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Sensor Data Fusion

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REASONING AND OBJECT-ORIENTED DATA PROCESSING FOR MULTISENSOR DATA FUSION

Advanced information technologies provide indispensable contribution to peacekeeping and other crisis response operations. Sensor grids, flexible communications networks and Web-based services provide for early warning, increased situational awareness, shorter decision cycles and flexible use of force. Remote, or 'stand-off,' monitoring saves lives. Unattended ground, sea and air sensor systems become vital tools for sensing movement or presence of persons, vehicles, weapon platforms and armed force formations in their vicinity. Alerting peacekeepers in a timely manner, modern sensor systems are the main source of information for adequate situation and treat assessment and for rapid deployment of force.

Number of commercial-off-the-shelf sensor systems already proved their efficiency in recent peacekeeping missions. Utilizing newly developed methods and computer hardware, they provide highly intelligent information processing, saving manpower and time. Despite considerable accomplishments in the field, the explosive growth of environmental complexity and uncertainty raise demands for higher degree of automation and more 'embedded intelligence.' Current technological advances in radar, infrared, electro-optical and laser sensors are paralleled by developments in image and data processing methods and systems to provide for effective monitoring.

The contest for efficient environmental sensing has focused current R&D on qualitatively new data processing methods and algorithms, thus establishing a new ground for efficient decision support, in particular for situation and threat assessment. The shift of scientific efforts in recent years towards advanced multisensor data fusion (MSDF) applications is of particular interest for the scientific community, and to the readership of '*Information & Security*.'

This issue of the journal presents latest achievements in two important and promising areas of Multisensor Data Fusion: (a) plausible and paradoxical reasoning in the context of state and parameters estimation and (b) object-oriented sensor data processing. Currently, there are two essential problems in the implementation of information processing systems. The first one stems from the lack of efficient algorithms for uncertainty management that is produced by the difficulties of automatic knowledge processing. The second one is the problem of "productivity" of developed algorithms, demonstrated as acute shortage of productive computational resources. The common understanding of the scientific community is that the first problem might be overcome by developing a new theory of reasoning based on wiser understanding of human cognitive processes. The second problem is subject of active study by two groups of innovators: (a) algorithm developers, that provide computationally efficient algorithms, and (b) designers of super computers that successfully create new computational facilities and advanced computer networks.

Presenting reasoning methods and algorithms, we have the outstanding opportunity to present the article *Foundations for a new theory of plausible and paradoxical reasoning*, kindly provided for publication by Jean Dezert from ONERA, France. Introducing the readers into the new theory, the author presents an advanced rule of combining sources of information in a very general framework, where information can be both uncertain and paradoxical. In this new theory, the rule of combination, that takes into account explicitly both conjunctions and disjunctions of assertions in the fusion process, appears to be more simple and general than the Dempster's rule of combination. Through numerous examples the author demonstrates the strong ability of this new approach to solve difficult practical problems, where the Dempster-Shafer theory usually fails.

Another work in this area is the presentation of *Fuzzy Logic Approach to Estimating Tendencies in Target Behavior*, written by Albena Tchamova and Tzvetan Semerdjiev, both from the Central Laboratory for Parallel Processing of the Bulgarian Academy of Sciences. This approach exploits the existence of attribute data that is usually available simultaneously with kinematic data. The approach is promising in real-world situations when kinematic data is not available or is not sufficient to provide right decision or/and accurate estimates. The available data is usually incomplete, inconsistent and vague, so the problem of overcoming arising uncertainties in such cases is of high importance. The objective of the provided paper is to present an approach to estimate the tendency of target behavior. The respective algorithm is presented and evaluated in detail. Fuzzy Logic principles are applied to conventional passive radar amplitude measurements. A set of fuzzy models is used to describe the tendencies of target behavior. The authors apply a noise reduction procedure and, using computer simulations, estimate the performance of the developed algorithm in the presence of noise.

Discussing the reasoning methods and algorithms, we are unavoidably touching another important area of research interest - information integration. Today, two processes are commonly recognized as general tendencies of the social development – integration of the existing information systems in one global *System of systems* and mass transition of human mental functions to computer systems and robots. Mankind’s difficulties and the need of many scientists to solve them are the main reason for evolving global processes of information integration. Only a "*shortening of distances*" based on development of information technologies will make it possible to solve the problems of the 21st century using the power of the integrated human minds for efficient reasoning. One original view on this set of problems is presented in the article *The Genetic Program: A Technocratic Hypothesis on the Paradigm of Civilization*. The proposed hypothesis for the evolution of the human civilization examines primarily information and information technologies as components of the global technological program of the universal mind. Assuming the *a priori* existence of ontological information nucleus in the genetic code, inherited by new generations, the hypothesis offers an explanation of technology, processes, and realization of a global algorithm for mastering our part of space and time, building an eternal incubator of wisdom – a colony for accumulation of knowledge and reduction of entropy in the universe. Taking into account the impact of social factors in the global models, as well as the lack of universal concept for sustainable development, we ascertain an *acute paradigmatic deficiency*. The interpretation of the hypothesis in the framework of general historic window provides global classification of the phases of technological development of humanity as a metamodel. Evolving towards the information society, the human civilization naturally advances to new ‘informational’ forms of warfare, treated under this hypothesis as paradigmatic deformations in the global relationship between humanness and violence.

These three articles presented in the first section of the issue give an excellent opportunity to obtain a common view both on the concept of modeling human reasoning and on the concept of universal mind.

The second section of the issue is devoted to latest developments in object-oriented sensor data processing. Design, implementation, and assessment of computationally efficient tracking algorithms are essential part of sensor data processing that raises many complex problems. One way to alleviate these problems is to provide the designer with an environment, facilitating the creation of different test scenarios, automating implementation of algorithms and the evaluation of their measures of performance. Such an environment is a complex software program that could be simplified by using object-oriented design and programming. Unifying data and functions that operate on the data, the overall program organization can be improved. In their contribution *Object-Oriented Environment for Assessing Tracking Algorithms*, E. Djerassi and P. Konstantinova propose a set of classes divided into three groups considering the modeling part, the processing part, and the organization of the statistical analysis to assess performance.

Following the direction of efficient tracking algorithm design, the next paper *On the Generalized Input Estimation* by V. Jilkov and X. Rong Li from the Department of Electrical Engineering in the University of New Orleans, US, presents some original assumptions. The "input estimation" (IE) is one of the competing methods for tracking maneuvering targets. The presentation aims to clarify the interrelation between the standard IE method and a recently proposed "generalized input estimation" (GIE). It is shown that the GIE can be obtained as a particular case of the conventional IE with a "constant input" and "time - varying transition matrix" of the input. This fact could be used in a straightforward manner for further optimization of existing GIE algorithms.

One original application of newly developed sensor data processing algorithms for tracking is presented in the paper *Contact Transitions Tracking During Force-Controlled Compliant Motion Using an Interacting Multiple Model Estimator*, contributed by a research team form the Katholieke Universiteit Leuven, Belgium. The work, which may be seen as spin-off of advanced defense research, addresses both monitoring of contact transitions and estimation of unknown first-order geometric parameters during force-controlled motions. A robotic system is required to move an object among a sequence of contact configurations with the environment, under partial knowledge of geometric parameters (positions and orientations) of the manipulated objects and of the environment itself. The authors consider a compliant motion task with multiple contacts, namely that of moving a cube into a corner. It is shown that by describing the contact configurations with different models and using the multiple model approach, it is possible: (a) to detect effectively current contact configuration and (b) to estimate accurately the unknown parameters. The reciprocity constraints between ideal reaction forces and velocities are used as measurement equations. An Interacting Multiple Model (IMM) estimator is implemented and its performance is evaluated based on experimental data.

The following two papers directly address the problem of computational load. L. Bojilov from CLPP-

BAS presents *An Improved Version of an Algorithm for Multiple Targets Tracking*. Some of the well-known data association rules and algorithms are changed and carried out. Performing an exhaustive set of experiments, the author shows that his algorithm provides a plausible alternative to the well-known algorithms for finding the first K-best hypotheses. Currently, the obtained result is prepared for implementation in the framework of the Multi Hypothesis Tracking approach, potentially allowing for new applications of these computationally intensive algorithms.

Similar achievement is reported in the contribution of L. Bojilov, K. Alexiev, and P. Konstantinova on *An Accelerated IMM-JPDA Algorithm for Tracking Multiple Maneuvering Targets in Clutter*. Theoretically, the most powerful approach for tracking multiple targets is known to be Multiple Hypothesis Tracking (MHT) approach. However, it leads to combinatorial explosion and computational overload. By using an algorithm for finding the K-best assignments, the MHT approach can be optimized in terms of computational load. A much simpler alternative of the MHT approach can be the Joint Probabilistic Data Association (JPDA) algorithm combined with Interacting Multiple Models (IMM) approach. Even though it is much simpler, this approach can be computationally overwhelming as well. To overcome this drawback, an algorithm due to Murty and optimized by Miller, Stone and Cox is embedded in IMM-JPDA algorithm for determining a ranked set of K-best hypotheses instead of all feasible hypotheses. The presented algorithm assures continuous maneuver detection and adequate estimation of maneuvering targets in heavy clutter. This results in a good overall target tracking performance with limited computational and memory requirements. The corresponding numerical results are presented in the article.

Specific Features of IMM Tracking Filter Design is considered in the paper provided by Iliyana Simeonova and Tzvetan Semerdjiev. As the interacting multiple model (IMM) algorithm is one of the most cost–effective and simple schemes for tracking maneuvering targets, so the knowledge of the specifics of its design is important to achieve more accurate parameter estimates. This paper presents the specifics of the IMM tracking filter design. Results, conclusions and experience of different authors have been generalized. Through this investigation the user is provided with a fast and easy way to determine advantages and the potential of different IMM structures given the target motion scenario. In addition, the behavior of three IMM configurations has been studied, using a specially developed MATLAB tool.

Today, computer simulation is an important instrument for design, analysis, and testing of complex systems, whose state and parameters cannot be easily estimated. Such simulation includes input data generation, modeling of system dynamic, and state estimation with proper result visualization. The most complex target tracking algorithms can be easily coded in Matlab environment. The Matlab language can be learnt quickly and provides high productivity for algorithm design and evaluation. This set of issues is discussed in the article *A MATLAB Tool for Development and Testing of Track Initiation and Multiple Target Tracking Algorithms*, contributed by Kiril Alexiev. The author describes a particular simulation tool for design and analysis of radar data processing systems. Its architecture and techniques are organized around the main stream of the process of algorithms generation, simulation, analysis and evaluation. It is reported that this is an effective instrument, which could be of benefit for radar data processing specialists and scientists.

