# Bicameral Winning Coalitions and Equilibrium Federal Legislatures

by

**Tasos Kalandrakis**

**Yale University**

**Abstract:** We analyze the legislative interaction of representatives from big and small states in a Bicameral legislature that decides for the allocation of a fixed resource among the states. We assume that the two houses are mal-apportioned and big states are under-represented in the upper house. We study the effect of this and other institutional features on the relative welfare of big and small states and on equilibrium coalitions. Contrary to common belief, an increase in the representation of small states may reduce their expected payoff, *ceteris paribus*. Also, contrary to interpretations of minimum winning coalition theorems, excess majorities may occur in one of the two houses. When proposal making tends to be dominated by big (small) states, excess majorities occur in the upper (lower) house. We find that higher proposal power increases the payoff of a group of states. Changes in the majority requirements in the two houses and expansion to encompass more small (big) states have a non-monotonic effect on the relative welfare of the two groups. We conclude our analysis with an empirical application using calibrations results for the 103d US Congress and the legislative institutions of the European Union before and after the Treaty of Nice.

**Keywords**: Bicameralism, European Union, Federalism, Legislative Bargaining, Minimum Winning Coalitions, Navette, One-round Navette, Supermajorities, US Congress.

**Correspondence Address:** Department of Political Science, Yale University, PO Box 208301, New Haven, CT 06520-8301. E-mail: kalandrakis@yale.edu.

"The equality of representation in the senate is another point, which, being evidently the result of compromise between the opposite pretensions of the large and the small states, does not call for much discussion." James Madison, Federalist, no. 62, 416–19, 27 Feb. 1788.

"Time: Did the battle over the relative weight of big and small countries overshadow more important matters at Nice?

Verhofstadt: It was absolutely necessary. What they tried to do in Nice was make a directorate of the big countries. The European Union can't survive like that." Interview of Guy Verhofstadt, prime-minister of Belgium with TIME magazine, December 25, 2000, Vol. 156, No. 26

## 1. INTRODUCTION

As the above quotations underline, the competing interests of big and small regions (countries, states, or provinces) constitute a systematic source of political disagreement in the formative stages of supra-regional entities such as Federations or Unions[1]. This tension is fundamentally premised on small states' concern that legislatures elected on the basis of the one man, one vote principal are likely to afford clear majorities to the more populous states.

Historically, constitutional engineers have responded to this conflict between big and small states by designing bicameral legislative institutions with mal-apportionement in (at least) one of the two houses. Indeed, one of the most common forms of Bicameralism involves the over-representation of small states in the upper house. Small regions are over-represented in the upper houses of the US, the European Union (EU), Germany, and Russia, to name a few instances of

[1]Although the analysis pertains to any political entity with region-based representation, from now on we will refer to the political entity as a Federation and the regions as states, unless the specificity of the reference demands otherwise.

federal structures with strong Bicameral institutions[2].

Underlying these institutional choices are assumptions about the types of coalitions that prevail among states in different legislative environments. Intuitively, the over-representation of small states in the upper house increases their (*ex ante*) welfare because it increases the likelihood that they are included in the winning coalition. But a number of additional intuitions dilute or counter the above.

For example, when considering the likelihood of inclusion of small states in the winning coalition it appears relevant to account for the relative numbers of big and small states and/or the majority requirements. Also, in at least some types of policy space such as distributive legislation, conflict of interest exists both within and between the groups of big and small states. If big states wish to exclude states of similar size from the winning coalition, then outcomes may be favorable to small states even in unicameral, well-apportioned legislatures. Finally, numerous studies of legislative bargaining in the tradition of Romer and Rosenthal, 1978, and Baron and Ferejohn, 1989, emphasize the role of proposal power in the determination of legislative outcomes. Thus, in order to assess the effect of particular institutional configurations, unicameral or bicameral, it appears essential that we also consider the assignment of proposal rights among states.

In what follows we offer a systematic study of the effect of Bicameral legislative institutions[3] on legislative outcomes and the welfare of big and small states. Our results are relevant in cases when the underlying policy space is the division of a fixed resource (a budget) among states. Representatives in both legislatures aim to maximize the share of the resource that accrues to their state and small states are over-represented in the upper house compared to their representation

[2]In chapter 2, Tsebelis and Money, 1997, provide an exhaustive list of existing Bicameral legislatures.

[3]Although we assume a Bicameral legislature our analysis extends directly to unicameral legislatures as special cases.

in the lower house. We characterize a stationary Nash equilibrium of this game as in Baron and Ferejohn, 1989, and Banks and Duggan, 2000[4]. We focus on the types of coalitions that form among states and the share of the resource received by each type of state.

Contrary to widely held beliefs regarding the effect of mal-apportionement, we find that an increase in the over-representation of small states may reduce their expected payoff. The mechanism behind this counter-intuitive effect has to do with the direction of associated changes in the equilibrium number of small and big states included in the winning coalition. For example, increasing the representation of small states in the upper house implies that the same number of votes can be obtained with fewer such states. Thus it is possible that both the number of small states in the winning coalition and their expected payoff decrease as a result of more representation.

Another important expectation arising from theories of coalition building which we show does not necessarily supported in our analysis is that the size of winning majorities in distributive spaces is minimum[5]. Instead, in our analysis excess majorities may prevail in equilibrium in one of the two houses, and this likelihood varies systematically with probabilities of recognition. Lower house excess majorities occur when big states have low proposal power, while upper house excess majorities occur when small states have low proposal power.

We note that excess majorities occur even though coalitions are minimum winning in the spirit of Riker, 1962, *i.e.* equilibrium coalitions are such that if funds are removed from one state the proposal fails passage. Our result lends perspective to empirical studies of coalition size in bicameral legislatures. At a minimum, our findings imply that in empirical tests, the null hypothesis

[4]Among models in the Baron Ferejohn tradition, ours is mostly related to those in Banks and Duggan, 2000, and McCarty, 2000a,b. McCarty analyzes bargaining with more than one bodies that have veto power, while Banks and Duggan consider general agreement rules.

[5]Among theoretical studies, MWCs are also predicted in Shepsle, 1974, Auman and Kurz, 1977, Ferejohn, Fiorina, and McKelvey, 1987, Baron and Ferejohn, 1989 (closed rule), to name a few. See Melissa Collie, 1988, for a review of the empirical literature.

of minimum winning coalitions (MWCs) should be specified in a manner that is consistent with observed majorities larger than 50% in one of the two houses.

Much like the effect of mal-apportionement, most other institutional features we analyze have a non-monotonic effect on the distribution of resources between the groups of small and big states. An exception is the effect of proposal power: the *ex ante* expected payoff of each type of state increases with higher overall probabilities of recognition for that group[6]. Not only is the effect of probabilities of recognition monotonic, it is also significant. Irrespective of the remaining features of the legislative environment (bicameralism, majority requirements, apportionment, etc.), the equilibrium expected share of funds received by the group of small states spans the entire range of possibilities between zero and one by appropriate assignment of proposal rights between big and small states. This finding underscores the importance of other institutional dimensions of constitutional bargaining among states if proposal rights can be explicitly guaranteed in the constitution[7].

Finally, we analyze the effect of two additional institutional features. First, we find that an increase in the majority requirement in one of the two houses also has a non-monotonic effect, although generally it favors (does not harm) the type of states (big or small) this house over-represents. Second, we consider the effect of changes in the number of states within the groups of big and small states. We find that expansion to include more states of a particular type reduces the relative payoff of that group. For example, an increase in the number of small states reduces the probability that any one small state is included in the winning coalition. This effect may be reversed if expansion brings about a concomitant increase in the probabilities of recognition for that group. These results are particularly relevant for federations that consider expansion, as is

[6]This is consistent with the findings of Eraslan, 2001, for unicameral legislatures.

[7]For example, the presidency of the EU's Council of ministers alternates among members every six months, thus granting equal proposal power over time, while many big countries would favor an allocation of this role in proportion to population.

the case currently for the EU.

While our agenda is primarily theoretical, we also consider empirical applications of the model using data on the institutional configuration of actual legislatures. We apply a calibration procedure to the institutions of the 103rd US Congress and the EU before and after the recent Treaty of Nice. *If* the remaining features of inter-cameral bargaining follow the assumptions of our model, we find that small EU countries lose from the recent reforms decided at the Nice Summit, at least with regard to distributive legislation. Relative losses also occur for small states with EU's pending expansion, since candidate countries are predominantly small.

Before we move to our analysis, we discuss related contributions. Cremer and Palfrey, 1996, 1999, study the preferences of the populations of a number of states over patterns of centralization and representation. Unlike our distributive space, Cremer and Palfrey consider a one-dimensional ideological policy space with preference heterogeneity across and within states. They find that, unconditionally, all states prefer representation on the basis of a one-state one-vote principal, but preferences over apportionment differ conditional on the level of centralization. Small states favor representation on the basis of population if centralization is low while big states prefer population based representation if centralization is high.

In a model that differs in specification and focus from ours, Diermeier and Myerson, 1999, also study the legislative institutions of Bicameralism. They use a vote-buying model to show that presidential power and bicameral separation can encourage the legislative chamber to create internal veto players via supermajority rules. A critical difference between their model and the one we propose is that members of the two legislatures in their analysis require separate amounts (contributions from lobbies) in order to approve proposals, while in our model legislators represent the same state in both chambers and hence can be induced to vote yes if a sufficient (common) amount is allocated to their state.

Among related empirical contributions, a number of studies offer systematic evidence that the over-representation of small states in the US Senate favors small states. These studies use data from federal spending programs and compare them with outcomes that would prevail according to the benchmark of equal *per capita* allocation. Atlas et al., 1995 use biannual aggregate spending data from the 1972-1990 period, while in a series of detailed studies Lee, 1998, Lee and Oppenheimer, 1999, and Lee, 2000, extend these results using data disaggregated by program, and provide additional evidence for the micro-details of coalition building in the US Congress.

A brief outline of the paper before we proceed with our analysis. In section 2, we describe the basic assumptions regarding the institutional make-up of the two houses and legislative interaction. We characterize a stationary equilibrium of the resultant game in section 3. In section 4, we present additional equilibrium comparative statics. We present our calibration results in section 5, and conclude with section 6.

## 2. THE MODEL

**States:** There are $s > 1$ small and $b > 1$ big states. In an obvious notation, denote the type of state by $T \in \{S, B\}$. The assumption that there are only two types of states is a simplification justified by our focus on the study of conflict between big and small states and the political significance of this dimension of conflict for the emergence and prevalence of historically important legislative institutions. As our introductory quotations illustrate, our model replicates the heuristics used by actual political actors in these environments whose political motivation and decisions are often guided exactly by the crude distinction between big and small states. It is thus interesting to study this clean yet naught straightforward to analyze model without assuming heterogeneity within groups.

