beta_5 \cdot sn_j \cdot p_j / sp_n + \beta_6 \cdot p_j / sp_n + \beta_3 \cdot sn_T \cdot p_j / p_n + \beta_4 \cdot sa_T \cdot p_j / p_n$
$+ \beta_x \cdot \text{controls} + \varepsilon$

I test the model with an OLS regression, including clustered standard errors for electoral districts (cantons) and elections.[22] In the second and third model, sub-apparentments' effects are tested in analogy to the formula on apparentments (table 4).[23]

xxx Include table 4 about here. xxx

The results of all three models widely confirm my expectations. In the first model, I tested only the variables that were included in my model concerning apparentments, and neglected effects of sub-apparentments. Sub-apparentments were subsequently included in the second model, and in the third model, control variables were also included. All effects remained stable throughout all models. -The two central variables in the models demonstrated highly significant results, of a magnitude that my hypothesis anticipated.

A summary of key findings

- It appears that, with every additional party in an apparentment ($n_j * p_j / p_n$) or sub-apparentment ($sn_j * p_j / sp_n$), the apparentment wins approximately half a seat, and this gain is allocated within the apparentment or sub-apparentment proportionally to the parties' vote

[22] The standard errors are clustered for parties competing in the same year in the same constituency. If one party wins an additional seat due to an apparentment, another one must lose one in the same canton and election year. I expect the random element to be fairly strong, and the series of elections to be reasonably short, and that there may not be a strong autocorrelation of the panel data.

[23] An alternative model, including only n_j, p_j, n_T, and a_T leads to completely different results, and explains only 8.95% of the variance.

shares. This effect remains stable when the number of apparentments (n_j) is included as a control variable. The coefficients deviate only marginally and non-significantly from the expected magnitude of 0.5.[24] As expected, I find a negative impact of (p_j / p_n) and of (p_j / sp_n); both are close to the expected magnitude of 0.5.

- The results confirm that the relative gains brought about by list apparentments are paid for mostly by the largest parties. Every party included in an apparentment has a negative effect on all parties' number of seats. Parties are affected by this effect proportionally to their vote share (n_T * p_j). Additionally, the estimated effect does not deviate statistically significantly from -0.5 and the inclusion of n_T as control variable does not highly affect the effect. This result confirms my expectation that losses will be distributed proportionally to parties' vote shares—meaning that large parties suffer more from others' apparentments than small parties. The effect of sub-apparentments on other parties (sn_T * p_j / p_n) is similar, but not as strong, and is not statistically significant. Once again, this trend is accompanied by an effect of (a_T * p_j) and (sa_T * p_j / p_n) in the opposite direction; after controlling with a_T, the magnitude of (a_T * p_j) is slightly lower.
- The parties' vote share (p_j), and other control variables, do not affect the outcomes substantially.

Taken alone, the apparentments explain about 19% of the variance, which is respectable, given that I expected a substantial random component. The inclusion of sub-apparentments (model 2) increases the explanatory power by an additional 2.5%. The control variables (model 3) do not add substantially to the explanation.

Conclusions

The effect of apparentments under PR rules is often transfigured and misunderstood by political practitioners, and the instrument has largely been neglected by the scholarly community. In the few academic works concerning them, apparentments are assumed to be beneficial for small parties (Lijphart, 1994, p. 134), or to have almost random effects, akin to a lottery (Pukelsheim and Leutgäb, 2009). This study elaborates upon this view. My empirical tests illustrate the mechanical effect list apparentments have had upon the four most recent Swiss national parliament elections, and confirm the expected systematic partisan effects of apparentments.

[24] However, it appears the benefits of apparentments appear somewhat lower than expected here. This could simply be a random result. However, there are a few effects that are not controlled for that might bias the results slightly. Some of the lists that join apparentments are extremely small, and present no chances for parties to win any additional seats. Even if some of them have but a very small vote share, I assume that their joining an apparentment might allow the apparentment to win an additional mandate every second time. This may be an overestimation, and therefore it should cause astonishment if certain coefficients are slightly below the expected 0.5.

First, the results reveal that apparentments can help fractionalised political blocs to be represented more proportionally. The smaller the cumulative vote share of the parties united in an apparentment, the stronger this effect will be. Hence, the smaller and more fractionalised a political bloc is, the more it stands to gain from allying in a comprehensive apparentment.