For those, interested to learn more about the problems considered in this issue, we present a list of selected recent publications. Some of them present in-depth studies of the multi target tracking problem. Others contain thorough examination of latest achievements and description of particular implementations. Useful references and considerable number of papers, devoted to MSDF, is available on the Internet site of the Bulgarian Information Society Center of Excellence for Education, Science and Technology in the 21st Century. Brief information about one particular work package of this center, named "Real-time Data processing in Adaptive Sensor Interfaces," is also presented. We find these publications useful for students, specialists and PhD applicants involved in the study of MSDF. Additionally, a short list of Internet links is given for everyone who is interested in latest news.

We hope this issue will help to develop new interrelations within the MSDF research community. The common interest in solving information processing problems using MSDF technologies will provide new opportunities for fruitful cooperation and consideration of future joint R&D projects.

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FOUNDATIONS FOR A NEW THEORY OF PLAUSIBLE AND PARADOXICAL REASONING

Jean DEZERT

1. Introduction

The processing of uncertain information has always been a hot topic of research since mainly the 18th century. Up to the middle of the 20th century, most theoretical advances have been devoted to the theory of probabilities through the works of eminent mathematicians like J. Bernoulli (1713), A. De Moivre (1718), T. Bayes (1763), P. Laplace (1774), K. Gauss (1823), S. Poisson (1837), E. Borel (1909), R. Fisher (1930), A. Kolmogorov (1933), B. De Finetti (1958), L. Savage (1967), T. Fine (1973), E. Jaynes (1995) to name just few of them. With the development of computer science, the second half of the 20th century has became very prolific for the development of new original theories dealing with uncertainty and imprecise information. Mainly, three major theories are available now as alternative to the theory of probabilities for the automatic plausible reasoning in expert systems: the fuzzy set theory developed by L. Zadeh in sixties (1965), the Shafer's theory of evidence in the seventies (1976) and the theory of possibilities by D. Dubois and H. Prade in eighties (1985) and, very recently, the unifying *avant-gardiste* neutrosophy theory proposed by F. Smarandache (2000).[54,55,2] This paper is focused on the development of a new theory of plausible and paradoxical reasoning which can be interpreted as a generalization of the theory of evidence. After a brief presentation of the Dempster-Shafer theory in section 2, we set up the foundations of our new theory in section 3 and discuss the justification of the new rule of combination of uncertain and paradoxical sources of evidences. Several illustrative examples of the power and the usefulness of our new theory are also presented and compared with results drawn from the classical Dempster-Shafer theory.

2. The Dempster-Shafer Theory of Evidence

We present here briefly the basis of the Dempster-Shafer theory (DST) or the Mathematical Theory of Evidence (MTE), [47,10] sometimes also called the theory of probable or evidential reasoning. The DST is usually considered as a generalization of the

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Bayesian theory of subjective probability [52] that offers a simple and direct representation of ignorance. The DST has shown its compatibility with the classical probability theory, with boolean logic and has a feasible computational complexity [44] for problems of small dimension. The DST is a powerful theoretical tool which can be applied for the representation of incomplete knowledge, belief updating, and for combination of evidence [41,19] through the Demspter-Shafer's rule of combination presented in the following. The Dempster-Shafer model of representation and processing of uncertainty has led to a huge number of practical applications in a wide range of domains, for example for the pattern classification,[12] the integration of knowledge from heterogeneous sources for object identification and tracking,[46] autonomous navigation,[14] technical and medical diagnosis under unreliable measuring devices, information retrieval, multisensor image segmentation, network reliability computation, safety control in large plants, and map construction and maintenance, just to mention a few.

2.1. Basic Belief Masses

Let $\Theta = \{\theta_i, i = 1, \dots, n\}$ be a finite discrete set of *exhaustive* and *exclusive* elements (hypotheses) called elementary elements. Θ has been called the frame of discernment of hypotheses or universe of discourse by G. Shafer. The cardinality (number of elementary elements) of Θ is denoted $|\Theta|$. The power set $\mathcal{P}(\Theta)$ of Θ which is the set of all subsets of Θ is usually denoted by $\mathcal{P}(\Theta) = 2^\Theta$ because its cardinality is exactly $2^{|\Theta|}$. Any element of 2^Θ is then a composite event (disjunction) of the frame of discernment.

Definition 1. *The DST starts by defining a map associated to a body of evidence $\mathcal{B}$ (source of information), called basic belief assignment (bba)* [1] or information granule $m(.) : 2^\Theta \rightarrow [0, 1]$ such that

$$m(\emptyset) = 0, \tag{1}$$

$$\sum_{A \in 2^\Theta} m(A) \equiv \sum_{A \subseteq \Theta} m(A) = 1. \tag{2}$$

$m(.)$ represents the strength of some evidence provided by the source of information under consideration. Condition (1) reflects the fact that no belief ought to be committed to $\emptyset$ and condition (2) reflects the convention that one's total belief has measure one.[47] $m(A)$ corresponds to the measure of the partial belief that is committed *exactly* to A (degree of truth supported exactly by A) by the body of evidence $\mathcal{B}$ but not the total belief committed to A. All subsets A for which $m(A) > 0$ are called focal elements of m. The set of all focal elements of $m(.)$ is called the core $\mathcal{K}(m)$ of m. Note that $m(A_1)$ and $m(A_2)$ can be both equal to zero even if $m(A_1 \cup A_2) \neq 0$. Even more

[1] This terminology suggested by Professor Philippe Smets to the author appears to be less confusing than the basic probability assignment terminology (bpa) originally adopted by Glenn Shafer.

peculiar, note that $A \subset B \nRightarrow m(A) < m(B)$ (i.e. $m(.)$ is not monotone to inclusion). Hence, the bba $m(.)$ is in general different from a probability distribution $p(.)$.

Example 1. Consider $\Theta = \{\theta_1, \theta_2, \theta_3\}$, then $2^\Theta = \{\emptyset, \theta_1, \theta_2, \theta_3, \theta_1 \cup \theta_2, \theta_1 \cup \theta_3, \theta_2 \cup \theta_3, \theta_1 \cup \theta_2 \cup \theta_3\}$. An information granule $m(.)$ on this frame of discernment Θ could be defined as

$$\begin{aligned} m(\emptyset) &\triangleq 0 & m(\theta_1 \cup \theta_2 \cup \theta_3) &= 0.05 \\ m(\theta_1) &= 0.40 & m(\theta_1 \cup \theta_2) &= 0.10 \\ m(\theta_2) &= 0.20 & m(\theta_2 \cup \theta_3) &= 0.10 \\ m(\theta_3) &= 0.05 & m(\theta_1 \cup \theta_3) &= 0.10 \end{aligned}$$

In this particular example $\mathcal{K}(m) = \{\theta_1, \theta_2, \theta_3, \theta_1 \cup \theta_2, \theta_1 \cup \theta_3, \theta_2 \cup \theta_3, \theta_1 \cup \theta_2 \cup \theta_3\}$ and note that $\theta_1 \subset \{\theta_1 \cup \theta_2\}$ with $m(\theta_1) > m(\theta_1 \cup \theta_2)$.

2.2. Belief Functions

Definition 2. *To measure the total belief committed to $A \in 2^\Theta$, Glenn Shafer has defined the belief (or credibility) function Bel$(.) : 2^\Theta \rightarrow [0,1]$ associated with bba $m(.)$ as*

$$\text{Bel}(A) = \sum_{B \subseteq A} m(B). \tag{3}$$

Bel(A) summarizes all our reasons to believe in A (i.e. the lower probability to believe in A). More generally, a belief function Bel$(.)$ can be characterized without reference to the information granule $m(.)$ if Bel$(.)$ satisfies the following three conditions $\forall n > 0, \forall A_1, \ldots, A_n \subset \Theta$,

$$\text{Bel}(\Theta) = 1, \tag{4}$$

$$\text{Bel}(\emptyset) = 0, \tag{5}$$

$$\text{Bel}(A_1 \cup \ldots \cup A_n) \geq \sum_{\substack{I \subset \{1,\ldots,n\} \\ I \neq \emptyset}} (-1)^{|I|+1} \text{Bel}(\bigcap_{i \in I} A_i). \tag{6}$$

For any given belief function Bel$(.)$, one can always associate an unique information granule $m(.)$, called the Möbius inverse of the belief function,[42] and defined by [47]

$$\forall A \subseteq \Theta, \qquad m(A) = \sum_{B \subseteq A} (-1)^{|A-B|} \text{Bel}(B). \tag{7}$$

Definition 3. *The vacuous belief function having Bel$(\Theta) = 1$ but Bel$(A) = 0$ for all $A \neq \Theta$ describes the full ignorance on the frame of discernment Θ. The corresponding bba $m_v(.)$ is such that $m_v(\Theta) = 1$ and $m_v(A) = 0$ for all $A \neq \Theta$.*

Proposition 1. *For any given belief function Bel$(.)$ defined on Θ, one has*

$$\forall A, B \subseteq \Theta,\ \max(0, \mathrm{Bel}(A) + \mathrm{Bel}(B) - 1) \leq \mathrm{Bel}(A \cap B) \leq \min(\mathrm{Bel}(A), \mathrm{Bel}(B))$$

Definition 4. *Any belief function satisfying Bel$(\emptyset) = 0$, Bel$(\Theta) = 1$ and Bel$(A \cup B) =$ Bel(A) + Bel(B) whenever $A, B \subset \Theta$ and $A \cap B = \emptyset$ is called a bayesian belief function.*

In this case, (6) coincides exactly with the well-known Poincaré's equality

$$P\{A_1 \cup \ldots \cup A_n\} = \sum_{\substack{I \subset \{1,\ldots,n\} \\ I \neq \emptyset}} (-1)^{|I|+1} P\{\bigcap_{i \in I} A_i\}. \tag{8}$$

Proposition 2. *If Bel$(.)$ is a bayesian belief function, then all focal elements are only single points of $\mathcal{P}(\Theta)$. The basic belief assignement $m(.)$ commits a positive number $m(\theta_i)$ only to some elementary $\theta_i \in \Theta$ (possibly all θ_i) and zero to all possible disjunctions of $\theta_1, \ldots, \theta_n$. In other words, there exists a bayesian bba $m(.) : \Theta \to [0, 1]$ such that*

$$\sum_{\theta_i \in \Theta} m(\theta_i) = 1 \qquad \text{and} \qquad \forall A \subseteq \Theta, \quad \mathrm{Bel}(A) = \sum_{\theta_i \in A} m(\theta_i). \tag{9}$$

2.3. Plausibility Functions

Since the degree of belief Bel(A) does not reveal to what extent one believes its negation A^c, G. Shafer has introduced the degree of doubt of A as the total belief of A^c. The degree of doubt is less useful than the plausibility Pl(A) of A which measures the total belief mass that can move into A (interpreted sometimes as the *upper probability* of A).