**Upper House:** We assume an Upper House where each small state is represented by one legislator, while big states have $c \geq 1$ legislators, *i.e.* there are a total of $N_U = s + cb$ representatives in the upper house. $M_U$ votes are required in order for a bill to be approved with $\frac{1}{2} < m_U \equiv \frac{M_U}{N_U} < 1$.

**Lower House:** Big states receive relatively higher representation in the lower house, which has a total of $N_L = s + kb$ representatives and big states receive $k > c$ each. $M_L$ votes are required for a bill to be approved, with $\frac{1}{2} < m_L \equiv \frac{M_L}{N_L} < 1$.

Note that the assumption that small states receive one representative in both the upper and lower house is only a normalization. Also, we assume $k$ and $c$ to be integer valued, but this assumption is not necessary for the interpretation of the model. Our results also hold if we assume that $k$ and $c$ reflect voting weights and take any value larger than one.

**Legislative Outcomes:** The task before this Bicameral legislature is to divide a fixed budget (a dollar) among the states. Let the set of these outcomes be denoted by $X$. Represent elements of $X$ by vectors $\mathbf{x} = (x_1, ..., x_s, x_{s+1}, ..., x_{s+b})$; the first $s$ coordinates correspond to the amount allocated to each of the small states and the remaining $b$ represent the allocation to big states. Obviously, $x_i$ are non-negative and $\sum_{i=1}^{s+b} x_i = 1$.

**Legislators:** There is a set $N$ with a total of $|N| = N_U + N_L$ legislators and generic element legislator $l$. These legislators only care about the amount of funds received by their state and discount the future by a common discount factor, $\delta$, such that $0 < \delta < 1$. Thus, legislators from the $i$-th small state in either the lower or upper house have utility function $u_i^S(\mathbf{x}, t) = \delta^{t-1} x_i$, where $t$ is the period in which a decision is reached, and similarly legislators from the $j$-th big state have utility $u_j^B(\mathbf{x}, t) = \delta^{t-1} x_{s+j}$. Thus, our analysis pertains to situations when allocation of funds takes place at the state level (e.g. EU allocations, US highway appropriations, etc.). Our analysis also applies to cases when allocation takes place at a lower level of geographic aggregation, but due to electoral incentives (closed list PR at the state level) or the nature of the allocated projects, representatives care about the sum of funds allocated to their state and not specifically about funds allocated within a particular region within their state.

**Legislative Interaction:** In each period one of the legislators is recognized to propose an allocation of funds across states. Having observed the proposal legislators in both houses vote *yes* or *no*. If the proposed allocation obtains the required majority in both houses, then it is implemented and the game ends. If the proposal fails in either house, the game moves to the next period and a legislator is chosen anew to make a proposal.

Recognition is probabilistic as in Baron and Ferejohn, 1989. Unlike their formulation, it is convenient for the purposes of this analysis to refer to the probabilities of recognition of states, not individual representatives. Specifically, in each period big states are *recognized* (meaning that any one representative from a given big state is chosen to make a proposal) with probability $p > 0$, while small states are recognized with probability $q > 0$. As a consequence, $p$ and $q$ must be such that $sq + bp = 1$. We will often refer to the overall probability, $P \equiv bp$, that big states are recognized and similarly that for small states, $Q \equiv sq$.

Important institutional features are buried under these assumptions. For instance, the reader may have noticed that we make no particular reference to the order via which each house is voting on the proposal, nor whether the proposer is drawn from the lower or the upper house. This does not matter in our analysis because probabilities of recognition are identical in each period. Thus, one interpretation of our model is that the resolution of inter-cameral conflict takes place via sequential voting and origination of proposals by the two houses as provided by (infinite round) *navette* institutions, but the probabilities of recognition of small and big states are identical in the lower and upper houses.

For other bicameral institutions we do not need the assumption that probabilities of recognition are identical in the two houses. Under one such instance the two houses convene in joint session so that votes are taken simultaneously, but a majority in each house is still required for a proposal to pass. This is the essence of interaction in a *Conference Committee* under a 'unit rule', as is provided for the resolution of disagreements between the two houses in the US Congress. A particular version of Navette institutions also fits this description, when legislation must originate from one of the two houses and there is only one round of inter-cameral negotiation.

Finally, while our assumptions are restrictive in some respects, they allow for more generality in others. Thus, as we point out in Remark 1 of section 3, our basic results extend directly to more complex institutional environments of bargaining among big and small states when it comes to requirements for approval of proposals. With those remarks on the interpretation of the model, we can proceed to the analysis of this game.

## 3. EQUILIBRIUM ANALYSIS

In this section we first develop some additional notation and define the equilibrium concept on the basis of which we solve this bicameral legislative bargaining game. We then state our equilibrium results in Proposition 1.

We focus equilibrium analysis on stationary, no-delay, *Nash* equilibria. For a justification of this equilibrium refinement see Baron and Kalai, 1993. General, existence of such equilibria is not at issue due to the results of Banks and Duggan, 2000. By no-delay we mean that equilibrium proposals always obtain majorities in both houses so that the first legislator recognized proposes an allocation that is accepted and the game ends in the first period. The stationarity assumption amounts to the restriction that players choose the same action in every structurally equivalent subgame.

A pure stationary *proposal* strategy for each legislator, $l$, is a proposal $\mathbf{z}_l \in X$, when $l$ is recognized in any given proposal period. A stationary voting strategy is an acceptance set $V_l = \{\mathbf{x} \in X \mid l \text{ votes } yes\}$. Given that legislators from the same type of state are *ex ante* identical, we further restrict our analysis to symmetric stationary equilibria. Under such equilibria, a proposal strategy for a proposer from, say, a big state is to allocate an amount $x_B$ to $\beta_B$ (out of the remaining

$(b-1)$) randomly chosen big states, an amount $x_S$ to $\sigma_T$ (out of a total of $s$) randomly chosen small states and retain $1-(\beta_B x_B + \sigma_B x_S)$ for her own state. We represent such stationary, symmetric proposal strategies for a proposer of type $T \in \{S, B\}$ by a pair of non-negative integers $(\sigma_T, \beta_T)$, which denote the number of big and small states that receive funds, respectively. For example, proposal strategy $(\sigma_B, \beta_B) = (4, 3)$ implies that legislators from big states choose 3 other big states and 4 small states and allocate positive funds $x_S$ and $x_B$ to each.

Existence of equilibrium requires mixtures of such proposals[8]. Denote such mixed, stationary, symmetric proposal strategies by $\mu_T$ and denote the probability with which proposal $(\sigma_T, \beta_T)$ is chosen under mixed strategy $\mu_T$ by $\mu_T[\sigma_T, \beta_T]$. Define the *expected number* of small states chosen under strategy $\mu_T$ by a proposer of type $T \in \{S, B\}$ as $\overline{\sigma}_T = \sum_{(\sigma_T,\beta_T)} \sigma_T \mu_T[\sigma_T, \beta_T]$[9], and similarly for $\overline{\beta}_T = \sum_{(\sigma_T,\beta_T)} \beta_T \mu_T[\sigma_T, \beta_T]$. For example, $\mu_S[3, 4] = \frac{1}{2}$ and $\mu_S[5, 3] = \frac{1}{2}$ implies that under strategy $\mu_S$ legislators from small states build coalitions with 4 big and 3 other small states 50% of the time and 3 big and 5 other small states in the remaining cases. In this example, $\overline{\sigma}_S = 4$ while $\overline{\beta}_S = 3.5$.

The *continuation value* for players of each type of state is defined as *the expected utility from the game if this happens to move in the next proposal period*. Denote this continuation value by $v_T$, $T \in \{S, B\}$. For any proposal strategies $\mu_B, \mu_S$ this continuation value can be written as:

$$\begin{aligned} v_B &= p \sum_{(\sigma_B,\beta_B)} \mu_B[\sigma_B, \beta_B](1-\sigma_B x_B - \beta_B x_B) + \\ & (P-p)\left(\sum_{(\sigma_B,\beta_B)} \mu_B[\sigma_B, \beta_B]\frac{\beta_B}{b-1}\right)x_B + Q\left(\sum_{(\sigma_S,\beta_S)} \mu_S[\sigma_S, \beta_S]\frac{\beta_S}{b}\right)x_B \end{aligned} \tag{1}$$

for legislators from big states. Factoring out terms, equation (1) reduces to:

$$v_B = p\left(1-\overline{\sigma}_B x_B - \overline{\beta}_B x_B\right) + (P-p)\frac{\overline{\beta}_B}{b-1}x_B + Q\frac{\overline{\beta}_S}{b}x_B \tag{2}$$

[8]Notice that proposal strategies of the form $(\sigma_T, \beta_T)$ already represents mixed strategies since states included in the coalition are chosen randomly from the respective group.

[9]Summation is justified since there are only a finite number of possible coalition pairs.

Similarly, the continuation value for legislators from small states can be derived as:

$$v_S = q\left(1 - \overline{\sigma}_S x_B - \overline{\beta}_S x_B\right) + P\frac{\overline{\sigma}_B}{s} x_S + (Q-q)\frac{\overline{\sigma}_S}{s-1} x_S \tag{3}$$

We can now state the definition of the equilibrium concept as a set of three conditions:

**Definition 1** *A symmetric, 'no-delay,' stationary Nash equilibrium in stage undominated voting strategies is a set of voting strategies, $V_l^*$, and proposal strategies $\mu_T^*$ with allocated amounts $x_T^*$, $T \in \{S, B\}$such that:*

$$\mu_T^*\left[\sigma_T, \beta_T\right] > 0 \Longrightarrow \tag{Condition 1}$$

$$(\sigma_T, \beta_T) \in \arg\max_{\beta_T, \sigma_T} \{1 - \beta_T x_B^* - \sigma_T x_S^*\} \text{ subject to}$$

$$\beta_T \leq \begin{cases} b-1 & if \quad T = B \\ b & if \quad T = S \end{cases} \tag{4a}$$

$$\sigma_T \leq \begin{cases} s & if \quad T = B \\ s-1 & if \quad T = S \end{cases} \tag{4b}$$

$$\sigma_T + c\beta_T \geq \begin{cases} M_U - c & if \quad T = B \\ M_U - 1 & if \quad T = S \end{cases} \tag{4c}$$

$$\sigma_T + k\beta_T \geq \begin{cases} M_L - k & if \quad T = B \\ M_L - 1 & if \quad T = S \end{cases} \tag{4d}$$

$$\sigma_T, \beta_T \geq 0 \tag{4e}$$

$$\sigma_T, \beta_T \text{ integer} \tag{4f}$$

$$x_T^* = \delta v_T \tag{Condition 2}$$

$$\mathbf{x} \in V_l^* \Longleftrightarrow \begin{cases} x_i \geq \delta v_S & if \quad l \text{ represents } i-th \text{ small state} \\ x_{s+j} \geq \delta v_B & if \quad l \text{ represents } j-th \text{ big state} \end{cases} \tag{Condition 3}$$

Equilibrium Condition 3 requires that voters vote *yes* only if the amount that is allocated to their state is no smaller than what they expect to receive in equilibrium if the game moves in the next period, *i.e.* it is equivalent to the requirement that players play only stage-undominated (Baron and Kalai, 1993) voting strategies. This restriction implies that only proposals that a winning coalition (weakly) prefer can be the policy decision along any path of play. Condition 2 requires that proposers allocate to states in their coalition exactly the bare minimum that is required to obtain a positive vote. It is straightforward and not particularly instructive to show that if Condition 1 and Condition 3 are met then proposers optimize only if they allocate the amount $x_T^*$ specified in equilibrium Condition 2. Lastly, given amounts $x_T^*$, equilibrium Condition 1 requires that the proposer must choose the number of coalition partners optimally subject to a series of constraints.