This does not mean that small, fractionalised parties are necessarily the main beneficiaries of list apparentments. Indeed, my analysis points to the opposite conclusion. Even if alliances of small parties are among the main beneficiaries of list apparentments, the gains of these apparentments are more likely to be accorded to large parties within these apparentments (see also Pukelsheim and Leutgäb, 2009). More precisely, within apparentments, net benefits (and net losses) are allocated approximately proportionally to the vote share of each partner, meaning that large parties can profit most.

With this in mind, it remains questionable as to whether list apparentments can help correct disproportionalities caused by the D'Hondt allocation rule. Indeed, large parties, which can already expect to win a seat share that outnumbers their vote share under the D'Hondt method, are poised to win *additional* mandates if they participate in list apparentments. The potential benefits for small and systematically under-represented parties is much smaller.

Apparentments seem likely to reduce the mechanical effect of the D'Hondt allocation rule, and to assuage representational disproportionalities, in two situations. First, this could happen if small parties – learning from the effect of apparentments – were to act strategically, and participate only in alliances among parties of roughly the same size. However, this seems highly impractical and unlikely. Indeed, if this were practiced universally, the political legitimacy of the instrument would suffer;[25] parties would have to align themselves solely according their relative size, ignoring their bedrock political characteristics. Voters would no loner be sure if their votes were supporting a party of a particular political bloc, or if they were being re-allocated to a political enemy. A second remedy against the redistribution of seats from small to large parties through list apparentments comes in the form of sub-apparentments. Since they allow small parties to ally with each other within larger apparentments in the same bloc, they are a means to prevent large parties from getting all the benefits of apparentments. Sub-apparentments, however, might well be perceived as an overly complex tool of electoral systems.

If list apparentments cannot help to remedy against the disproportionalities of the D'Hondt rule, political practitioners might well evaluate whether it is worth introducing them, or if the same goal could not be more easily achieved by switching to a more proportional seat allocation rule, such as Sainte-Laguë.

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[25] The Netherlands use a different rule, deciding seat allocation according to LR-Hare within the apparentments (Andeweg, 2005, p. 496). This means that small parties can profit equally (proportionally) from seats won by the apparentment as large ones. However, there is still a problem with this approach: if no seat is won, the apparentment could well have the effect of re-distributing a large parties' seats to another party within the apparentment.

Appendix: Inclusion of an additional party into an apparentment

This appendix investigates political parties' strategic choices to engage in apparentments, and the reasons why inter-party sub-apparentments are created.

Suppose we have n_j parties in the apparentment, with a cumulated vote share of p_n. There are n_m other parties, not yet included in the apparentment, but they might be suitable coalition partners. Up to which vote share p_m of these parties is an extension of the apparentment beneficial for the already allied parties? The following formulas show the expected benefits from apparentments, before and after the inclusion of additional alliance partners.

before inclusion: $$g_j = \frac{p_j \cdot (n_j - 1)}{p_n \cdot 2} - \frac{p_j \cdot (n_T - a_T)}{2}$$

after inclusion: $$g_j' = \frac{p_j \cdot (n_j + n_m - 1)}{(p_n + p_m) \cdot 2} - \frac{p_j \cdot (n_T + n_m - a_T)}{2}$$

The net benefit from inclusion of additional parties n_m can be established as follows:

$$diff = g_j' - g_j = \frac{p_j}{2} \cdot \left(\frac{n_j + n_m - 1}{p_n + p_m} - n_T - n_m + a_T - \frac{n_j - 1}{p_n} + n_T - a_T \right)$$

$$diff = \frac{p_j}{2} \cdot \left(\frac{p_n \cdot n_m - p_m \cdot n_j + p_m - n_m \cdot p_n^2 - n_m \cdot p_m \cdot p_n}{p_n \cdot (p_n + p_m)} \right)$$

An extension of the apparentment is beneficial if $diff \succ 0$.

$$\frac{p_j}{2} \cdot \left(\frac{p_n \cdot n_m - p_m \cdot n_j + p_m - n_m \cdot p_n^2 - n_m \cdot p_m \cdot p_n}{p_n \cdot (p_n + p_m)} \right) \succ 0$$

This can be resolved to p_m:

$$p_m \prec \frac{n_m \cdot p_n \cdot (1 - p_n)}{n_j + n_m \cdot p_n - 1}$$

With an increasing number of parties in the apparentment n_j, it becomes unattractive to accept additional parties if they are large. The larger the number of parties already allied in the apparentment (n_j), and the smaller the cumulated vote share of the already allied parties (p_n), the smaller a potential new party needs to be (at maximum p_m) for it to be beneficial to the already allied parties.