Definition 5. *More precisely, the plausibility Pl(A) of any assertion $A \subset 2^\Theta$ is defined by*

$$\mathrm{Pl}(A) \triangleq 1 - \mathrm{Bel}(A^c) = \sum_{B \subseteq \Theta} m(B) - \sum_{B \subseteq A^c} m(B) = \sum_{B \cap A \neq \emptyset} m(B). \tag{10}$$

The dual of (6) implies $\forall n > 0, \forall A_1, \ldots, A_n \subset \Theta$,

$$\mathrm{Pl}(A_1 \cap \ldots \cap A_n) \leq \sum_{\substack{I \subset \{1,\ldots,n\} \\ I \neq \emptyset}} (-1)^{|I|+1} \mathrm{Pl}(\bigcup_{i \in I} A_i). \tag{11}$$

The comparison of (3) with (10) indicates that $\forall A \subseteq \Theta$, $\text{Bel}(A) \leq \text{Pl}(A)$.

Proposition 3. *For any given plausibility function Pl$(.)$ defined on frame of discernment Θ, the following inequality holds* [47]

$$\forall A, B \subseteq \Theta,\ \mathit{max}(\mathit{Pl}(A), \mathit{Pl}(B)) \leq \mathit{Pl}(A \cup B) \leq \mathit{min}(1, \mathit{Pl}(A) + \mathit{Pl}(B)). \qquad (12)$$

Let Θ be a given frame of discernment and $m(.)$ a general bba (neither a vacuous bba, nor a bayesian bba) provided by a body of evidence, then it is always possible to build the following *pignistic* [2] probability [56,62] (bayesian belief function) by choosing

$\forall \theta_i \in \Theta, P\{\theta_i\} = \sum_{B \subseteq \Theta | \theta_i \in B} \frac{1}{|B|} m(B)$.

In such case, one always has

$$\forall A \subseteq \Theta, \qquad \text{Bel}(A) \quad \leq \quad [P(A) = \sum_{\theta_i \in A} P\{\theta_i\}] \quad \leq \quad \text{Pl}(A). \qquad (13)$$

Since $\text{Bel}(A)$ summarizes all our reasons to believe in A and $\text{Pl}(A)$ expresses how much we should believe in A if all currently unknown were to support A, the true belief in A is somewhere in the interval $[\text{Bel}(A), \text{Pl}(A)]$. Now suppose that the true value of a parameter under consideration is known with some uncertainty $[\text{Bel}(A), \text{Pl}(A)] \subseteq [0, 1]$, then its corresponding bba $m(A)$ can always be constructed by choosing

$$m(A) = \text{Bel}(A), \qquad m(A \cup A^c) = \text{Pl}(A) - \text{Bel}(A), \qquad m(A^c) = 1 - \text{Pl}(A).$$

2.4. The Dempster's Rule of Combination

Glenn Shafer has proposed the Dempster's rule of combination (orthogonal summation), symbolized by the operator $\oplus$, to combine two so-called distinct bodies of evidences $\mathcal{B}_1$ and $\mathcal{B}_2$ over the same frame of discernment Θ. Let $\text{Bel}_1(.)$ and $\text{Bel}_2(.)$ be two belief functions over the same frame of discernment Θ and $m_1(.)$ and $m_2(.)$ their corresponding bba masses. The combined global belief function $\text{Bel}(.) = \text{Bel}_1(.) \oplus \text{Bel}_2(.)$ is obtained from the combination of the information granules $m_1(.)$ and $m_2(.)$ as follows: $m(\emptyset) = 0$ and for any $C \neq \emptyset$ and $C \subseteq \Theta$,

$$\begin{aligned} m(C) \triangleq [m_1 \oplus m_2](C) &= \frac{\sum_{A \cap B = C} m_1(A) m_2(B)}{\sum_{A \cap B \neq \emptyset} m_1(A) m_2(B)} \\ &= \frac{\sum_{A \cap B = C} m_1(A) m_2(B)}{1 - \sum_{A \cap B = \emptyset} m_1(A) m_2(B)}. \qquad (14) \end{aligned}$$

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[2] We adopt here the historical definition of the pignistic probability coined by P. Smets, but in the meantime proposed independently.[13] New pignistic probabilities have recently been proposed by J. Sudano.[65,66]

$\sum_{A\cap B=C}$ represents the sum over all $A, B \subseteq \Theta$ such that $A \cap B = C$ (the interpretation for other summation notations follows directly by analogy). The orthogonal sum $m(.)$ is a proper bba if $K \triangleq 1 - k = 1 - \sum_{A\cap B=\emptyset} m_1(A)m_2(B) \neq 0$. If $K = 0$, which means $\sum_{A\cap B=\emptyset} m_1(A)m_2(B) = 1$ then the orthogonal sum $m(.)$ *does not exist* and the bodies of evidences $\mathcal{B}_1$ and $\mathcal{B}_2$ are said to be totally (flatly) contradictory or in *full contradiction*. Such case arises whenever the cores of $\text{Bel}_1(.)$ and $\text{Bel}_2(.)$ are disjoint or, equivalently, when there exists $A \subset \Theta$ such that $\text{Bel}_1(A) = 1$ and $\text{Bel}_2(A^c) = 1$. The same problem of existence has already been pointed out in the presentation of the optimal bayesian fusion rule.[15] The quantity $\log 1/K$ is called the *weight of conflict* between the bodies of evidences $\mathcal{B}_1$ and $\mathcal{B}_2$. It is easy to show that the Dempster's rule of combination is commutative ($m_1 \oplus m_2 = m_2 \oplus m_1$) and associative ($[m_1 \oplus m_2] \oplus m_3 = m_1 \oplus [m_2 \oplus m_3]$). The vacuous belief function such that $m_v(\Theta) = 1$ and $m_v(A) = 0$ for $A \neq \Theta$ is the identity element for $\oplus$ fusion operator, i.e. $m_v \oplus m = m \oplus m_v \equiv m$. If $\text{Bel}_1(.)$ and $\text{Bel}_2(.)$ are two combinable belief functions and if $\text{Bel}_1(.)$ is bayesian, then $\text{Bel}_1 \oplus \text{Bel}_2$ is a bayesian belief function.

This *ad hoc* rule of combination proposed by G. Shafer [47] (see also the discussion [49]) has been strongly criticized in the past decades but is now accepted since the axiomatic of the transferable belief model developed by Smets [57,18,27,60,61] from an idea initiated by Cheng and Kashyap.[5] Another approach for the justification of Dempster's rule of combination based on the Mathematical Theory of Hint (MTH) has been also proposed by Kohlas.[32] Justifications and interpretations of the DST and the Dempster's rule of have been discussed at length.[17,30,31,33,41,43,69] An interesting discussion on the justification of Dempster's rule of combination from the information entropy viewpoint based on the measurement projection and balance principles can be found in [67]. Connection of the DST with the fuzzy set theory is available in [3,63] and the relationship between foundations of the fuzzy set theory and the probability theory is discussed in [8]. The relationship between experimental observations and the DST belief functions is currently a hot topic of research. Several models have been developed for fitting belief functions with experimental data. A very recent detailed presentation and discussion on this problem is also available.[70]

In the bayesian framework, if we consider M independent sources of information (bodies of evidence) $\mathcal{B}_1, \ldots, \mathcal{B}_M$ providing M *subjective* probability functions $P_1\{.\}, \ldots, P_M\{.\}$ over the same space Θ, then the optimal bayesian fusion rule is given by (see [15] for a general and theoretical justification)

$$P_{1,\ldots,M}\{\theta_i\} \triangleq [P_1 \oplus \ldots \oplus P_M]\{\theta_i\} = \frac{p_i^{1-M} \prod_{m=1,M} P_m\{\theta_i\}}{\sum_{i=1,n} p_i^{1-M} \prod_{m=1,M} P_m\{\theta_i\},} \tag{15}$$

where p_i is the prior probability of θ_i. It is easy to check (when the fusion rule is numerically well defined) that this optimal rule of combinations reduces to

$$P_{1,\ldots,M}\{\theta_i\} = [P_1 \oplus \ldots \oplus P_M]\{\theta_i\} = \frac{\prod_{m=1,M} P_m\{\theta_i\}}{\sum_{i=1,n}\prod_{m=1,M} P_m\{\theta_i\}}, \qquad (16)$$

if we admit the principle of indifference (by setting all $p_i = 1/n$).

In the last case, one can see a strong similarity between the Dempster's rule and the optimal bayesian fusion rule. Actually, the classical bayesian inference $P\{A \mid B\} = P\{B \mid A\}P\{A\}/P\{B\}$ can be interpreted as a special case of bayesian rule of combination (16) between two sources of information (between prior and posterior information). The Dempster's and Bayes' fusion rules coincide exactly when $m_1(.)$ and $m_2(.)$ become bayesian basic probability mass assignments and if we accept the principle of indifference within the optimal bayesian fusion rule.

The complexity of DS rule of combination is important in general (when we deal with large frames of discernment) since the computational burden for finding all pairs A and B of subsets of Θ such that $A \cap B = C$ is $o(2^{|\Theta|-|C|} \times 2^{|\Theta|-|C|})$ becomes a huge number. For example, if $|\Theta| = 10$ and $|C| = 2$, we will have to perform $o(2^{16}) = o(65536)$ tests to find $\{A \cap B | A \cap B = C\}$. Fortunately, there exists a fast Móbius transform which allows an efficient implementation of DS rule of combination [25,26] to deal with problems of high dimension.

Example 2. A simple example of the Dempster's rule of combination

Consider the simple frame of discernment $\Theta = \{S(\text{unny}), R(\text{ainy})\}$ about the true nature of the weather at a given location L for the next day and let consider two independent bodies of evidence $\mathcal{B}_1$ and $\mathcal{B}_2$ providing the following weather forecasts at L

$$m_1(S) = 0.80 \qquad m_1(R) = 0.12 \qquad m_1(S \cup R) = 0.08$$

$$m_2(S) = 0.90 \qquad m_2(R) = 0.02 \qquad m_2(S \cup R) = 0.08$$

The Dempster's rule yields the following result (where $K = 1 - 0.108 - 0.016$)

$$\begin{aligned} m(S) &= (m_1 \oplus m_2)(S) = (0.72 + 0.072 + 0.064)/K \approx 0.977 \\ m(R) &= (m_1 \oplus m_2)(R) = (0.0024 + 0.0096 + 0.0016)/K \approx 0.016 \\ m(S \cup R) &= (m_1 \oplus m_2)(S \cup R) = 0.0064/K \approx 0.007 \end{aligned}$$

Hence, in this example, the fusion of the two sources of evidence reinforces the belief that tomorrow will be a sunny day at location L (assuming that both bodies of evidence are equally reliable).