<<Insert Figure 1 about here>>

It is instructive as to the nature of the equilibrium that prevails to motivate the optimization problem in Condition 1 in a graphical manner. We do so using Figure 1. The vertical axis of Figure 1 represents the number of big states that receive funds, $\beta_B$, while the horizontal axis represents the corresponding number of small states, $\sigma_B$. The first constraints in Condition 1, equations (4c) and (4d), ensure that proposed coalitions obtain majorities in both houses. These constraints are represented with the solid grey lines in Figure 1. Note that as a result of the disproportional representation of big states in the two houses, the negative of the slope of the upper house majority constraint (equal to $\frac{1}{c}$), is larger than that of the lower house majority constraint (equal to $\frac{1}{k}$).

The two dotted lines in Figure 1 represent constraints (4a) and (4b). These constraints simply require that the maximum number of big and small states that may be part of the winning coalition cannot be larger than the number of existing big and small states (minus the proposers's state). Finally constraints (4e) and (4f)) restrict proposals on a lattice in the positive orthant. As a result of all the above constraints, only the coalitions highlighted in Figure 1 are feasible choices for the proposer.

The proposer's task then is to choose that among the feasible proposals that minimizes her 'cost':

$$C \equiv \beta_T \delta v_B + \sigma_T \delta v_S \tag{5}$$

Figure 1 depicts the cost function in (5) for two possible levels of cost $C_1$, and $C_2$. Note that the slope of these curves is given by the ratio of continuation values $w \equiv \frac{v_S}{v_B}$. It is obvious that among all curves with slope $w = \frac{v_S}{v_B}$, the one associated with cost $C_1$ minimizes the proposer's cost and, hence, the proposer optimizes by choosing the coalition represented by point $L_1$. In other words, no feasible coalition involves a smaller cost level than $C_1$.

The geometric exposition in Figure 1 suggests that the optimum of the proposer depends directly on the value of the ratio of continuation values, $w$. In particular, coalition $L_1$ is proposed for large values of $w$, while coalition $L_3$ for small values of this quantity. For values of $w$ between these extremes, the proposer's optimum coalitions (effectively) span all points in the line segment defined by points $L_1$, $L_2$, and $L_3$, with convex combinations between consecutive pairs of these points attained by mixing between coalitions $L_1$, $L_2$, and $L_2$, $L_3$.

But, in turn, the ratio of continuation values, $w$, depends on which one(s) of the feasible coalitions are chosen, as is obvious by inspection of equations (2) and (3). Thus, an equilibrium in this game reduces to the following 'fixed point' calculation: the optimal proposals from legislators of both big and small states must determine a value for the ratio of continuation values, $w$, such that the optimality of these proposals is preserved *i.e.*, given $w = \frac{v_S}{v_B}$, all pairs $(\sigma_T, \beta_T)$ chosen with positive probability constitute optima for the optimization problem of the proposer in Condition 1.

In what follows, we show that such a fixed point exists for all values of the probability of recognition of big states $P$. Furthermore, both the incidence of oversized coalitions and the distributional outcome between small and big states change in a systematic way with this parameter.

In order to state these results we need a last definition. Specifically we define proposals that induce an *excess majority* as follows:

**Definition 2** *A proposal induces an* excess majority *in the upper (lower) house if a different proposal that is otherwise identical except for the fact that funds are removed from a single state obtains majority approval in the upper (lower) house.*

We can now state the following equilibrium result:

**Proposition 1** *There exists a symmetric, 'no-delay,' stationary Nash equilibrium in stage-undominated voting strategies for all values of the overall probability of recognition of big states, $P$, in the interval $(0,1)$. It is such that:*

1. *The expected number of small, $\overline{\sigma}_T$, (big, $\overline{\beta}_T$) states included in the coalition by proposers from either type of state is non-decreasing in the probability of recognition of big states, $P$.*

2. *The ratio of continuation values $w = \frac{v_S}{v_B}$ is continuous and non-increasing in the overall probability of recognition of big states, $P$, and takes all values in $(0, +\infty)$.*

3. *Each proposal $(\sigma_T, \beta_T)$ played with positive probability may induce* excess majorities *in one of the two houses, but not in both.*

4. *If $P' \in [0,1]$ is the maximum (sup) value of $P$ for which excess majorities occur in the lower house by proposers of type $T \in \{S, B\}$, then expected majority in the lower house $(\overline{\sigma}_T + k\overline{\beta}_T)$ does not decrease as $P$ decreases for $P < P'$.*

5. *If $P'' \in [0,1]$ is the minimum (inf) value of $P$ for which excess majorities occur in the upper house by proposers of type $T \in \{S, B\}$, then expected majority in the upper house $(\overline{\sigma}_T + c\overline{\beta}_T)$ does not decrease as $P$ increases for $P > P''$.*

The proof of Poposition 1 appears in the Appendix. Parts 1, 2, 4, and 5 state equilibrium comparative statics with respect to the overall probability, $P$, that big states are recognized. Part 2 states that the *ex-ante* payoff of big states changes continuously and increases (weakly) monotonically with increases in their probability of recognition $P$. Part 2 also insures that virtually all possible distributions of the dollar between small and big states are attainable in equilibrium for appropriate values of $P$, including cases when the (*ex-ante* or expected) share of funds received by each type of state gets arbitrarily close to zero. Part 1 states that this negative effect of increases in $P$ on the ratio of continuation values comes at the cost of a smaller or equal expected number of big states (and a larger or equal expected number of small states) included in the winning coalition. Thus, in equilibrium there is a trade-off between the *ex-ante* welfare of each group of states and the (expected) number of these states included in the winning coalition. These results follow directly from the diagrammatic analysis in Figure 1.

Parts 3, 4, and 5 refer to the size of winning coalitions. Part 3 asserts that coalitions that are chosen with positive probability may involve *excess majorities* in only one of the two houses. Parts 4 and 5 provide additional information as to what determines the incidence of excess majorities in each House. In particular if oversized majorities occur in the lower (upper) house, then the expected number of votes in favor of equilibrium proposals for each type of proposer increase as the probability of recognition of big states, $P$, increases (decreases). Oversized coalitions occur when *proposal making tends to be dominated by one of the two types of states.* Also, *oversized majorities that occur when one type of state has less access to proposal making are encountered in the House this state is over-represented*: Lower House excess majorities tend to occur when big states have small overall recognition probability (low $P$), and Upper House oversized majorities tend to occur when small states have small recognition probability (high $P$).

We note that the characterized equilibrium (and hence many of the above findings) are consistent with additional restrictions as to the number of big and/or small states required for approval of a proposal. Indeed, the proof of proposition 1 allows a more general result:

**Remark 1** *The equilibrium constructed in the proof of proposition 1 and parts 1 and 2 of that proposition hold if we impose arbitrary additional constraints on the combinations of* numbers *of small and big states that are necessary for approval of proposals. It suffices to specify arbitrary, common, non-empty set of winning coalitions of the form* $(\sigma_T, \beta_T)$ *for each type of proposer* $T \in \{S, B\}$, *not just the feasible coalitions determined by constraints (4a) to (4f).*

Remark 1 is useful when we consider the institutions of the EU, among others. As specified in the Treaty of Amsterdam (article 251) one requirement on winning coalition in addition to those we have introduced in this model is that at least ten member-countries approve a proposal in the Council of Ministers. The equilibrium with this and similar additional requirements on winning coalitions can be calculated with the algorithm in the proof of Proposition 1, a fact we exploit in the calculations of section 5.

## 4. REQUIRED MAJORITIES, APPORTIONMENT, & EXPANSION

In the last subsection we characterized a stationary equilibrium and analyzed the effect of changes in the overall probability of recognition of big and small states on the composition of winning coalitions and the distribution of resources between big and small states. In this section we consider other institutional features that may influence the distributional outcome between the two types of states. Specifically, we analyze the effect of changes in the majority requirements in the two houses, $M_L$ and $M_U$, as well as the pattern of over- or under-representation of small and/or big states. The latter feature of Bicameral legislatures we capture in the model by the number of big state representatives in the upper and lower houses, $c$ and $k$ respectively. Finally, we also consider the effect of changes in the number of small and big states $s$ and $b$ for cases when expansion to new geographical units is in order as, for instance, is currently the case for the EU.

Deciphering the effect of these parameters on the *ex-ante* distribution of resources between small and big states is more involved. One difficulty is substantive. Unlike the case with probabilities of recognition, the effect of changes in these parameters is not monotonic. This makes for some counter-intuitive findings, which we discuss in due course. Preceding this difficulty, though, is a technical one. Since the equilibrium involves optimization via integer programming and many of these parameters are integer valued, standard calculus arguments are not directly applicable. While we were able to overcome this difficulty when analyzing comparative statics with regard to probabilities of recognition, $P$,and excess majorities, a similar direct analytic approach becomes extremely cumbersome for most of the remaining parameters we consider.

Thus, we offer two alternatives to analytic results. First, we calculate comparative statics for the continuous version of the model, *i.e.* ignoring the restriction to integer solutions for the proposers' optimization problem. As can be verified by inspection of Figure 1, the solutions of the linear programming problem of the proposer with and without the integer restriction are very similar. Thus, with the obvious caveat that these results do not constitute exact comparative statics, we assume that the continuous approximations capture the effect of these parameters, especially for large legislatures[10].

A second alternative, which we also pursue, is numerical simulation. The proof of Proposition 1 provides an algorithm for the construction of the equilibrium on the basis of which we can perform exact equilibrium calculations for any configuration of the model's parameters. Results of such numerical calculations have the inherent disadvantage that they are only valid for the specific parameter values used. Of the (large) number of possible calculations we choose to report empirically relevant ones that involve parameter values calibrated to emulate actual legislatures.

We won't present these simulations before section 5. In the remainder of this section, we first discuss the effect of changes in the majority requirement in the two Houses. Then we analyze

[10]See Samualson, 1983, page 447, for a related argument in decision-theoretic contexts.

changes in the degree of over(under)-representation of big states. Lastly we analyze the effects of the expansion of the legislature to incorporate more big or small sates.