However, the problem of an unattractive large alliance partner can be mitigated via inter-party sub-apparentments, allowing small parties to engage in sub-alliance—which can benefit them—within a larger apparentment. For two allied parties S and T, it is never negative to join in an apparentment with one or several additional large parties L, if S and T can agree in a sub-apparentment within the large apparentment S-T-L.[26]

Using data from the four Swiss national elections 1995-2007, I have investigated the matter of whether these strategic considerations are reflected in the alliance behaviour of the Swiss parties. I hypothesise that inter-party sub-apparentments are created most frequently in situations where they are needed to make an apparentment attractive for all partners. If one party within the apparentment is so large that the other parties would rather form an apparentment without the larger party, then the odds are high that these other parties will attempt to join in a sub-apparentment. As my analysis has shown, this happens when one of the parties in an apparentment is particularly large in relation to the cumulated vote share of the other parties, or when there are particularly numerous parties unified in an apparentment. Since sub-apparentments are only allowed in cases where parties run under a common label, it is reasonable to expect parties to be rather reserved when it comes to joining inter-party sub-apparentments. However, large parties sometimes try to hinder their smaller allies from joining a comprehensive sub-apparentment.

An inter-party sub-apparentment can only be formed when there are more than two parties in an apparentment. In the four investigated elections, there were 84 apparentments overall that included three or more parties. In 28 out of these 84 cases, parties agreed on an inter-party sub-apparentment. These cases are distinguished through a clear and statistically highly significant dominance of the largest party within the apparentment: In the group with sub-apparentments, the largest party typically had a clearly dominant position, controlling 66.4% of the votes within the apparentment (standard deviation: 14.0%). In the cases with no inter-party sub-apparentments, the largest party had a much less dominant position, controlling only 54.2% of the votes (standard deviation 14.9%). The second-degree alliances include the largest party of the apparentment in just two out of the 28 cases of inter-party sub-apparentments.

I have explained the probability of the creation of a inter-party sub-apparentment (dummy variable) via a logistic regression based on the three discussed variables: the vote share of the parties united in the apparentment except for the largest party (p_n), the relative vote share of the largest party within the apparentment ($p_m / (p_n+p_m)$), and the overall number of parties in the apparentment ($n_j + 1$). The vote share of the parties in the apparentment p_n and the number of parties n_j+1 is logarithmised.

[26] This is directly related to the fact that an apparentment of two or several parties is never negative for the allied parties, compared to the situation without such an apparentment.

Sub-apparentments are agreed upon if the benefits of the small parties within the apparentment would otherwise be small (table A1). The probability that parties will join a sub-apparentment increases if the vote share of the largest party in the apparentment becomes dominant. Apart from that, the overall vote share of the apparentment without the large party, and the overall number of parties in the apparentment, have a positive effect on the formation of sub-apparentments.[27]

xxx Include table A1 about here. xxx

[27] The two cases where the largest party was included in the sub-apparentment were not included, because there it is obvious that the sub-apparentment serves other aims than securing the small parties their benefit from the apparentment.

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Tables

Number of seats in district: $h = 9$ **No apparentments**				
Party j	vote share p_j	Divisor v	Seats $p_j \cdot h \cdot v$	rounded off
A	43%	1.3	5.03	5
B	25%	1.3	2.93	2
C	23.5%	1.3	2.75	2
D	8.5%	1.3	0.99	0
Total	100%			9

Number of seats in district: $h = 9$ **Apparentment of parties B-C-D**				
Party j	vote share p_j	Divisor v	Seats $p_j \cdot h \cdot v$	rounded off
A	43%	1.1	4.26	4
B-C-D	57%	1.1	5.64	5
Total	100%			9
Seat allocation within apparentment B-C-D				$h' = 5$
B	25%/57%	1.31	2.95	2
C	23.5%/57%	1.31	2.77	2
D	8.5%/57%	1.31	1.00	1
Total	57%/57%			5

Table 1: Exemplary seat allocation in a district with 9 seats on 4 parties, without and with a list apparentment. Example taken from Schmid (1961, p. 51).

	centre-left parties									centre-right parties							
election year	SPS	GPS	EVP	GLP	LdU	PdA	FGA	Sol.	CSP	SVP	FDP	CVP	LPS	SD	EDU	FPS	Lega
1995	4	0	0	–	1	0	1	0	1	-2	-3	-3	0	0	0	1	0
1999	1	2	1	–	1	0	0	0	1	-4	-1	0	0	-1	0	0	0
2003	-3	2	0	–	–	1	1	1	1	-2	-2	-1	-1	1	1	0	1
2007	0	4	0	1	–	0	0	0	1	-3	-4	1	0	0	0	0	0
Total	2	8	1	1	2	1	2	1	4	-11	-10	-3	-1	0	1	1	1
mean national vote share 1995-2007	21.8%	6.8%	2.1%	1.4%	1.3%	0.9%	0.6%	0.4%	0.4%	23.3%	18.3%	15.4%	2.2%	1.6%	1.3%	1.3%	0.7%

Table 2: Net benefits or losses in mandates through apparentments; calculation based on parties' voting results in elections 1995-2007; mean national vote share (for the election years where parties ran). – LdU and GLP ran only in certain election years.