Example 3. Another simple but disturbing example

In 1982, Lofti Zadeh [74] has given to Philippe Smets during a dinner at Acapulco the following example of using the Dempster's rule which shows an unexpected result drawn from the DST. Two doctors examine a patient and agree that it suffers from either meningitis (M), concussion (C) or brain tumor (T). Thus, $\Theta = \{M, C, T\}$. Assume that the doctors agree in their low expectation of a tumor, but disagree in likely cause and provide the following diagnosis

$$m_1(M) = 0.99, \quad m_1(T) = 0.01 \quad \text{and} \quad m_2(C) = 0.99, \quad m_2(T) = 0.01.$$

If we now combine belief functions using Dempster's rule of combination, one gets the unexpected final conclusion $m(T) = \frac{0.0001}{1-0.0099-0.0099-0.9801} = 1$ which means that the patient suffers with certainty from brain tumor!. This unexpected result arises from the fact that the two bodies of evidence (doctors) agree that the patient does not suffer from tumor but are in almost full contradiction in regard to the other causes of the disease. This very simple but practical example shows the limitations of practical use of the DST for automated reasoning. Some extreme caution on the degree of conflict of the sources must always be taken before taking a final decision based on the Dempster's rule of combination. A justification of non effectiveness of the Dempster's rule in such kind of example based on an information entropy argument has already been reported.[67]

Example 4. Blackman's example

Let's consider now the example,[3] provided by Samuel Blackman.[4, pp. 207–209] Consider only two attribute types corresponding to the frame of discernment $\Theta = \{\theta_1, \theta_2\}$ and the assignment problem for a single observation and two tracks (T_1 and T_2). Assume now the following two predicted bba for the two tracks:

$$m_{T_1}(\theta_1) = 0.5 \qquad m_{T_1}(\theta_2) = 0.5 \qquad m_{T_1}(\theta_1 \cup \theta_2) = 0$$

$$m_{T_2}(\theta_1) = 0.1 \qquad m_{T_2}(\theta_2) = 0.1 \qquad m_{T_2}(\theta_1 \cup \theta_2) = 0.8$$

Now assume to receive the following new bba drawn from attribute observation Z of the system

$$m_Z(\theta_1) = 0.5 \qquad m_Z(\theta_2) = 0.5 \qquad m_Z(\theta_1 \cup \theta_2) = 0$$

The observation bba $m_Z(.)$ fits perfectly with the predicted bba $m_{T_1}(.)$, whereas $m_Z(.)$ has some disagreement with the predicted bba $m_{T_2}(.)$. If we use the DST to solve this very simple assignment problem between the observation and several predicted bba,

[3] This example has been pointed out to the author by Dr. Albena Tchamova from CLPP, Bulgarian Academy of Sciences, during NM&A 02 Conference in Borovetz, Bulgaria, August 2002.

one gets from the DS rule of combination exactly the same result, i.e. for $m_{T_1 Z} \triangleq m_{T_1} \oplus m_Z$ and $m_{T_2 Z} \triangleq m_{T_2} \oplus m_Z$:

$$m_{T_1 Z}(\theta_1) = 0.5 \qquad m_{T_1 Z}(\theta_2) = 0.5 \qquad m_{T_1 Z}(\theta_1 \cup \theta_2) = 0$$
$$m_{T_2 Z}(\theta_1) = 0.5 \qquad m_{T_2 Z}(\theta_2) = 0.5 \qquad m_{T_2 Z}(\theta_1 \cup \theta_2) = 0$$

From these two same results only, it is impossible to find the correct solution of this simple assignment problem. Moreover the weights of conflict between sources for the two combinations of evidences are respectively equal to

$$k_{T_1 Z} = 0.5 \qquad \text{for the fusion} \quad m_{T_1} \oplus m_Z$$

$$k_{T_2 Z} = 0.1 \qquad \text{for the fusion} \quad m_{T_2} \oplus m_Z$$

Therefore the resultant conflict terms provide a larger discrepancy between observation bba m_Z with the predicted bba m_{T_1} than with the predicted bba m_{T_2}, despite the fact that their bba are equal. Within such conditions, the search for the minimum weight of conflict between sources cannot be taken as a reliable solution for the assignment problem. To solve this anomaly, S. Blackman has proposed to use a relative, rather than an absolute, attribute likelihood function as follows

$$L(Z \mid T) \triangleq (1 - k_{TZ})/(1 - k_{TZ}^{\min}),$$

where $k_{TZ}^{\min}$ is the minimum conflict factor that could occur for either the observation Z or the track T in the case of *perfect* assignment (when $m_Z(.)$ and $m_T(.)$ coincide). By adopting this relative likelihhod function, one gets

$$L(Z \mid T_1) = (1-0.5)/(1-0.5) = 1 \quad \text{and} \quad L(Z \mid T_2) = (1-0.1)/(1-0.02) = 0.92.$$

Using the Blackman's approach, there is now a larger likelihood associated with the first assignment (hence the right assignment solution can be obtained now based on the max likelihood criteria) but the difference between the two likelihood values is not so big .... As reported by S. Blackman,[4] *more study in this area is required.* Dr. Tchamova has recently proposed, in a private communication to the author, to use the city-block and Euclidean distances $d_1(T, TZ) = \sum_{A \in 2^\Theta} \mid m_T(A) - m_{TZ}(A) \mid$ or $d_2(T, TZ) = \sqrt{\sum_{A \in 2^\Theta} [m_T(A) - m_{TZ}(A)]^2}$ to measure the closeness between $m_{T_1 Z}$ and m_{T_1} and between $m_{T_2 Z}$ and m_{T_2} and then to choose the assignment which corresponds to the minimum distance. Using her approach, one gets

$$d_1(T_1, T_1 Z) = d_2(T_1, T_1 Z) = 0 \qquad d_1(T_2, T_2 Z) = 1.6 \qquad d_2(T_2, T_2 Z) \simeq 0.98$$

The Tchamova's approach can therefore solve the *anomaly* of the DS result in this assignment problem.

Let's consider now the previous predicted gbba $m_{T_1}(.)$ and $m_{T_2}(.)$ but with an observation bba which agrees with $m_{T_2}(.)$ so that $Z \leftrightarrow T_2$ becomes now the correct assignment we are looking for. In other words, let's consider

$$m_Z(\theta_1) = 0.1 \qquad m_Z(\theta_2) = 0.1 \qquad m_Z(\theta_1 \cup \theta_2) = 0.8 \qquad m_Z(\theta_1 \cap \theta_2) = 0$$

Using the DS rule of combination, we get now the following results

$$\begin{array}{lll} m_{T_1Z}(\theta_1) = 0.5 & m_{T_1Z}(\theta_2) = 0.5 & m_{T_1Z}(\theta_1 \cup \theta_2) = 0 \\ m_{T_2Z}(\theta_1) = 0.173 & m_{T_2Z}(\theta_2) = 0.173 & m_{T_2Z}(\theta_1 \cup \theta_2) = 0.654 \end{array}$$

with resulting conflict factors $k_{T_1Z} = 0.1$ and $k_{T_2Z} = 0.02$. From these bba $m_{T_1Z}(.)$, $m_{T_1Z}(.)$ and conflict factors k_{T_1Z}, k_{T_2Z} it is clear that the assignment solution is directly given here by the fusion $m_{T_2} \oplus m_Z$ which has the minimum conflict factor. In this second case, we do not need to look for any additional approach to reach the right solution. Nevertheless, it is still interesting to examine the result of the distance approach in this case.
We get then the following distances:

$$d_1(T_1, T_1Z) = d_2(T_1, T_1Z) = 0 \quad d_1(T_2, T_2Z) = 0.292 \quad d_2(T_2, T_2Z) \simeq 0.1788$$

The decision drawn from the minimum distance criteria will yield here the wrong assignment if this approach is chosen.

Therefore, as seen in this simple example, there is no unique and reliable approach to solve the assignment problem based on DST for both cases. In general, we will always have to look for the suitable approach (minimum conflict, Blackman or Tchamova approaches) which allows us to get (hopefully) the correct solution of the problem. Given the difficulties in choosing the best approach to use, it can be rather difficult to find an automatic inference system depending on the complexity of the assignment problem. We will present at the end of this paper how our new theory of plausible and paradoxical reasoning can help to solve this assignment problem. By using only an unique and simple criteria based on our *generalized entropy like measure*, we will be able to provide the correct solution for the two cases of the assignment problem presented in this example.

2.5. Conditional Belief Functions

Let $m_B(A) = 1$ if $B \subseteq A$ and $m_B(A) = 0$ if $B \not\subseteq A$ (the subset B is the only focal element of Bel_B and its basic belief number is one). Then $\text{Bel}_B(.)$ is a belief function that focuses all of the belief on B (note that Bel_B is not in general a bayesian belief function unless $\mid B \mid = 1$).

Definition 6. *Consider now a belief function Bel defined on Θ and a specific belief function Bel_B, then the orthogonal sum denoted as Bel$(. \mid B) =$ Bel $\oplus$ Bel$_B$ is defined for all $A \subset \Theta$ by* [47]

$$\text{Bel}(A \mid B) = \frac{\text{Bel}(A \cup B^c) - \text{Bel}(B^c)}{1 - \text{Bel}(B^c)} \tag{17}$$

and

$$\text{Pl}(A \mid B) = \frac{\text{Pl}(A \cap B)}{\text{Pl}(B).} \tag{18}$$

Proposition 4. *If Bel$(.)$ is a bayesian belief function, then*

$$\text{Bel}(A \mid B) = \frac{\text{Bel}(A \cap B)}{\text{Bel}(B)} = \text{Pl}(A \mid B), \tag{19}$$

which coincides exactly with the classical conditional probability

$$P\{A \mid B\} = \frac{P\{A \cap B\}}{P\{B\}}. \tag{20}$$

3. A New Theory for Plausible and Paradoxical Reasoning

3.1. Introduction

As seen in the previous disturbing example by Zadeh, the use of the DST must be done only with extreme caution if one has to take a final and important decision from the result of the Dempter's rule of combination. In most practical applications based on the DST, some ad-hoc or heuristic recipes must be added to the fusion process to correctly manage or reduce the possibility of high degree of conflict between sources. Otherwise, the fusion results lead to a very dangerous conclusion (or cannot provide a reliable result at all). Even though the DST has provided fruitful results in many applications (mainly in artificial intelligence and systems expert areas) in past decades, we strongly argue that this theory is still too limited because it is based on the two following restrictive constraints :

- C1- The DST considers a discrete and finite frame of discernment based on a set of exhaustive and exclusive elementary elements.