**Majority Requirements:** With regard to the effect of changes in the majority requirement in the two houses, we can state the following[11]:

**Proposition 2** *An increase in the majority requirement in the upper house, $M_U$, (lower house, $M_L$) has no effect on the relative welfare of big and small states if an excess majority prevails in that house or when mixed proposal strategies are used. In the remaining cases, an increase in $M_U$ ($M_L$) increases the welfare of small states if $0 < \sigma_T < s$ ($0 < \beta_T < b$) and has a negative effect if $\sigma_S = \sigma_B = 0$ or $\sigma_S + 1 = \sigma_B = s$, ($\beta_S = \beta_B = 0$ or $\beta_S = \beta_B + 1 = b$), ceteris paribus.*

Obviously, (small) changes in the majority requirement in the upper or the lower house do not change the equilibrium in cases when the corresponding majority constraint (equations (4c) and (4d)) is not satisfied with equality. In those cases feasible coalitions remain unaltered in the relevant portion of the feasible set. This 'no effect' result also holds whenever the proposers use mixed proposal strategies; in those cases it is the mixture probabilities that are affected but not the relative welfare of the two types of states. In the remaining cases, increasing the majority requirements in any one house increases the relative welfare of the state this house over-represents, *unless all or none of these states are included in the equilibrium coalition.* For example, an increase in the majority requirement in the upper house increases the expected payoff of small states, except if all small states or none of the small states (besides the proposer) are part of winning coalitions in equilibrium. In the latter cases the additional votes that are required due to the increase in the majority requirement are drawn from big states, hence it is big states that benefit.

**Over(under)-representation of Big States:** The last remark and proposition 1, part

[11]The calculations in Proposition 2 to 4 are based on the continuous approximation of the equilibrium and are available upon request.

1, suggest that the direction of the effect of changes in the models' parameters on the welfare of each group of states depends on the resultant direction of change in the number of big or small states included in the winning coalition. This finding generalizes when we analyze the effect of the over(under)-representation of big states. One would expect that the larger the representation of big states in the upper or lower house (larger $c$ or $k$ respectively), the higher their expected share of the dollar. Due to a similar mechanism to the one outlined above, this is not always true.

Before we proceed we point out that the *ceteris paribus* caveat on which such comparative statics analyses are premised must be further qualified in this case. In particular, when considering increases in the representation of big states in the lower or upper house we cannot assume that the remaining parameters of the model remain constant. Rather, concomitant changes in the majority requirement in the corresponding house or in the probability of recognition of big states (or both) are likely to occur. Probabilities of recognition certainly change with changes in representation under the benchmark assumption of Baron and Ferejohn, 1989, that proposers are drawn randomly among legislators. If for example proposers are drawn randomly from the lower house ($P = \frac{kb}{s+kb}$) an increase in $k$ also increases the probability of recognition of big states $P$.

To account for the possibility of concomitant changes in majority requirements $M_U$, $M_L$, and probabilities of recognition , $P$, we report comparative statics according to a number of assumptions. First, throughout we assume that the fraction of the majority requirement in each house remains constant, *i.e.* $m_L \equiv \frac{M_L}{s+kb}$ and $m_U \equiv \frac{M_U}{s+cb}$ do not change with changes in $k$, $c$. With regard to the probability of recognition of big states, $P$, we analyze comparative statics under four benchmark cases: (i) the case the probability of recognition of big states $P$ remains constant after such changes; (ii) the case when proposers are drawn randomly among legislators in the Lower House ($P = \frac{kb}{s+kb}$), (iii) the case proposers are drawn randomly from the Upper House ($P = \frac{cb}{s+cb}$); and finally (iv) the case the proposer is drawn with equal probability among states ($P = \frac{b}{s+b}$). The resultant comparative statics for all possible cases are summarized below:

**Proposition 3** *The effect of an increase in the representation of big states in the lower house, k, or the upper house, c, on the expected payoff of big states holding required majority fractions $m_L$ and $m_U$ constant is as follows for each possible equilibrium configuration:*

| | *c* | | *k* | |
|---|---|---|---|---|
| *Equilibirum with* | *P constant*‡ | $P = \frac{cb}{s+cb}$ | *P constant*† | $P = \frac{kb}{s+kb}$ |
| $\beta_S = \beta_B + 1 = b$ *or* $\sigma_S + 1 = \sigma_B = s$* | + | + | 0 | + |
| $\beta_S = \beta_B = 0$ *or* $\sigma_S = \sigma_B = 0$* | - | +/- | 0 | + |
| *Mixing** | + | + | 0 | 0 |
| *MWC in both Houses* | +/- | +/- | +/- | +/- |
| *Mixing*** | 0 | 0 | + | + |
| $\beta_S = \beta_B = 0$ *or* $\sigma_S = \sigma_B = 0$** | 0 | + | - | +/- |
| $\beta_S = \beta_B + 1 = b$ *or* $\sigma_S + 1 = \sigma_B = s$** | 0 | + | + | + |

** A MWC occurs in Upper House only. ** A MWC occurs in Lower House only.*

‡ *Includes* $P = \frac{b}{s+b}$, *or* $P = \frac{kb}{s+kb}$. † *Includes* $P = \frac{b}{s+b}$, *or* $P = \frac{cb}{s+cb}$.

In most cases the *ex ante* share of the dollar received by big states increases (or does not decrease) with increases in their representation, as expected. Cases when the effect is negative involve situations when *a minimum winning coalition prevails in the House where the increase takes place and one of two possibilities:* either $\sigma_S = \sigma_B = 0$ in which case the increased representation of big states means that fewer big states are necessary for passage of a proposal than before, or when $\beta_S = \beta_B = 0$ in which case the concomitant increase in the majority requirement implies that more small states are now included in the winning coalitions than before. In the above cases the expected payoff of big states decreases if overall probability of recognition remains constant. These counter-intuitive comparative statics are partially (or fully) offset if along with increases in $k$, $c$ there is a concomitant increase in the probability of recognition of big states, $P$. The latter is clearly the manifestation of the monotonic effect of this variable established in part 2 of Proposition

1. Finally, there are cases when the effect of increases in $c$, $k$ may go in either direction. This is when MWCs prevail in both Houses.

**Expansion:** Finally, we consider the effect of an expansion of the federation to include more small states, $s$, or more big states, $b$. As for the analysis in the previous subsection we qualify these comparative statics depending on the assumption about concomitant changes in the probability of recognition of big states $P$ and assume the ratio of majority requirements in each House constant.

**Proposition 4** *The effect of an increase in the number of small states $s$ or big states $b$, on the expected payoff of big states holding required majority fractions $m_L$ and $m_U$ constant is as follows:*

| | $s$ | | $b$ | |
|---|---|---|---|---|
| *Equilibrium with* | *Constant $P$* | *Other Cases‡* | *Constant $P$* | *Other Cases‡* |
| $\beta_S = \beta_B + 1 = b$ *or* $\sigma_S + 1 = \sigma_B = s$ | + | - | - | + |
| $\sigma_S = \sigma_B = 0$ *or* $\beta_S = \beta_B = 0$ | + | + | - | - |
| *Mixed Strategies* | *0* | *0* | *0* | *0* |
| *MWC in both Houses* | + | +/- | - | +/- |

‡ $P = \frac{b}{s+b}$, *or* $P = \frac{cb}{s+cb}$, *or* $P = \frac{kb}{s+kb}$.

The results reported in Proposition 4 are more straightforward. If the probability of recognition of big states remains constant, then *expansion harms the group of states that increase in number*. Small states lose if the expansion is to small states, and big states lose if expansion is to big states. In a nutshell, an increase in the number of big (small) states implies that the probability that any one big (small) state makes part of the winning coalition decreases; hence the *ex ante* expected payoff of the respective group decreases. The direction of the effect of expansion may change if probabilities of recognition of big (small) states are responsive to this change. This may happen if MWCs occur in both Houses or if all big or all small states are included in the winning coalition. Obviously, the reversal in the direction of the effect is in part due to the monotonic effect of probabilities of recognition established in part 2 of proportion 1.

The reader may consider the above result counter-intuitive and expect an increase in the size of a group of states to improve outcomes for that group. Underlying this intuition is the idea that the group can somehow coordinate and/or act as a single player in order to achieve collectively better outcomes. This mechanism is not at work in our analysis. It is applicable if we revoke the non-cooperative foundation of our analysis, or expand the set of admissible equilibria to include non-stationary behavior and similar strong coordination assumptions on the part of legislators from a group of states. In the current analysis we separate the forces that arise from such complex, coordinated behavior from the pure effects that emerge in an environment where such cooperative behavior is not present, and focus on studying the latter.

## 5. APPLICATIONS WITH CALIBRATED LEGISLATURES

We conclude our analysis with numerical calculations of equilibria for particular parameterizations of the model. Our investigation is both empirical and theoretical in nature. We provide results that apply to actual legislatures calibrated to fit the model's parameters. We emphasize that the obvious caveat applies that the calculations we are about to report are performed under the assumption that inter-cameral bargaining in the legislatures we calibrate is identical to bargaining in the model. Since actual legislative institutions differ in many respects from the institutions we assume, these calculations should be taken with a grain of salt. They are interesting substantively to the extend that they serve as counter-factual predictions of real world outcomes under the institutions of our model. Also, because they allow the reader to develop a sense of the range of predictions that arise from this model in familiar legislative environments.

The calculations involve two steps: first, we estimate the model's parameters ($b$, $s$, $c$, $k$, $N_L$, $N_U$, $M_L$, $M_U$) using data from the composition of actual legislatures. The details of this calibration procedure are in Appendix B. Then, we use the algorithm for the construction of the equilibrium in the proof of proposition 1 in order to calculate the equilibria[12] for these calibrated parameter values, under different assumptions about the probabilities of recognition of big states, $P$.

We apply the above procedure to the 103rd US Congress and the Council of ministers and European Parliament of the EU. The application in the case of the EU has particular importance given the institutional turmoil that characterizes it. In the current and last decade the EU has and/or will implement institutional changes in virtually all the parameters of our model. First, the EU is considering expansion; currently, a total of 12 countries have started or await initiation of formal negotiations to join the Union[13]. Second, in the recent Intergovernmental Conference at Nice the leaders of the current members of the Union took the complex task to, among other things, adjust the voting weights in the Coucnil of ministers and the representation of countries in the European Parliament to the new realities after expansion.

<<Insert Table 1 about here>>

Thus, we calibrate the model's parameters with a total of seven distinct institutional arrangements. Two for the 103rd US Congress under the assumptions of simple majority or filibuster-proof majority required in the Senate; and five for the EU including the status quo determined by the Treaty of Amsterdam, the Treaty of Nice before and after expansion, and a hypothetical institution (also before and after expansion) proposed by the Commission but not adopted at Nice known as Simple Double Majority. Both the status quo and the Treaty of Nice require a super-majority in the Council of Ministers with an additional qualification as to the composition of the winning coalition. Simple double majority provides that supporters of a successful proposal in the Coucil of

[12]We have written software that implements that algorithm, which is available upon request.