Explanatory variable	**Expected effect**
$n_j * p_j / p_n$	positive impact, coefficient $\beta_1 \approx 0.5$
p_j / p_n	negative impact, coefficient $\beta_2 \approx - \beta_1$
$n_T * p_j$	negative impact, coefficient $\beta_3 \approx -0.5$
$a_T * p_j$	positive impact, coefficient $\beta_4 \approx - \beta_3$
p_T	control variable, no effect expected
n_T	control variable, no effect expected
n_j	control variable, no effect expected
constant	no effect expected, $\alpha \approx 0$
Sub-apparentments	
$sn_j * p_j / sp_n$	positive impact, coefficient $\beta_1 \approx 0.5$
p_j / sp_n	negative impact, coefficient $\beta_2 \approx - \beta_1$
$sn_T * p_j / p_n$	negative impact, coefficient $\beta_3 \approx 0.5$
$sa_T * p_j / p_n$	positive impact, coefficient $\beta_4 \approx - \beta_3$

Table 3: Explanatory variables, and their hypothesised effect (direction and magnitude).

Variable notation: p_j vote share of party in %; p_n cumulated vote share in % of parties that participate in the apparentment of which the party is party of ; n_j number of parties that participate in the apparentment in which the party participates; n_T overall number of parties participating in a inter-party apparentment in the canton and in the election where a party competes; a_T overall number of inter-party apparentments in the canton and in the election where the party competes; sp_n cumulated vote share in % of parties that participate in the sub-apparentment; sn_j number of parties that participate in the sub-apparentment; sn_T overall number of parties in an inter-party sub-apparentment within the apparentment in which a party participates; sa_T overall number of inter-party sub-apparentments within the apparentment of which the party participates.

Regression model Dependent variable: Net benefits from apparentments	(1) only apparentments			(2) with sub-apparentments			(3) with sub-apparentments and control variables		
	B	Std err	VIF	B	Std err	VIF	B	Std err	VIF
Constant	.009	.012		-.01	.012		-.01	.02	
$n_j * p_j / p_n$	.40**	.08	8.23	.41**	.09	10.61	.42**	.09	14.24
p_j / p_n	-.57**	.19	7.62	-.57**	.20	8.80	-.60**	.21	9.84
$n_T * p_j$	-.32**	.09	22.54	-.33**	.09	22.76	-.24**	.12	36.37
$a_T * p_j$	.33	.22	21.48	.35	.22	21.72	.11	.31	38.31
n_j							-.002	.007	2.45
n_T							-.016(*)	.009	13.60
a_T							.045(*)	.027	13.03
p_j							.0005	.0014	4.05
$sn_j * p_j / sp_n$				.49*	.19	23.23	.49**	.18	23.39
p_j / sp_n				-.69(*)	.41	22.79	-.68(*)	.40	22.82
$sn_T * p_j / p_n$				-.12	.29	35.90	-.13	.29	35.98
$sa_T * p_j / p_n$				.12	.65	34.94	.13	.65	35.03
N	730		14.97	730		22.59	730		20.76
cantons (clusters)	82			82			82		
R^2	0.188			0.213			0.216		

Table 4: OLS regression to estimate the benefits from apparentments by party, robust standard errors, cases clustered by electoral districts and election.
**significant at p < 0.01; *significant at p < 0.05; (*) significant at p < 0.1.

Logit regression model Dependent variable: formation of inter-party sub-apparentments	B	Std err
Constant	-14.16	3.83
$\log(p_n)$	1.35*	.66
$p_m/(p_n+p_m)$	8.11**	2.19
$\log(n_j+1)$	3.04*	1.55
N	82	
LR chi2(3)	26.64	
pseudo R^2	0.260	

Table A1: OLS regression to estimate the benefits from apparentments by party, robust standard errors, cases clustered by electoral districts and election.
**significant at p < 0.01; *significant at p < 0.05; (*) significant at p < 0.1.