- C2- The bodies of evidence are assumed independent (each source of information does not take into account the knowledge of other sources) and provide a belief function on the power set 2^Θ.

These two constraints are very strong in many practical problems involving uncertain and probable reasoning and dealing with fusion of uncertain, imprecise and paradoxical information. This important remark has already been discussed.[34,35,45] Schubert has proposed a new partitioning management technique to overcome mainly the C2 constraint.[45] The first constraint is very severe actually since it does not allow paradoxes on elements of the frame of discernment Θ. The DST accepts as foundation the commonly adopted principle of the third exclude. Even if, at first glance, it makes sense in the traditional classical thought, we develop here a new theory which does not accept this principle of the third exclude and accepts and deals with paradoxes. This is the main purpose and innovation of our new theory referred to as the DSmT (standing for Dezert-Smarandache Theory of paradoxical reasoning).[55]

The constraint C1 assumes that each elementary hypothesis of the frame of discernment Θ is finely and precisely defined and we are able to discriminate between all elementary hypotheses without ambiguity and difficulty. We argue that this constraint is too limited and that it is not always possible in practice to choose and define a frame of discernment satisfying C1 even for some very simple problems wherein each elementary hypothesis corresponds to a fuzzy or vague concept or attributes. In such cases, the elementary elements of the *frame of discernment* cannot be precisely separated without ambiguity such that no refinement of the frame of discernment satisfying the first constraint is possible.

Example 5. As a simple example, consider an armed robbery situation having a witness and the frame of discernment (associated to the possible size of the thief) having only two elementary imprecise classes $\Theta = \{\theta_1 = \text{small}, \theta_2 = \text{tall}\}$. An investigator asks the witness about the size of the thief and the witness declares that the thief was tall with bba number $m(\theta_2) = 0.80$, small with bba number $m(\theta_1) = 0.15$ and is uncertain (either tall or small) with $m(\theta_1 \cup \theta_2) = 0.05$. The investigator will have to deal only with this information although the smallness and the tallness have not been precisely defined. The use of this testimony by the investigator (having also some additional information about the thief from other sources) to infer on the true size of the thief is delicate especially with the important missing information about the size of the witness (who could be either a basketball player, a dwarf or, most probably, is of average size. Actually, these two hypotheses are not incompatible since some dwarfs really enjoy to play basketball).

Hence, in many situations the frame of discernment Θ can only be described in terms of imprecise elements which cannot be clearly separated and which cannot be considered as fully disjoint so that the refinement of the initial frame into a new one satisfying C1 is like a graal quest that cannot be accomplished. Our last remark about C1 constraint concerns the universal nature of the frame of discernment. It is clear that, in general, the *same* frame of discernment is interpreted differently by the bodies of evidence or experts. Some subjectivity, or at least some fortuitious biases, on the information provided by a source of information is almost unavoidable, otherwise this would assume, as within the DST, that all bodies of evidence have an objective/universal (possibly uncertain) interpretation or measure of the phenomena under consideration. This vision seems to be too restrictive because usually independent bodies of evidence provide their beliefs about some hypotheses only with respect to their own worlds of knowledge and experience. We do not go deeper here in the techniques of refinements and coarsenings of compatible frame of discernments which is a prerequisite to the Dempster's rule of combination (see [47] for details). We just want to emphasize here that the DST cannot be used at all in all cases where C1 cannot be satisfied and we have more generally to accept the idea to deal directly with paradoxical information.

To convince the reader to accept our radically new way of thought, just think about the true nature of a photon? For experts working in particle physics, photons look like particles, for physicists working in electromagnetic field theory, photons are only considered as electromagnetic waves. Both interpretations are true, there is no unicity on the true nature of the photon and actually a photon holds both aspects which appears as a paradox for most human minds. This notion has been accepted in modern physics only with great difficulty and many vigorous discussions about this fundamental question were held at the beginning of the 20th century between all eminent physicists at the time.[40]

The constraint C2 hides also a strong difficulty. To apply the Dempster's rule for two independent bodies of evidence $\mathcal{B}_1$ and $\mathcal{B}_2$, it is necessary that both frames of discernment Θ_1 and Θ_2 (related to each source $\mathcal{B}_1$ and $\mathcal{B}_2$) have to be compatible and to correspond to the same *universal vision* of the possibilities of the answer of the question under consideration. Actually, this constraint itself is very difficult to satisfy since each source of information has usually only its own (and maybe biased) interpretation of elements of frame of discernment. The belief provided by each local source of information mainly depends on the own knowledge frame of the source without reference to the (inaccessible) absolute truth of the space of possibilities. Therefore, C2 is, in many cases, also a too strong hypothesis to accept as foundations for a general theory of probable and paradoxical reasoning. A general theory should include the possibility to deal with evidences arising from different sources of information which have no access to absolute interpretation of the elements of the frame of discernment Θ under consideration. This yields to accept paradoxical information as basis for a new general theory of probable reasoning. Actually, we will show in the forthcoming examples that the paradoxical information arising from the fusion of several bodies of evidence is very informative and can be used to help us take a legitimate final decision.

In other words, our new theory can be interpreted as a general and direct extension of probability theory and the Dempster-Shafer theory in the following sense. Let $\Theta = \{\theta_1, \theta_2\}$ be the simpliest frame of discernment involving only two elementary hypotheses (with no more additional assumptions on θ_1 and θ_2), then

- the probability theory deals with basic probability assignments $m(.) \in [0, 1]$ such that
$$m(\theta_1) + m(\theta_2) = 1;$$
- the Dempster-Shafer theory extends the probability theory by dealing with basic belief assignments $m(.) \in [0, 1]$ such that
$$m(\theta_1) + m(\theta_2) + m(\theta_1 \cup \theta_2) = 1;$$
- our general theory extends the two previous theories by accepting the possibility for paradoxical information and deals with new basic belief assignments $m(.) \in [0, 1]$ such that
$$m(\theta_1) + m(\theta_2) + m(\theta_1 \cup \theta_2) + m(\theta_1 \cap \theta_2) = 1.$$

3.2. Notion of Hyper-Power Set

Let $\Theta = \{\theta_1, \ldots, \theta_n\}$ be a set of n elementary elements considered as exhaustive which cannot be precisely defined and separated so that no refinement of Θ in a new larger set Θ_{ref} of disjoint elementary hypotheses is possible and let's consider the classical set operators $\cup$ (disjunction) and $\cap$ (conjunction). The exhaustive hypothesis about Θ is not a strong constraint since when $\theta_i, i = 1, n$ does not constitute an exhaustive set of elementary possibilities, we can always add an extra element θ_0 such that $\theta_i, i = 0, n$ describes now an exhaustive set. We will assume therefore, from now on, that Θ characterizes an exhaustive frame of discernment. Θ will be called a *general* frame of discernment in the sequel to emphasize the fact that Θ does not satisfy the Dempster-Shafer C1 constraint.

Definition 7. *The classical power set $\mathcal{P}(\Theta) = 2^\Theta$ has been defined as the set of all proper subsets of Θ when all elements θ_i are disjoint. We extend here this notion and define now the hyper-power set D^Θ as the set of all composite possibilities built from Θ with $\cup$ and $\cap$ operators such that $\forall A \in D^\Theta, B \in D^\Theta, (A \cup B) \in D^\Theta$ and $(A \cap B) \in D^\Theta$.*

Obviously, one would always have $D^\Theta \subset 2^{\Theta_{ref}}$ if the refined power set $2^{\Theta_{ref}}$ could be defined and accessible which, as already argued, is not possible in general.

The cardinality of D^Θ is majored by 2^{2^n} when Card$(\Theta) = \mid \Theta \mid = n$. The generation of hyper-power set D^Θ corresponds to the famous Dedekind's problem on enumerating the set of monotone Boolean functions (i.e., functions expressible using only AND and OR set operators).[9] This problem is also related to the Sperner systems [64,37] based on finite poset, called also *antichains* in literature.[6] The number of antichains on the n-set Θ are equal to the number of monotonic increasing Boolean functions of n variables, and also the number of free distributive lattices with n generators.[20,22,28,29,38,53] Determining these numbers is exactly the Dedekind's problem. The choice of letter D in our notation D^Θ to represent the hyper-power set of Θ is in honor of the great mathematician R. Dedekind. The general solution of the Dedekind's problem (for $n > 10$) has not been found yet. We just know that the cardinality numbers of D^Θ follow the integers of the Dedekind's sequence minus one when $\text{Card}(\Theta) = n$ increases.

Example 6.

1. for $\Theta = \{\}$ (empty set), $D^\Theta = \{\emptyset\}$ and $\mid D^\Theta \mid= 1$
2. for $\Theta = \{\theta_1\}$, $D^\Theta = \{\emptyset, \theta_1\}$ and $\mid D^\Theta \mid= 2$
3. for $\Theta = \{\theta_1, \theta_2\}$, $D^\Theta = \{\emptyset, \theta_1, \theta_2, \theta_1 \cup \theta_2, \theta_1 \cap \theta_2\}$ and $\mid D^\Theta \mid= 5$
4. for $\Theta = \{\theta_1, \theta_2, \theta_3\}$,

$$\begin{gathered} D^\Theta = \{\emptyset, \theta_1, \theta_2, \theta_3, \\ \theta_1 \cup \theta_2, \theta_1 \cup \theta_3, \theta_2 \cup \theta_3, \theta_1 \cap \theta_2, \theta_1 \cap \theta_3, \theta_2 \cap \theta_3, \theta_1 \cup \theta_2 \cup \theta_3, \theta_1 \cap \theta_2 \cap \theta_3, \\ (\theta_1 \cup \theta_2) \cap \theta_3, (\theta_1 \cup \theta_3) \cap \theta_2, (\theta_2 \cup \theta_3) \cap \theta_1, (\theta_1 \cap \theta_2) \cup \theta_3, (\theta_1 \cap \theta_3) \cup \theta_2, (\theta_2 \cap \theta_3) \cup \theta_1, \\ (\theta_1 \cup \theta_2) \cap (\theta_1 \cup \theta_3) \cap (\theta_2 \cup \theta_3)\} \end{gathered}$$

and $\mid D^\Theta \mid= 19$.

It is not difficult, although tedious, to check that $\forall A \in D^\Theta, B \in D^\Theta, (A \cup B) \in D^\Theta$ and $(A \cap B) \in D^\Theta$ (see appendix for the proof).