[13]On 31 March 1998, accession negotiations were started with six applicant countries - Hungary, Poland, Estonia, the Czech Republic, Slovenia and Cyprus. On 13 October 1999, the Commission recommended Member States to open negotiations with Romania, the Slovak Republic, Latvia, Lithuania, Bulgaria and Malta (source: http://europa.eu.int/).

ministers must constitute a simple majority of member states and a majority of EU's population. The calibrated parameters for these seven institutions and the exact details of the institutional provisions are reported in Table 1. Note that impressionistic political assessments of the group of big states in the EU and the results of our calibration procedure coincide.

**103rd US Congress:** We report the calculated equilibria for the US Congress in Table 2. We only report results for a discount factor $\delta = .9$, since findings do not differ markedly for lower values of this parameter. The probability of recognition of big states, $P$, ranges from 0.1 to 0.9. We report the ratio of the *ex ante* expected payoffs for the two types of states, $w$, the size of *expected* winning majorities[14] in the Senate and House of Representatives, as well as the expected number of big and small states receiving funds.

<<Insert Tables 2 and 3 about here>>

As expected from part 2 of Proposition 1 distribution becomes less favorable for small states as big states are recognized with higher probability. Yet, small states perform better than predicted under the benchmark of equal per capita allocation ($w = \frac{1}{5.04} \simeq .2$) for values of $P$ as high as 70%. In other words, the institutional configuration in the US Congress (mal-apportionement, majority requirements, etc.) is so favorable to small states so that representatives from the seven big states would have to make proposals at least 70% of the time in order to gain equal *ex ante* expected *per capita* allocations for their population. Since representatives from big states are likely to make proposals less than 70% of the time, these patterns are consistent with the finding of Atlas et al., 1995, and Lee, 1998, 2000, and Oppenheimer and Lee, 1999, that small states receive more funds *per capita*.

Also in accordance with proposition 1, we observe that excess majorities prevail in the Senate when big states are recognized with high probability, $P$, while oversized majorities occur in the House of representatives for low values. Expected majorities are as high as 73% in the Senate and

[14]Defined as $P\ \overline{\sigma}_B + c\overline{\beta}_B\ + (1-P)\ \overline{\sigma}_S + c\overline{\beta}_S$ .

67% in the House of Representatives if we assume that filibuster proof majorities are required in the Senate. Under the benchmark assumption that proposals arise from the Senate excess majorities occur in the House of Representatives while the opposite occurs under the benchmark assumption that proposals arise from the House of Representatives. Likewise, we observe a shift in the expected number of small states included in the winning coalition. As the probability of recognition of big states increases, small states become less expensive and are included in higher numbers. The expected number of small states receiving funds ranges from 21 when $P = 0.1$ to 35 when $P = 0.9$.

**European Union:** In the case of the EU our primary focus is on the distributional outcome between big and small states. The balance of power between big and small states was at the center of negotiations that led to the Treaty of Nice. We report our calculations in Table 3. Again, we report results assuming $\delta = .9$ throughout. As we have already pointed out, the institutions of the EU are somewhat more complex than those we assume in section 2, in that successful majorities in the Council of Ministers must satisfy additional constraints besides the majority constraints (4d) and (4c). This poses no problems for our analysis as we have already stated in Remark 1. It is interesting, though, to assess the consequence of these additional requirements, hence we calculate equilibria both with and without these additional constraints being in effect. Our findings can be summarized as follows.

First, under all alternative institutions we consider before and after expansion, small states receive a higher expected payoff than the one predicted under equal *per capita* allocation, unless we assume that big states completely dominate proposal making ($P$ close to .9). The second finding that stands out from these calculations is that the advantage of small states diminishes under the institutional changes adopted in Nice. Although the magnitude of this expected loss may vary depending on the parameters of the model, this finding is robust. Small states lose for virtually the entire range of values for probabilities of recognition ($P < .9$) and irrespective of whether we assume that the EU expands or not.

On the other hand, the effect of the institution of simple double majority proposed by the Commision changes significantly depending on assumptions about probabilities of recognition. If we assume that big states make proposals more than roughly 50% of the time, then big states benefit from simple double majority more than they would benefit under the institutions that prevailed in Nice. For values of $P$ below (roughly) 50%, though, simple double majority is better for small states even compared to the outcomes under the status quo provision of the Treaty of Amsterdam assuming no expansion.

Third, because expansion is disproportionately to small states[15], small member states of the EU stand to lose from expansion under both the institutions adopted in Nice or the alternative of simple double majority. We point out, of course, that this is a *relative* loss: small states receive a smaller fraction of the dollar compared to big states. The net effect of EU expansion for distributive allocations depends on the concomitant change on the size of the dollar. Relative and absolute losses coincide only if the dollar increases proportionately with expansion.

A final finding has to do with the consequences of the additional constraints imposed by the EU Treaties for decisions in the Coucil of Ministers. A glimpse at Table 3 suggests clearly that the requirement that ten member states approve decisions in the Council under the Treaty of Amsterdam favors small states. On the other hand, the requirement in the Treaty of Nice that member states approving proposals account for 62% percent of EU's population has no consequence. This threshold is too low to have an effect in equilibrium. Finally, the equivalent requirement under simple double majority that states approving proposals account for 50% of EU's population favors big countries.

[15]Of the 12 countries that have been invited for accession negotiations, only Poland is considered big. Note that calculations in Table 3 reflect concomitant changes in the representation parameters as well.

## 6. CONCLUSIONS

We analyzed bargaining for the distribution of a fixed resource among big and small states in a bicameral legislature where the two groups may be unequally represented in each house. We found that institutional features have important and sometimes unexpected consequences for a number of aspects of legislative interaction. One counter-intuitive result of our analysis is that an increase in the representation of small states may actually reduce their expected payoff.

With regard to equilibrium coalitions, we showed that excess majorities may occur in one of the two houses of the bicameral legislature. This is because voting by representatives in the two houses is correlated and, as a result, bare majorities need only occur in one of the two. Assuming that big states are under-represented in the upper house, excess majorities tend to occur in that house if big states make proposals with high probability while excess majorities tend to occur in the lower house if small states make proposals with high probability. Small states are more likely to form part of the winning coalition if they make proposals with lower probability.

Probabilities of recognition have important additional consequences for the relative welfare of big and small states. As probabilities of recognition increase, the expected payoff of the corresponding group increases. In fact, every possible *ex ante* division of the dollar between the two types of states can be achieved in equilibrium by appropriate assignment of proposal power.

Other institutional features induce effects that are not monotonic, much like the effect of mal-apportionement. An increase in the majority requirement in either House generally favors the group of states the corresponding house over-represents. The effect is reversed if equilibrium proposals involve coalitions that exclude or include all *other* states from that group besides the proposer.

Finally, if the legislature expands to represent more big (small) states then the expected relative payoff of big (small) states decreases, since any one state in that group has smaller chances of being included in the winning coalition. This effect may be reversed if the expansion increases the proposal making power of that group.

We coupled these theoretical findings with simulations for the 103rd US Congress and the legislative institutions of the EU. We calibrated the parameters of our model with the institutional design of these legislatures and calculated equilibria in accordance with our theoretical analysis. Under the model's assumptions about inter-cameral bargaining, we found that the institutions of the US Congress and the EU favor small states relative to the benchmark of equal per capita allocations. We also found that the pending expansion of the EU is to the advantage of big countries when it comes to distributive legislation. The institutional changes adopted in the Nice Summit similarly reduce the power of small states, irrespective of whether the Union expands or not. The unsuccessful institutional alternative of simple double majority proposed by the Commission would be more favorable to small EU countries only if they collectively control at least as much proposal power as big countries do.

Our theoretical findings suggest important caveats for empirical studies of the effect of legislative institutions. First, studies of the size of winning coalitions should account for the fact that the MWCs hypothesis is consistent with excess majorities in one of the houses of bicameral legislatures. Also, the non-monotonicity of the effect of changes in institutional features of the legislative environment has important implications for empirical studies if observed institutions have been chosen to serve particular goals. For example, if over-representation of small states in a bicameral legislature is a conscious choice to protect small states, then it should come as no surprise if a positive effect is estimated. Yet, theoretically informed studies must account for the possibility that outside the range of observed institutional configurations institutional features may have the opposite effects.

Lastly, we point out that while our model is rich in some respects, it is coarse in others. This enters obvious caveats for our theoretical and empirical findings. First, our conclusions for the balance of power between big and small states apply in the context of distributive politics. Our analysis is silent about ideological battles within bicameral legislatures since in these cases the outcome depends crucially on the alignment of ideological preferences among regions. Similarly, our analysis of the consequences of expansion neglects other political dimensions of expansion outside the realm of distributive politics. For example, expansion may increase the size of the budget more than enough in order to compensate for the losses of a particular group.

Furthermore, we have only considered a subset of existing Bicameral institutions. One important direction where our study merits extension involves the analysis of inter-cameral bargaining under different assumptions for the resolution of disagreements among houses. Yet another involves a more explicit analysis of the rules for agenda formation within each legislature, along the lines analyzed in Duggan, 2001.

## REFERENCES


1. Atlas, Cary M., Thomas W. Gilligan, Robert J Hendershott, and Mark A. Zupan. 1995. "Slicing the Federal Government Net Spending Pie: Who Wins, Who Loses, and Why." *American Economic Review* 85(June): 624-9.

2. Banks Jeffrey S., and John Duggan. 2000. "A Bargaining Model of Collective Choice." *American Political Science Review* 94(March): 73-88.

3. Baron, David. 1989. "A Noncooperative Theory of Legislative Coalitions." *American Journal of Political Science* 33: 1048-84.

4. Baron, David P., and John A. Ferejohn. 1989. "Bargaining in Legislatures." *American Political Science Review* 85(December): 137-64.

5. Baron, David P. and Ehud Kalai. 1993. "The Simplest Equilibrium of a Majority Rule

Game." *Journal of Economic Theory* 61: 290-301.

6. Collie Melissa P. 1988. "The Legislature and Distributive Policy Making in Formal Perspective." *Legislative Studies Quarterly* 13 (November): 427-58.

7. Cremer, Jacques and Thomas Palfrey. 1999. "Political Confederation," *American Political Science Review* 93(March): 69-83.

8. ____________________________. 1996. "In or Out?: Centralization by Majority Vote," *European Economic Review* 40(1): 43-60.

9. Diermeier, D. and R. Myerson. 1999. "Bicameralism and Its Consequences for the Internal Organization of Legislatures," *American Economic Review,* 89(5):1182-97.