The extension to a larger frame of discernment is possible but entails a higher computational load. The general and direct analytic computation of $\mid D^\Theta \mid$ for a n-set Θ with $n > 10$ is not known and is still under investigation by the mathematical community. Cardinality numbers $\mid D^\Theta \mid$ follow the Dedekind's sequence (minus one), $1, 2, 5, 19, 167, 7580, 7828353, \ldots$ when $\text{Card}(\Theta) = n = 0, 1, 2, 3, 4, 5, 6, \ldots$.

3.3. The General Basic Belief Masses $m(.)$

Definition 8. *Let Θ be a general frame of discernment of the problem under consideration. We define a map $m(.) : D^\Theta \rightarrow [0, 1]$ associated to a given body of evidence $\mathcal{B}$ which can support paradoxical information, as follows*

$$m(\emptyset) = 0 \qquad \text{and} \qquad \sum_{A \in D^\Theta} m(A) = 1. \tag{21}$$

The quantity $m(A)$ is called A's *general basic belief number* (gbba) or the general basic belief mass for A.

As in the DST, all subsets $A \in D^\Theta$ for which $m(A) > 0$ are called focal elements of $m(.)$ and the set of all focal elements of $m(.)$ is also called the core $\mathcal{K}(m)$ of m.

Definition 9. *The belief and plausibility functions are defined in the same way as in the DST, i.e.*

$$\text{Bel}(A) = \sum_{B \in D^\Theta, B \subseteq A} m(B); \tag{22}$$

$$\text{Pl}(A) = \sum_{B \in D^\Theta, B \cap A \neq \emptyset} m(B). \tag{23}$$

Note that we do not define here explicitly the complementary A^c of a proposition A since $m(A^c)$ cannot be precisely evaluated from $\cup$ and $\cap$ operators on D^Θ since we include the possibility to deal with a complete paradoxical source of information such that $\forall A \in D^\Theta, \forall B \in D^\Theta, m(A \cap B) > 0$. These definitions are compatible with the DST definitions when the sources of information become uncertain but rational (they do not support paradoxical information). We still have $\forall A \in D^\Theta, \text{Bel}(A) \leq \text{Pl}(A)$.

3.4. Construction of Pignistic Probabilities from gbba $m(.)$

The construction of a pignistic probability measure from the general basic belief masses $m(.)$ over D^Θ with $|\Theta| = n$ is still possible and is given by the general expression of the form

$$\forall i = 1, \ldots, n \qquad P\{\theta_i\} = \sum_{A \in D^\Theta} \alpha_{\theta_i}(A) m(A), \tag{24}$$

where $\alpha_{\theta_i}(A) \in [0, 1]$ are weighting coefficients that depend on the inclusion or non-inclusion of θ_i with respect to proposition A. No general analytic expression for $\alpha_{\theta_i}(A)$ has been derived yet even if $\alpha_{\theta_i}(A)$ can be obtained explicitly for simple examples. When general bba $m(.)$ reduces to classical bba (i.e., the DS bba without paradox), then $\alpha_{\theta_i}(A) = \frac{1}{|A|}$ if $\theta_i \subseteq A$ and therefore one gets

$$\forall i = 1, \ldots, n \qquad P\{\theta_i\} = \sum_{A \subseteq \Theta | \theta_i \in A} \frac{1}{|A|} m(A). \tag{25}$$

We present here an example of a pignistic probabilities reconstruction from a general and non degenerated bba $m(.)$ (i.e. $\nexists A \in D^\Theta$ with $A \neq \emptyset$ such that $m(A) = 0$) over D^Θ.

Example 7. If $\Theta = \{\theta_1, \theta_2\}$ then

$$P\{\theta_1\} = m(\theta_1) + \frac{1}{2}m(\theta_1 \cup \theta_2) + \frac{1}{2}m(\theta_1 \cap \theta_2)$$

$$P\{\theta_2\} = m(\theta_2) + \frac{1}{2}m(\theta_1 \cup \theta_2) + \frac{1}{2}m(\theta_1 \cap \theta_2)$$

Example 8. If $\Theta = \{\theta_1, \theta_2, \theta_3\}$ then

$$\begin{aligned}
P\{\theta_1\} =& m(\theta_1) + \frac{1}{2}m(\theta_1 \cup \theta_2) + \frac{1}{2}m(\theta_1 \cup \theta_3) + \frac{1}{2}m(\theta_1 \cap \theta_2) + \frac{1}{2}m(\theta_1 \cap \theta_3) \\
&+ \frac{1}{3}m(\theta_1 \cup \theta_2 \cup \theta_3) + \frac{1}{3}m(\theta_1 \cap \theta_2 \cap \theta_3) \\
&+ \frac{1/2 + 1/3}{3}m((\theta_1 \cup \theta_2) \cap \theta_3) + \frac{1/2 + 1/3}{3}m((\theta_1 \cup \theta_3) \cap \theta_2) \\
&+ \frac{1/2 + 1/2 + 1/3}{3}m((\theta_2 \cup \theta_3) \cap \theta_1) \\
&+ \frac{1/2 + 1/2 + 1/3}{5}m((\theta_1 \cap \theta_2) \cup \theta_3) \\
&+ \frac{1/2 + 1/2 + 1/3}{5}m((\theta_1 \cap \theta_3) \cup \theta_2) \\
&+ \frac{1 + 1/2 + 1/2 + 1/3}{5}m((\theta_2 \cap \theta_3) \cup \theta_1) \\
&+ \frac{1/2 + 1/2 + 1/3}{4}m((\theta_1 \cup \theta_2) \cap (\theta_1 \cup \theta_3) \cap (\theta_2 \cup \theta_3))
\end{aligned}$$

$$\begin{aligned}
P\{\theta_2\} =& m(\theta_2) + \frac{1}{2}m(\theta_1 \cup \theta_2) + \frac{1}{2}m(\theta_2 \cup \theta_3) + \frac{1}{2}m(\theta_1 \cap \theta_2) + \frac{1}{2}m(\theta_2 \cap \theta_3)\\
&+ \frac{1}{3}m(\theta_1 \cup \theta_2 \cup \theta_3) + \frac{1}{3}m(\theta_1 \cap \theta_2 \cap \theta_3)\\
&+ \frac{1/2+1/3}{3}m((\theta_1 \cup \theta_2) \cap \theta_3)\\
&+ \frac{1/2+1/2+1/3}{3}m((\theta_1 \cup \theta_3) \cap \theta_2)\\
&+ \frac{1/2+1/3}{3}m((\theta_2 \cup \theta_3) \cap \theta_1)\\
&+ \frac{1/2+1/2+1/3}{5}m((\theta_1 \cap \theta_2) \cup \theta_3)\\
&+ \frac{1+1/2+1/2+1/3}{5}m((\theta_1 \cap \theta_3) \cup \theta_2)\\
&+ \frac{1/2+1/2+1/3}{5}m((\theta_2 \cap \theta_3) \cup \theta_1)\\
&+ \frac{1/2+1/2+1/3}{4}m((\theta_1 \cup \theta_2) \cap (\theta_1 \cup \theta_3) \cap (\theta_2 \cup \theta_3))
\end{aligned}$$

$$\begin{aligned}
P\{\theta_3\} =& m(\theta_3) + \frac{1}{2}m(\theta_1 \cup \theta_3) + \frac{1}{2}m(\theta_2 \cup \theta_3) + \frac{1}{2}m(\theta_1 \cap \theta_3) + \frac{1}{2}m(\theta_2 \cap \theta_3)\\
&+ \frac{1}{3}m(\theta_1 \cup \theta_2 \cup \theta_3) + \frac{1}{3}m(\theta_1 \cap \theta_2 \cap \theta_3)\\
&+ \frac{1/2+1/2+1/3}{3}m((\theta_1 \cup \theta_2) \cap \theta_3) + \frac{1/2+1/3}{3}m((\theta_1 \cup \theta_3) \cap \theta_2)\\
&+ \frac{1/2+1/3}{3}m((\theta_2 \cup \theta_3) \cap \theta_1)\\
&+ \frac{1+1/2+1/2+1/3}{5}m((\theta_1 \cap \theta_2) \cup \theta_3)\\
&+ \frac{1/2+1/2+1/3}{5}m((\theta_1 \cap \theta_3) \cup \theta_2) + \frac{1/2+1/2+1/3}{5}m((\theta_2 \cap \theta_3) \cup \theta_1)\\
&+ \frac{1/2+1/2+1/3}{4}m((\theta_1 \cup \theta_2) \cap (\theta_1 \cup \theta_3) \cap (\theta_2 \cup \theta_3))
\end{aligned}$$

The evaluation of weighting coefficients $\alpha_{\theta_i}(A)$ has been obtained from the geometrical interpretation of the relative contribution of the distinct parts of A with the proposition θ_i under consideration. For example, consider $A = (\theta_1 \cap \theta_2) \cup \theta_3$ which corresponds to the area $a_1 \cup a_2 \cup a_3 \cup a_4 \cup a_5$ on the following Venn diagram.

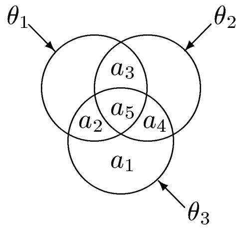

Figure 1 : Representation of $A = (\theta_1 \cap \theta_2) \cup \theta_3 \equiv a_1 \cup a_2 \cup a_3 \cup a_4 \cup a_5$.

a_1 which is shared only by θ_3 will contribute to θ_3 with weight 1; a_2 which is shared by θ_1 and θ_3 will contribute to θ_3 with weight 1/2; a_3 which is not shared by θ_3 will contribute to θ_3 with weight 0; a_4 which is shared by θ_2 and θ_3 will contribute to θ_3 with weight 1/2; a_5 which is shared by both θ_1,θ_2 and θ_3 will contribute to θ_3 with weight 1/3. Since, moreover, one must have $\forall A \in D^\Theta$ with $m(A) \neq 0$, $\sum_{i=1}^n \alpha_{\theta_i}(A)m(A) = m(A)$, it is necessary to normalize $\alpha_{\theta_i}(A)$. Therefore, $\alpha_{\theta_1}(A)$, $\alpha_{\theta_2}(A)$ and $\alpha_{\theta_3}(A)$ will be given by

$$\alpha_{\theta_1}(A) = \alpha_{\theta_2}(A) = \frac{1/2 + 1/2 + 1/3}{5}, \qquad \alpha_{\theta_3}(A) = \frac{1 + 1/2 + 1/2 + 1/3}{5}.$$

All $\alpha_{\theta_i}(A), \forall A \in D^\Theta$ entering in derivation of the *pignistic* probabilities $P\{\theta_i\}$ can be obtained in a similar manner.