10. Duggan, John. 2001. "Endogenous Amendment Agendas and Open Rule Legislatures," working paper. University of Rochester.

11. Eraslan, H. 2002. "Uniqueness of Stationary Equilibrium Payoffs in the Baron-Ferejohn Model," *Journal of Economic Theory,*103 (1): 11-30.

12. Ferejohn, John A., Morris P. Fiorina, and Richard D. McKelvey. 1987. "Sophisticated Voting and Agenda Independence in the Distributive Politics Setting." *American Journal of Political Science* 31: 169-93.

13. Lee, Frances. 1998. "Representation and Public Policy: The Consequences of Senate Apportionment for the Geographic Distribution of Federal Funds." *Journal of Politics* 60(February): 34-62.

14. ___________. 2000. "Senate Representation and Coalition Building in Distributive Politics." *American Political Science Review* 94 (March): 59-72.

15. Lee, Frances and Bruce Oppenheimer. 1999. *Sizing up the Senate: The Unequal Consequences of Equal Representation* Chicago: University of Chicago Press.

16. McCarty, Nolan.2000a. "Presidential Pork: Executive Veto Power and Distributive Politics." *American Political Science Review* 94 (March): 117-130.

17. ____________. 2000b. "Proposal Rights, Veto rights, and Political Bargaining." *American Journal of Political Science* 44 (3): 506-522.

18. Riker, W. H. 1962. *The Theory of Political Coalitions*. New Haven: Yale University Press.

19. Romer, Thomas and Howard Rosenthal. 1978. "Political Resource Allocation, Controlled Agendas, and the Status Quo," *Public Choice* 33: 27-44.

20. Samuelson, A. Paul. 1983. *Foundations of Economic Analysis*. Cambridge: Harvard University Press.

21. Shepsle, K. 1974. "On the Size of Winning Coalitions." *American Political Science Review* 68(June): 505-18.

22. Tsebelis, George and Jeannette Money. 1997. *Bicameralism*. Cambridge: Cambridge University Press.

## APPENDIX A: EQUILIBRIUM ANALYSIS

First, equilibrium Condition 2 in conjunction with (2), (3), and $P = bp = 1 - Q$ allows us to solve for the continuation values:

$$v_B = \frac{P\left(s - \delta\overline{\sigma}_B\right)}{\left(sb - sQ\delta\overline{\beta}_S - bP\delta\overline{\sigma}_B\right)} \tag{6a}$$

$$v_S = \frac{Q\left(b - \delta\overline{\beta}_S\right)}{\left(sb - sQ\delta\overline{\beta}_S - bP\delta\overline{\sigma}_B\right)} \tag{6b}$$

On the basis of the above we obtain the ratio of continuation values $w = \frac{v_S}{v_B}$ as:

$$w = \frac{v_S}{v_B} = \frac{1-P}{P} \frac{b - \delta \overline{\beta}_S}{s - \delta \overline{\sigma}_B} \tag{7}$$

We now state some non-equilibrium comparative statics for this ratio of continuation values:

**Lemma 1** *For any given expected coalition, $w$ is a continuous function of $P \in (0,1)$ and as $P$ increases, $w$ decreases.*

**Proof.** From equation (7) note that $\frac{b-\delta\overline{\beta}_S}{s-\delta\overline{\sigma}_B} > 0$, because of constraints (4a) and (4b) and the fact that $0 < \delta < 1$. Hence $\frac{\partial w}{\partial P} = -\frac{1}{P^2} \frac{b-\delta\overline{\beta}_S}{s-\delta\overline{\sigma}_B} < 0$. ■

**Lemma 2** $\lim\limits_{P \to 0} w = +\infty$ *and* $\lim\limits_{P \to 1} w = 0$.

**Proof.** We have $\lim\limits_{P \to 0} \frac{1-P}{P} = +\infty$, $\lim\limits_{P \to 1} \frac{1-P}{P} = 0$, and $\frac{b-\delta\overline{\beta}_S}{s-\delta\overline{\sigma}_B} > 0$ is bounded from above – by constraints (4a), (4b), and (4e) and the fact that $0 < \delta < 1$. ■

We can now prove Proposition 1.

**Proof of Proposition 1.** By construction (the reader is referred to Figure 1 for a visual aid in the analysis to follow). Let $F_T$ be the set of coalitions that satisfy constraints (4a) to (4f) for proposer of type $T$. This set is non-empty since unanimous coalitions are always an element of this set. Let $H(F_T)$ be the convex hull of $F_T$ and $\overline{H(F_T)}$ the boundary of $H(F_T)$. Define $K_T = \{(\sigma_T, \beta_T) \mid \beta_T = \arg\min \{x \mid (\sigma_T, x) \in F_T\}, \sigma_T = \arg\min \{y \mid (y, \beta_T) \in F_T\}\}$, *i.e.* the set of points that have minimum abscissa (ordinate) among all points with the same ordinate (abscissa), and let $O_T = K_T \cap \overline{H(F_T)}$, which is clearly non-empty. It follows that for distinct $(\sigma_T, \beta_T), (\sigma'_T, \beta'_T) \in O_T$ we have $\beta_T \neq \beta'_T, \sigma_T \neq \sigma'_T$; also $\beta_T > \beta'_T \Leftrightarrow \sigma_T < \sigma'_T$. Thus, we can assign an index $i = 1, ..., |O_T|$ to each element of $O_T$ and write $(\sigma^i_T, \beta^i_T)$, so that $i > j \implies \beta^i_T < \beta^j_T$ and $\sigma^i_T > \sigma^j_T$. Define the slope, $\alpha^i_T$, between consecutive – according to the above enumeration – points in $O_T$, as $\alpha^i_T \equiv \frac{(\beta^i_T - \beta^{i+1}_T)}{(\sigma^i_T - \sigma^{i+1}_T)} < 0, i = 1, ..., |O_T| - 1$, and set $\alpha^{|O_T|}_T \equiv 0$, $\alpha^0_T \equiv +\infty$. Obviously,

$\left|\alpha_T^0\right| > \left|\alpha_T^1\right| \geq ... \geq \left|\alpha_T^i\right| \geq ... \geq \left|\alpha_T^{|O_T|-1}\right| > \left|\alpha_T^{|O_T|}\right|$. Now inductively construct $G$ pairs of coalitions $I_g$ for proposers from big and small states respectively with $I_1 = \left[\left(\sigma_B^1, \beta_B^1\right), \left(\sigma_S^1, \beta_S^1\right)\right]$ and for $I_g = \left[\left(\sigma_B^i, \beta_B^i\right), \left(\sigma_S^j, \beta_S^j\right)\right]$, $I_{g+1} = \begin{cases} \left[\left(\sigma_B^{i+1}, \beta_B^{i+1}\right), \left(\sigma_S^j, \beta_S^j\right)\right] & if \quad \left|\alpha_B^i\right| > \left|\alpha_S^j\right| \\ \left[\left(\sigma_B^i, \beta_B^i\right), \left(\sigma_S^{j+1}, \beta_S^{j+1}\right)\right] & if \quad \left|\alpha_B^i\right| < \left|\alpha_S^j\right| \\ \left[\left(\sigma_B^{i+1}, \beta_B^{i+1}\right), \left(\sigma_S^{j+1}, \beta_S^{j+1}\right)\right] & if \quad \left|\alpha_B^i\right| = \left|\alpha_S^j\right| \end{cases}$.

For each $I_g = \left[\left(\sigma_B^i, \beta_B^i\right), \left(\sigma_S^j, \beta_S^j\right)\right]$, let $\alpha_g = \max\left\{\left|\alpha_B^i\right|, \left|\alpha_S^j\right|\right\}$, and set $\beta_S^g = \beta_S^j$, and $\sigma_B^g = \sigma_B^i$. By construction $\alpha_1 \geq ... \geq \alpha_g \geq ... \geq \alpha_{G-1} > \alpha_G = 0$, while $\beta_S^g \geq \beta_S^{g+1}$ and $\sigma_B^g \leq \sigma_B^{g+1}$ with one of the two inequalities strict. Applying equation (7) solve $w = \alpha_g$ for $P$ to get $P_g^g = \frac{d_g}{d_g+\alpha_g}$, where $d_g = \frac{b-\delta\beta_S^g}{s-\delta\sigma_B^g}$, and solve $w = \alpha_{g-1}$ to get $P_g^{g-1} = \frac{d_g}{d_g+\alpha_{g-1}}$. Since $d_g < d_{g+1}$, we have $0 \equiv P_1^0 < P_1^1 < P_2^1 \leq P_2^2 < P_3^2 \leq ... \leq P_g^g < P_{g+1}^g \leq P_{g+1}^{g+1} < ... \leq P_G^G = 1$. Finally, consider 'mixed' proposal strategies that involve proposers from big or small states choosing the corresponding element of $I_g$ with probability $\mu_g$ and that of $I_{g+1}$ with $\left(1-\mu_g\right)$, and for such proposals solve $w = \alpha_g$ for$\mu_g$:

$$\mu_g = \frac{(1-P)\left(b-\delta\beta_S^{g+1}\right) - \alpha_g P\left(s-\delta\sigma_B^{g+1}\right)}{\delta\left[(1-P)\left(\beta_S^g - \beta_S^{g+1}\right) - \alpha_g P\left(\sigma_B^g - \sigma_B^{g+1}\right)\right]} \tag{8}$$

Then the following constitute optimal equilibrium proposals for all $P \in (0,1)$: (a) For $P \in \left(P_g^{g-1}, P_g^g\right)$, $I_g = \left[\left(\sigma_B^i, \beta_B^i\right), \left(\sigma_S^j, \beta_S^j\right)\right]$ are proposed with probability 1. (b) For $P \in \left[P_g^g, P_{g+1}^g\right]$, $I_g$ are proposed with probability $\mu_g$, and $I_{g+1}$ with probability $\left(1-\mu_g\right)$.