3.5. General rule of Combination of Paradoxical Sources of Evidence

Let's consider now two distinct (but potentially paradoxical) bodies of evidences $\mathcal{B}_1$ and $\mathcal{B}_2$ over the same frame of discernment Θ with belief functions $\text{Bel}_1(.)$ and $\text{Bel}_2(.)$ associated with information granules $m_1(.)$ and $m_2(.)$.

Definition 10. *The combined global belief function Bel$(.)$ = Bel$_1(.) \oplus$ Bel$_2(.)$ is obtained through the combination of the granules $m_1(.)$ and $m_2(.)$ by the simple rule*

$$\forall C \in D^\Theta, \qquad m(C) \triangleq [m_1 \oplus m_2](C) = \sum_{A,B \in D^\Theta, A \cap B = C} m_1(A)m_2(B). \tag{26}$$

Since D^Θ is closed under $\cup$ and $\cap$ operators, this new rule of combination guarantees that $m(.) : D^\Theta \to [0,1]$ is a proper general information granule statisfying (21). The global belief function $\text{Bel}(.)$ is then obtained from the granule $m(.)$ through (22). This rule of combination is commutative and associative and can always be used for the fusion of paradoxical or rational sources of information (bodies of evidence). Obviously, the decision process will have to be more cautious in making a final decision based on the general granule $m(.)$ when internal paradoxical conflicts arise.

It is important to note that any fusion of sources of information generates either uncertainties, paradoxes or *more generally, both*. This is intrinsic to the general fusion process itself. For instance, let's consider the frame of discernment $\Theta = \{\theta_1, \theta_2\}$ and the following very simple examples:

Example 9. Consider the rational information granules

$$m_1(\theta_1) = 0.80 \qquad m_1(\theta_2) = 0.20 \qquad m_1(\theta_1 \cup \theta_2) = 0 \qquad m_1(\theta_1 \cap \theta_2) = 0$$

$$m_2(\theta_1) = 0.90 \qquad m_2(\theta_2) = 0.10 \qquad m_2(\theta_1 \cup \theta_2) = 0 \qquad m_2(\theta_1 \cap \theta_2) = 0$$

then

$$m(\theta_1) = 0.72 \qquad m(\theta_2) = 0.02 \qquad m(\theta_1 \cup \theta_2) = 0 \qquad m(\theta_1 \cap \theta_2) = 0.26$$

Example 10. Consider the uncertain information granules

$$m_1(\theta_1) = 0.80 \qquad m_1(\theta_2) = 0.15 \qquad m_1(\theta_1 \cup \theta_2) = 0.05 \qquad m_1(\theta_1 \cap \theta_2) = 0$$

$$m_2(\theta_1) = 0.90 \qquad m_2(\theta_2) = 0.05 \qquad m_2(\theta_1 \cup \theta_2) = 0.05 \qquad m_2(\theta_1 \cap \theta_2) = 0$$

then

$$m(\theta_1) = 0.805 \qquad m(\theta_2) = 0.0175 \qquad m(\theta_1 \cup \theta_2) = 0.0025 \qquad m(\theta_1 \cap \theta_2) = 0.175$$

Example 11. Consider the paradoxical information granules

$$m_1(\theta_1) = 0.80 \qquad m_1(\theta_2) = 0.15 \qquad m_1(\theta_1 \cup \theta_2) = 0 \qquad m_1(\theta_1 \cap \theta_2) = 0.05$$

$$m_2(\theta_1) = 0.90 \qquad m_2(\theta_2) = 0.05 \qquad m_2(\theta_1 \cup \theta_2) = 0 \qquad m_2(\theta_1 \cap \theta_2) = 0.05$$

then

$$m(\theta_1) = 0.72 \qquad m(\theta_2) = 0.0075 \qquad m(\theta_1 \cup \theta_2) = 0 \qquad m(\theta_1 \cap \theta_2) = 0.2725$$

Example 12. Consider the uncertain and paradoxical information granules

$$m_1(\theta_1) = 0.80 \qquad m_1(\theta_2) = 0.10 \qquad m_1(\theta_1 \cup \theta_2) = 0.05 \qquad m_1(\theta_1 \cap \theta_2) = 0.05$$

$$m_2(\theta_1) = 0.90 \qquad m_2(\theta_2) = 0.05 \qquad m_2(\theta_1 \cup \theta_2) = 0.03 \qquad m_2(\theta_1 \cap \theta_2) = 0.02$$

then

$$m(\theta_1) = 0.789 \qquad m(\theta_2) = 0.0105 \qquad m(\theta_1 \cup \theta_2) = 0.0015 \qquad m(\theta_1 \cap \theta_2) = 0.199$$

Note that this general fusion rule can also be used with intuitionist logic in which the sum of bba is allowed to be less than one ($\sum m(A) < 1$) and with the paraconsistent logic in which the sum of bba is allowed to be greater than one ($\sum m(A) > 1$) as well. In such cases, the fusion result does not provide in general $\sum m(A) = 1$.

For example, let's consider the fusion of the paraconsistent source $\mathcal{B}_1$ with $m_1(\theta_1) = 0.60$, $m_1(\theta_2) = 0.30$, $m_1(\theta_1 \cup \theta_2) = 0.20$, $m_1(\theta_1 \cap \theta_2) = 0.10$ with the intuitionist source $\mathcal{B}_2$ with $m_2(\theta_1) = 0.50$, $m_2(\theta_2) = 0.20$, $m_2(\theta_1 \cup \theta_2) = 0.10$, $m_2(\theta_1 \cap \theta_2) = 0.10$. In such case, the fusion result of these two sources of information yields the following global paraconsistent bba $m(.)$:

$$m(\theta_1) = 0.46 \qquad m(\theta_2) = 0.13 \qquad m(\theta_1 \cup \theta_2) = 0.02 \qquad m(\theta_1 \cap \theta_2) = 0.47$$

which yields $\sum m = 1.08 > 1$.

In practice, for the sake of fair comparison between several alternatives or choices, it is better and more simple to deal with normalized bba to take a final important decision for the problem under consideration. A nice property of the new rule of combination of non-normalized bba is its invariance to the pre- or post-normalization process as we will show right now. In the previous example, the post-normalization of bba $m(.)$ will yield the new bba $m'(.)$

$$m'(\theta_1) = \frac{0.46}{1.08} \approx 0.426 \qquad m'(\theta_2) = \frac{0.13}{1.08} \approx 0.12$$

$$m'(\theta_1 \cup \theta_2) = \frac{0.02}{1.08} \approx 0.019 \qquad m'(\theta_1 \cap \theta_2) = \frac{0.47}{1.08} \approx 0.435$$

The fusion of pre-normalization of bba $m_1(.)$ and $m_2(.)$ will yield the same normalized bba $m'(.)$ since

$$m'_1(\theta_1) = \frac{0.6}{1.2} = 0.50 \qquad m'_1(\theta_2) = \frac{0.3}{1.2} = 0.25$$

$$m'_1(\theta_1 \cup \theta_2) = \frac{0.2}{1.2} \approx 0.17 \qquad m'_1(\theta_1 \cap \theta_2) = \frac{0.1}{1.2} \approx 0.08$$

$$m'_2(\theta_1) = \frac{0.5}{0.9} \approx 0.56 \qquad m'_2(\theta_2) = \frac{0.2}{0.9} \approx 0.22$$

$$m_2'(\theta_1 \cup \theta_2) = \frac{0.1}{0.9} \approx 0.11 \qquad m_2'(\theta_1 \cap \theta_2) = \frac{0.1}{0.9} \approx 0.11$$

$$m'(\theta_1) \approx 0.426 \qquad m'(\theta_2) \approx 0.12 \qquad m'(\theta_1 \cup \theta_2) \approx 0.019 \qquad m'(\theta_1 \cap \theta_2) \approx 0.435$$

It is easy to verify from the general fusion table that the pre- or post-normalization step yields always the same global normalized bba even for the general case (when $|\Theta| = n$), because the post-normalization constant $\sum m(A)$ is always equal to the product of the two pre-normalization constants $\sum m_1(A)$ and $\sum m_2(A)$.

3.6. Justification of the New Rule of Combination

Let's consider two bodies of evidence $\mathcal{B}_1$ and $\mathcal{B}_2$ characterized respectively by their bba $m_1(.)$,$m_2(.)$ and their cores $\mathcal{K}_1 = \mathcal{K}(m_1)$, $\mathcal{K}_2 = \mathcal{K}(m_2)$. Following Sun's notation,[67] each source of information will be denoted

$$\mathcal{B}_1 = \begin{bmatrix} \mathcal{K}_1 \\ m_1 \end{bmatrix} = \begin{bmatrix} f_1^{(1)} & f_2^{(1)} & \ldots & f_k^{(1)} \\ m_1(f_1^{(1)}) & m_1(f_2^{(1)}) & \ldots & m_1(f_k^{(1)}) \end{bmatrix}, \tag{27}$$

$$\mathcal{B}_2 = \begin{bmatrix} \mathcal{K}_2 \\ m_2 \end{bmatrix} = \begin{bmatrix} f_1^{(2)} & f_2^{(2)} & \ldots & f_l^{(2)} \\ m_2(f_1^{(2)}) & m_2(f_2^{(2)}) & \ldots & m_2(f_l^{(2)}) \end{bmatrix}, \tag{28}$$

where $f_i^{(1)}, i = 1, k$ are the focal elements of $\mathcal{B}_1$ and $f_j^{(2)}, j = 1, l$ are the focal elements of $\mathcal{B}_2$.