Before we show optimality, notice that the expression in (8) is a well-defined probability for $P \in \left[P_g^g, P_{g+1}^g\right]$, with $P = P_{g+1}^g \iff \mu_g = 0$ and $P = P_g^g \iff \mu_g = 1$. Also note that a proposer is indifferent between distinct $\left(\sigma_T^g, \beta_T^g\right)$ and $\left(\sigma_T^{g+1}, \beta_T^{g+1}\right)$ whenever $\beta_T^g\delta v_B^* + \sigma_T^g\delta v_S^* = \beta_T^{g+1}\delta v_B^* + \sigma_T^{g+1}\delta v_S^* \iff \frac{v_S^*}{v_B^*} = \frac{\left(\beta_T^g - \beta_T^{g+1}\right)}{\left(\sigma_T^g - \sigma_T^{g+1}\right)} \iff w = \alpha_g$, which is true by construction when proposers mix as in (b). Also by construction $\alpha_{g-1} \geq \frac{v_S^*}{v_B^*} = \alpha_g \geq \alpha_{g+1}$, when proposers mix as in (b), while Lemma 1 ensures $\alpha_{g-1} \geq \frac{v_S^*}{v_B^*} \geq \alpha_g$ for the pure strategies in (a). Now suppose these proposal strategies are not optima, when $\left(\sigma_T^g, \beta_T^g\right)$ is chosen with positive probability. Then, for the continuation values $v_B^*$, $v_S^*$ resulting from these strategies, there exists $\left(\widetilde{\sigma}_T, \widetilde{\beta}_T\right) \in F_T$ such that

$$\widetilde{\beta}_T \delta v_B^* + \widetilde{\sigma}_T \delta v_S^* < \beta_T^g \delta v_B^* + \sigma_T^g \delta v_S^* \tag{9}$$

It suffices to consider $\left(\widetilde{\sigma}_T, \widetilde{\beta}_T\right) \in K_T$ since if (9) holds for $\left(\widetilde{\sigma}_T, \widetilde{\beta}_T\right) \notin K_T$ then there exists another coalition in $K_T$ which involves an even smaller coalition building cost. There are three cases to consider:

Case 1: $\widetilde{\sigma}_T = \sigma_T^g$; then for equation (9) to hold, $\widetilde{\beta}_T < \beta_T^g$ which contradicts $\left(\sigma_T^g, \beta_T^g\right) \in K_T$.

Case 2: $\widetilde{\sigma}_T < \sigma_T^g$; then $\left(\sigma_T^g, \beta_T^g\right) \in K_T \Longrightarrow \widetilde{\beta}_T > \beta_T^g$, hence (9) implies $\left|\frac{\left(\beta_T - \beta_T^g\right)}{\left(\sigma_T - \sigma_T^g\right)}\right| < \frac{v_S^*}{v_B^*} \leq \alpha_h$, for all $h < g$, by construction. But then $\left(\sigma_T^g, \beta_T^g\right) \notin H\left(F_T\right)$ which implies $\left(\sigma_T^g, \beta_T^g\right) \notin K_T \subset F_T$, a contradiction.

Case 3: $\widetilde{\sigma}_T > \sigma_T^g$; then (9) implies $\widetilde{\beta}_T < \beta_T^g$ and $\left|\frac{\left(\beta_T - \beta_T^g\right)}{\left(\sigma_T - \sigma_T^g\right)}\right| > \frac{v_S^*}{v_B^*} \geq \alpha_h, h \geq g$. Then, $\left(\widetilde{\sigma}_T, \widetilde{\beta}_T\right) \notin H\left(F_T\right) \supset F_T$, a contradiction.

This completes the proof of existence. Now consider parts 1 to 5.

**Part 1:** Follows immediately from the ordering of proposals $(\sigma_T, \beta_T) \in O_T$.

**Part 2:** $P = P_{g+1}^g \Longleftrightarrow \mu_g = 0$ and $P = P_g^g \Longleftrightarrow \mu_g = 1$ and lemma 1 for the cases in (a) ensure continuity. Also, weak monotonicity follows from lemma 1 for any of the cases in (a), and by the fact that $w$ is constant by construction in any of the cases in (b). Finally, Lemma 2 and continuity ensure $w$ takes all values in $(0, +\infty)$.

**Part 3:** Since only $(\sigma_T, \beta_T) \in O_T$ are played with positive probability, excess majorities cannot occur in both Houses; if that were the case, then $(\sigma_T, \beta_T) \notin K_T$.

**Part 4:** let $P'$=sup $P$ *s.t. an excess majority occurs in the Lower house*; assuming $P' > 0$, for any $P < P'$, equilibrium proposals are either as in (a) or (b). In the former cases expected majority for each type of proposer is constant with changes in $P$. In the latter cases, two coalitions are played with positive probability, say $\left(\widehat{\sigma}_T, \widehat{\beta}_T\right)$ and $\left(\widetilde{\sigma}_T, \widetilde{\beta}_T\right)$, by at least one type of proposer. W.l.o.g. let $\widetilde{\beta}_T > \widehat{\beta}_T \Longrightarrow \widehat{\sigma}_T > \widetilde{\sigma}_T \Longrightarrow \widehat{\sigma}_T > 0$ which implies that the upper house majority constraint binds at $\left(\widetilde{\sigma}_T, \widetilde{\beta}_T\right)$ (else one small state can be removed from the coalition and majority passage is still possible in both houses contradicting part 3). Hence, we have $\widetilde{\beta}_T c + \widetilde{\sigma}_T \geq \widehat{\beta}_T c + \widehat{\sigma}_T \Longrightarrow (k >) \, c \geq \dfrac{(\widehat{\sigma}_T - \widetilde{\sigma}_T)}{\left(\widetilde{\beta}_T - \widehat{\beta}_T\right)} \Longrightarrow \widetilde{\beta}_T k + \widetilde{\sigma}_T > \widehat{\beta}_T k + \widehat{\sigma}_T$. The result follows from the last relationship since $\left(\widetilde{\sigma}_T, \widetilde{\beta}_T\right)$ is proposed with higher probability as $P$ decreases.

**Part 5:** *Mutatis mutandis* as in part 4. ■

## APPENDIX B: CALIBRATION PROCEDURE

In this Appendix we describe the procedure used to calculate the calibrated parameters reported in Table 1. Consider an actual Bicameral legislature with $n$ represented states and denote the population of state $i$ by $P_i$, the number of representatives of this state in the upper house by $U_i$, and that in the lower house by $L_i$. Hence, the actual size of the upper house is $N_U^a = \sum_{i=1}^n U_i$, that for the lower house is $N_L^a = \sum_{i=1}^n L_i$, with $M_U^a$, $M_L^a$ being the corresponding majority requirements. Our problem is to find values for the parameters of the model $b$,$s$,$c$,$k$,$N_L$,$N_U$, $M_L$, $M_U$ that most closely fit these data. As our 'closeness' measure, we use a least squares criterion.

Specifically, suppose for a moment we could partition the $n$ states into $\widehat{s} > 0$ small and $\widehat{b} > 0$ big $\left[\widehat{b} + \widehat{s} = n\right]$ and, without loss of generality, let states $1, 2, ..., \widehat{s}$, be small, and states $\widehat{s}+1, \widehat{s}+2, ..., \widehat{s}+\widehat{b}$ be big. The model assumes that states within each group have an *equal number of representatives*, say $U_S, U_B$ in the upper house and $L_S, L_B$ in the lower house. Imposing this restriction and applying our least squares criterion in, say, the upper house amounts to the following minimization problem:

$$\min_{\{U_S, U_B\}} \sum_{i=1}^{s} (U_i - U_S)^2 + \sum_{i=s+1}^{n} (U_i - U_B)^2 \tag{B.1}$$

i.e. to choose values for the 'typical' representation of small and big states $\widehat{U}_S, \widehat{U}_B$ in a way that minimizes the squared deviation of this choice from the actual number of representatives. This problem has the following simple solution $\widehat{U}_S = \dfrac{1}{\widehat{s}} \sum_{i=1}^{s} U_i$, and $\widehat{U}_B = \dfrac{1}{\widehat{b}} \sum_{i=s+1}^{n} U_i$.

Clearly, $\widehat{s}\widehat{U}_S + \widehat{b}\widehat{U}_B = N_U^a$. Observe, though, that the model imposes the restriction that small states have one representative each in the upper and lower houses. Incorporating that restriction amounts to setting $\widehat{N}_U = \frac{N_U^a}{U_S}$, $\widehat{M}_U = \frac{M_U^a}{U_S}$, and $\widehat{c} = \frac{U_B}{U_S}$. Similarly for the lower house we can set $\widehat{N}_L = \frac{N_L^a}{L_S}$, $\widehat{M}_L = \frac{M_L^a}{L_S}$, and $\widehat{k} = \frac{L_B}{L_S}$, where $\widehat{L}_S$, $\widehat{L}_B$ are the estimates derived from the same procedure as in the minimization program in B.1 but for the lower house data.

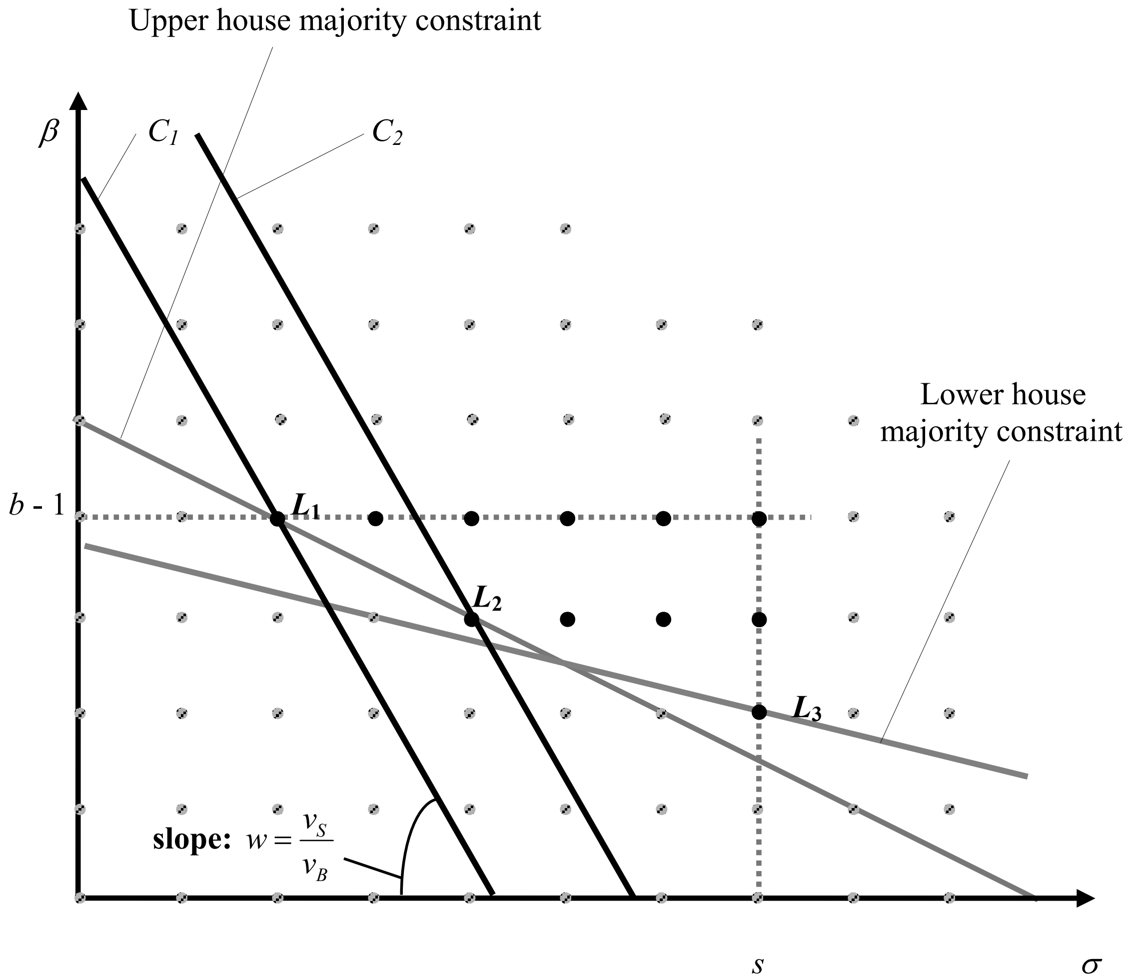
All that remains then is a criterion to determine the group of $\widehat{s}$ small and $\widehat{b}$ large states. This we can do on the basis of the population, $P_i$, of these states[16]. Again the same principal applies in that we conceive of states in each group as having the same population $\{P_S$ for small states and $P_B$ for big states) and then choose $P_S$, $P_B$ *and* separate the states into two groups in such a way so as to minimize the sum of squared deviations of actual population size from $P_S$, $P_B$ as appropriate. Specifically, if non-empty $\mathbf{S}$, $\mathbf{B}$ are sets that constitute a partition of the set of states $[\mathbf{S} \cap \mathbf{B} = \emptyset, \mathbf{S} \cup \mathbf{B} = \{1, ..., i, ..., n\}]$ we need solve the following minimization problem:

$$\min_{\{\mathbf{S}, \mathbf{B}, P_S, P_B\}} \sum_{i \in \mathbf{S}} (P_i - P_S)^2 + \sum_{i \in \mathbf{B}} (P_i - P_B)^2 \tag{B.2}$$

But this is computationaly straightforward since it is obvious that a solution to this problem must satisfy $\widehat{P}_S = \frac{1}{|\mathbf{S}|} \sum_{i \in \mathbf{S}} P_i$, $\widehat{P}_B = \frac{1}{|\mathbf{B}|} \sum_{i \in \mathbf{B}} P_i$, and $\max_{i \in S} P_i \leq \min_{i \in B} P_i$. The latter condition implies there are only $n - 1$ candidate partitions of the set of states so we can trivially calculate the optimal partition among this finite number of possible solutions.

[16]In the case of the US Congress, this procedure was applied on the data for representation in the Lower House, $L_i$, instead of the actual population of the states, $P_i$, since the two correlate highly.

**FIGURE 1: Optimal Coalitions for Proposers from Big States**




**key:** *Proposer is from big state, b = 5, s = 7,* $N_U = 17$*,* $N_L = 22$*,* $M_U = 12$*,* $M_L = 15$*, c = 2, k = 3. Curves* $C_1$*,* $C_2$ *satisfy* $\sigma_B \delta v_S + \beta_B \delta v_B = C_i$**,** *where* $C_i$ *, i = 1, 2 is the 'cost' of building a coalition with* $\sigma_B$ *small states and* $\beta_B$ *big states. Proposer aims to minimize that cost, hence optimum coalition proposal is* $L_1$*.*

**TABLE 1: Calibrated Parameters for Selected Bicameral Legislatures**

| Model Parameter | EU OF 15 | | | EXPANDED EU | | 103rd US CONGRESS | |
|---|---|---|---|---|---|---|---|
| | Amsterdam Treaty† | Nice Treaty‡ | Simple Double Majority♪ | Nice Treaty‡ | Simple Double Majority♪ | Simple Majority in Senate | Filibuster -Proof Majority in Senate |
| $s$ | 10 | 10 | 10 | 21 | 21 | 43 | 43 |
| $b$ | 5* | 5* | 5* | 6* | 6* | 7** | 7** |
| $c$ | 2.46 | 3.04 | 1.00 | 3.40 | 1.00 | 1.00 | 1.00 |
| $k$ | 4.20 | 4.29 | 4.29 | 4.58 | 4.58 | 5.04 | 5.04 |
| $M_U$ | 15.90 | 18.09 | 7.50 | 30.96 | 13.50 | 25.00 | 30.00 |
| $M_L$ | 15.50 | 15.74 | 15.74 | 24.25 | 24.25 | 39.22 | 39.22 |
| $N_U$ | 22.31 | 25.21 | 15.00 | 41.40 | 27.00 | 50.00 | 50.00 |
| $N_L$ | 30.99 | 31.47 | 31.47 | 48.49 | 48.49 | 78.26 | 78.26 |

***Sources:*** *Calculated by the author on the basis of procedure in APPENDIX B. Data on composition of legislatures reported in Congressional Quarterly for 103d Congress, and the Treaty of Amsterdam, draft of Treaty of Nice (http://europa.eu.int/) and The Economist, December 16th, 2000, for EU.*

*U: Upper house is Senate for US and Council of Ministers for EU.*

*L: Lower House is House of Representatives for US and the European Parliament for EU.*

** Big states are France, Germany, Italy, Spain, and UK, as well as Poland after expansion.*

*** Big states are CA, NY, TX, FL, PA, IL, and OH.*

*† Article 251 also requires that at least ten member-states approve a proposal in the Council of Ministers.*

*‡ Article 3.4 of Annex I of the Protocol on the enlargement of the Union, also requires that member-states approving a proposal in the Council of Ministers must account for 62% of EU's population.*

*♪ "Simple Double Majority" refers to an unsuccessful proposal by the Commission of the EU for inclusion in the Treaty of Nice. It also requires that member-states approving proposals in the Council of Ministers constitute a majority of EU's population.*

**TABLE 2: Expected Majorities and Distribution between Big and Small States for Calibrated 103d US Congress**

| *Discount Factor* $\delta$ *= 0.9* | *P* | *w†* | *Expected Majority* | | *Expected Coalition* | |
|---|---|---|---|---|---|---|
| | | | *Senate* | *House of Reps* | *Small States* | *Big States* |
| *50% Majority Required in Senate* | 0.10 | 1.00 | 0.52 | **0.58** | 21.2 | 4.8 |
| | $0.14^S$ | 0.90 | 0.52 | **0.54** | 22.0 | 4.0 |
| | 0.30 | 0.34 | 0.52 | **0.54** | 22.0 | 4.0 |
| | $0.45^H$ | 0.26 | **0.56** | 0.51 | 25.0 | 3.0 |
| | 0.50 | 0.21 | **0.56** | 0.51 | 25.0 | 3.0 |
| | 0.70 | 0.20 | **0.70** | 0.51 | 33.8 | 1.2 |
| | 0.90 | 0.07 | **0.73** | 0.51 | 35.5 | 0.9 |
| *60% Majority Required in Senate* | 0.10 | 1.00 | 0.62 | **0.67** | 25.7 | 5.3 |
| | $0.14^S$ | 1.00 | 0.62 | **0.62** | 26.7 | 4.3 |
| | 0.30 | 0.56 | 0.62 | **0.55** | 28.0 | 3.0 |
| | $0.45^H$ | 0.40 | **0.64** | 0.51 | 30.0 | 2.0 |
| | 0.50 | 0.33 | **0.64** | 0.51 | 30.0 | 2.0 |
| | 0.70 | 0.20 | **0.70** | 0.51 | 33.8 | 1.2 |
| | 0.90 | 0.07 | **0.73** | 0.51 | 35.5 | 0.9 |

**Sources:** *Compiled by the author on the basis of Table 1 and Proposition 1.*
*† Ratio of expected share of the dollar received by small states over expected share received by big states. Ratio of approximately equal per capita allocations is w = 0.20.*
*P: Probability that a representative from a large state is the proposer.*
$^H$: *Proposer Randomly drawn from House of Representatives.*
$^S$: *Proposer Randomly drawn from Senate.*

**TABLE 3: Ratio of Expected Payoff of Big and Small States† for EU Institutions**

| *Discount Factor* $\delta = 0.9$ | *P* | EU OF 15 | | | EXPANDED EU | |
|---|---|---|---|---|---|---|
| | | Amsterdam Treaty | Nice Treaty | Simple Double Majority | Nice Treaty | Simple Double Majority |
| Equilibrium imposing additional majority requirement in Council of Ministers‡ | 0.1 | 0.82 | 0.62 | 1.00 | 0.49 | 1.00 |
| | 0.3 | 0.50 | 0.33 | 0.98 | 0.33 | 0.69 |
| | 0.5 | 0.50 | 0.33 | 0.42 | 0.29 | 0.30 |
| | 0.7 | 0.50 | 0.33 | 0.25 | 0.29 | 0.13 |
| | 0.9 | 0.13 | 0.13 | 0.19 | 0.17 | 0.12 |
| | $P^S$ | 0.50 | 0.33 | 0.84 | 0.33 | 1.00 |
| | $P^U$ | 0.50 | 0.33 | 0.84 | 0.29 | 1.00 |
| | $P^L$ | 0.50 | 0.33 | 0.25 | 0.29 | 0.23 |
| Equilibrium without additional majority requirement in Council of Ministers‡ | 0.1 | 0.70 | 0.62 | 1.00 | 0.49 | 1.00 |
| | 0.3 | 0.40 | 0.33 | 0.98 | 0.33 | 0.69 |
| | 0.5 | 0.40 | 0.33 | 0.42 | 0.29 | 0.30 |
| | 0.7 | 0.40 | 0.33 | 0.33 | 0.29 | 0.22 |
| | 0.9 | 0.13 | 0.13 | 0.13 | 0.17 | 0.19 |
| | $P^S$ | 0.40 | 0.33 | 0.84 | 0.33 | 1.00 |
| | $P^U$ | 0.40 | 0.33 | 0.84 | 0.29 | 1.00 |
| | $P^L$ | 0.40 | 0.33 | 0.33 | 0.29 | 0.23 |

**Sources:** *Compiled by the author on the basis of Table 1 and Proposition 1.*
*† Ratio of expected share of the dollar received by small states over expected share received by big states. Ratio of approximately equal per capita allocations is w = 0.13 before expansion and w = 0.12 after expansion.*
*P: Probability that proposal arises from a big member state.*
$^S$*: Each state has equal probability to make proposals.*
$^U$*: Probability states make proposal determined by weight in the Council of Ministers.*
$^L$*: Probability states make proposal determined by weight (representation) in the European Parliament.*
*‡ Article 251 of the Treaty of Amsterdam requires that at least ten member-states approve a proposal in the Council of Ministers. Article 3.4 of Annex I of the Protocol on the enlargement of the Union of the Treaty of Nice also requires that member-states approving a proposal in the Council of Ministers must account for 62% of EU's population. "Simple Double Majority" requires that member-states approving proposals in the Council of Ministers constitute a majority of EU's population. The latter two constraints are imposed on the basis of a ratio of 0.13 and 0.12 of the average population of small to big states before and after expansion respectively.*