Let's consider now the combined information associated with a new body of evidence $\mathcal{B}$ resulting from the fusion of $\mathcal{B}_1$ and $\mathcal{B}_2$ having bba $m(.)$ with core $\mathcal{K}$. We denote $\mathcal{B}$ as

$$\mathcal{B} \triangleq \mathcal{B}_1 \oplus \mathcal{B}_2 = \begin{bmatrix} \mathcal{K} \\ m \end{bmatrix} = \begin{bmatrix} f_1^{(1)} \cap f_1^{(2)} & f_1^{(1)} \cap f_2^{(2)} & \ldots & f_k^{(1)} \cap f_l^{(2)} \\ m(f_1^{(1)} \cap f_1^{(2)}) & m(f_1^{(1)} \cap f_2^{(2)}) & \ldots & m(f_k^{(1)} \cap f_l^{(2)}) \end{bmatrix}. \tag{29}$$

The fusion of the two information granules can be represented with the general table of fusion as follows

$\oplus$	$m_1(f_1^{(1)})$	$m_1(f_2^{(1)})$	$\ldots$	$m_1(f_i^{(1)})$	$\ldots$	$m_1(f_k^{(1)})$
$m_2(f_1^{(2)})$	$m(f_1^{(1)} \cap f_1^{(2)})$	$m(f_2^{(1)} \cap f_1^{(2)})$	$\ldots$	$m(f_i^{(1)} \cap f_1^{(2)})$	$\ldots$	$m(f_k^{(1)} \cap f_1^{(2)})$
$m_2(f_2^{(2)})$	$m(f_1^{(1)} \cap f_2^{(2)})$	$m(f_2^{(1)} \cap f_2^{(2)})$	$\ldots$	$m(f_i^{(1)} \cap f_2^{(2)})$	$\ldots$	$m(f_k^{(1)} \cap f_2^{(2)})$
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
$m_2(f_j^{(2)})$	$m(f_1^{(1)} \cap f_j^{(2)})$	$m(f_2^{(1)} \cap f_j^{(2)})$	$\ldots$	$m(f_i^{(1)} \cap f_j^{(2)})$	$\ldots$	$m(f_k^{(1)} \cap f_j^{(2)})$
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
$m_2(f_l^{(2)})$	$m(f_1^{(1)} \cap f_l^{(2)})$	$m(f_2^{(1)} \cap f_l^{(2)})$	$\ldots$	$m(f_i^{(1)} \cap f_l^{(2)})$	$\ldots$	$m(f_k^{(1)} \cap f_l^{(2)})$

We look for the optimal rule of combination, i.e. the bba $m(.) = m_1(.) \oplus m_2(.)$ which maximizes the joint entropy of the two information sources. Jaynes [23,24] provides justification for the use of the Maxent criteria. Thus, one has to find $m(.)$ such that [67,68]

$$\max_m [H(m)] \equiv \max_m \left[- \sum_{i=1}^{k} \sum_{j=1}^{l} m(f_i^{(1)} \cap f_j^{(2)}) \log[m(f_i^{(1)} \cap f_j^{(2)})] \right]$$
$$\equiv - \min_m [-H(m)], \qquad (30)$$

satisfying both

1. the measurement projection principle (marginal bba), i.e. $\forall i = 1, \ldots, k$ and $\forall j = 1, \ldots, l$

$$m_1(f_i^{(1)}) = \sum_{j=1}^{l} m(f_i^{(1)} \cap f_j^{(2)}) \text{ and } m_2(f_j^{(2)}) = \sum_{i=1}^{k} m(f_i^{(1)} \cap f_j^{(2)}) \qquad (31)$$

   These constraints state that the marginal bba $m_1(.)$ is obtained by the summation over each column of the fusion table and the marginal bba $m_2(.)$ is obtained by the summation over each row of the table of fusion.

2. the measurement balance principle (the sum of all cells of the table of fusion must be unity)

$$\sum_{i=1}^{k} \sum_{j=1}^{l} m(f_i^{(1)} \cap f_j^{(2)}) = 1. \qquad (32)$$

Using the concise notation $m_{ij} \triangleq m(f_i^{(1)} \cap f_j^{(2)})$, the Lagrangian associated with this optimization problem under equality constraints is given by (we consider here the minimization of $-J(m)$ appearing in r.h.s of (30))

$$\mathcal{L}(m, \lambda) = \sum_{i=1}^{k} \sum_{j=1}^{l} m_{ij} \ln[m_{ij}]$$
$$+ \sum_{i=1}^{k} \lambda_i [m_1(f_i^{(1)}) - \sum_{j=1}^{l} m_{ij}] \qquad (33)$$
$$+ \sum_{j=1}^{l} \gamma_j [m_2(f_j^{(2)}) - \sum_{i=1}^{k} m_{ij}] \qquad (34)$$
$$+ \eta [\sum_{i=1}^{k} \sum_{j=1}^{l} m_{ij} - 1], \qquad (35)$$

which can be written more concisely as

$$\mathcal{L}(m,\lambda) = -H(m) + \lambda'\mathbf{g}(m), \tag{36}$$

where $m = [m_{11}\ m_{12} \ldots\ m_{kl}]'$ and

$$\lambda = \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_k \\ \gamma_1 \\ \vdots \\ \gamma_l \\ \eta \end{bmatrix} \qquad \text{and} \qquad \mathbf{g}(m) = \begin{bmatrix} m_1(f_1^{(1)}) - \sum_{j=1}^{l} m_{1j} \\ \vdots \\ m_1(f_k^{(1)}) - \sum_{j=1}^{l} m_{kj} \\ m_2(f_1^{(2)}) - \sum_{i=1}^{k} m_{i1} \\ \vdots \\ m_2(f_l^{(2)}) - \sum_{i=1}^{k} m_{il} \\ \sum_{i=1}^{k}\sum_{j=1}^{l} m_{ij} - 1 \end{bmatrix}. \tag{37}$$

Following the classical method of Lagrange multipliers, one has to find optimal solution (m^*,λ^*) such that

$$\frac{\partial\mathcal{L}}{\partial m}(m^*,\lambda^*) = \mathbf{0} \qquad \text{and} \qquad \frac{\partial\mathcal{L}}{\partial\lambda}(m^*,\lambda^*) = \mathbf{0}. \tag{38}$$

The first $k \times l$ equations express the general solution $m[\lambda]$ and the last $k+l+1$ equations determine λ^* and, therefore, by substitution into $m[\lambda]$ the optimal solution is $m^* = m[\lambda^*]$. One has to solve

$$\frac{\partial\mathcal{L}}{\partial m} = \begin{bmatrix} \frac{\partial\mathcal{L}}{\partial m_{11}} \\ \vdots \\ \frac{\partial\mathcal{L}}{\partial m_{ij}} \\ \vdots \\ \frac{\partial\mathcal{L}}{\partial m_{kl}} \end{bmatrix} = \begin{bmatrix} \ln(m_{11}) + 1 + \eta - \lambda_1 - \gamma_1 \\ \vdots \\ \ln(m_{ij}) + 1 + \eta - \lambda_i - \gamma_j \\ \vdots \\ \ln(m_{kl}) + 1 + \eta - \lambda_k - \gamma_l \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ \vdots \\ \vdots \\ 0 \end{bmatrix} = \mathbf{0}, \tag{39}$$

which yields $\forall i,j$,

$$m_{ij} = e^{-\eta-1}e^{\lambda_i}e^{\gamma_j} \tag{40}$$

and

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \begin{bmatrix} \frac{\partial \mathcal{L}}{\partial \lambda_1} \\ \vdots \\ \frac{\partial \mathcal{L}}{\partial \lambda_k} \\ \frac{\partial \mathcal{L}}{\partial \gamma_1} \\ \vdots \\ \frac{\partial \mathcal{L}}{\partial \gamma_l} \\ \frac{\partial \mathcal{L}}{\partial \eta} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} = \mathbf{0} \Leftrightarrow \begin{bmatrix} e^{-\eta-1}\sum_{j=1}^{l} e^{\lambda_1}e^{\gamma_j} \\ \vdots \\ e^{-\eta-1}\sum_{j=1}^{l} e^{\lambda_k}e^{\gamma_j} \\ e^{-\eta-1}\sum_{i=1}^{k} e^{\gamma_1}e^{\lambda_i} \\ \vdots \\ e^{-\eta-1}\sum_{i=1}^{k} e^{\gamma_l}e^{\lambda_i} \\ e^{-\eta-1}\sum_{i=1}^{k}\sum_{j=1}^{l} e^{\gamma_l}e^{\lambda_i} \end{bmatrix} = \begin{bmatrix} m_1(f_1^{(1)}) \\ \vdots \\ m_1(f_k^{(1)}) \\ m_2(f_1^{(2)}) \\ \vdots \\ m_2(f_l^{(2)}) \\ 1 \end{bmatrix}. \quad (41)$$

The last constraint in (41) can also be written as

$$e^{-\eta-1}\sum_{i=1}^{k}\sum_{j=1}^{l} e^{\gamma_l}e^{\lambda_i} = e^{-\eta-1}(\sum_{i=1}^{k} e^{\lambda_i})(\sum_{j=1}^{l} e^{\gamma_l}) = 1. \quad (42)$$

Now with basic algebraic manipulation, the optimal global bba $m_{ij}\ \forall i,j$ we are searching for, can be expressed as

$$\begin{aligned} m_{ij} &= e^{-\eta-1}e^{\lambda_i}e^{\gamma_j} \\ &= e^{-\eta-1}e^{\lambda_i}e^{\gamma_j} \times \overbrace{e^{-\eta-1}(\sum_{i=1}^{k} e^{\lambda_i})(\sum_{j=1}^{l} e^{\gamma_l})}^{1} \\ &= \underbrace{(e^{-\eta-1}e^{\lambda_i}\sum_{j=1}^{l} e^{\gamma_l})}_{m_1(f_i^{(1)})}\underbrace{(e^{-\eta-1}e^{\gamma_j}\sum_{i=1}^{k} e^{\lambda_i})}_{m_2(f_j^{(2)})} \end{aligned}$$

Thus, the solution of the maximisation of the joint entropy is obtained by choosing $\forall i,j$

$$m_{ij} = m(f_i^{(1)} \cap f_j^{(2)}) = m_1(f_i^{(1)})m_2(f_j^{(2)}) \quad (43)$$

Since several combinations yielding to the same focal element may exist, the bba of all focal elements equal to $f_i^{(1)} \cap f_j^{(2)}$ over the fusion space is

$$m(f_i^{(1)} \cap f_j^{(2)}) = \sum_{i,j} m_1(f_i^{(1)})m_2(f_j^{(2)}), \quad (44)$$

which coincides exactly with the new rule of combination expressed previously.

3.7. The Generalized Entropy Like Measure of a Source

The evaluation of the entropy $H(m)$ of a given source from the direct extension of its classical definition with convention [7] $0\ln(0) = 0$ and with bba $m(.)$, i.e.

$$H(m) = -\sum_{A \in D^{\Theta}} m(A)\ln(m(A))$$

seems not to be the best measure for the self-information of a general (uncertain and paradoxical) source of information because it does not catch the intrinsic informational strength (i.i.s. for short) $s(A)$ of each proposition A involved in the evaluation of the entropy of the source. An extension of the classical entropy in the DST framework had already been proposed in 1983 by R. Yager.[71] It is based on the weight of conflict between the belief function Bel and the certain support function Bel_A focused on each proposition A.

Definition 11. *In the classical definition (based only on a probability measure), one always has $s(A) \equiv |A| = 1$. This does not hold in our general theory of plausible and paradoxical reasoning and we propose to generalize the notion of entropy in the following manner to measure the self-information of a general source of information:*

$$H_g(m) = -\sum_{A \in D^{\Theta}} \frac{1}{s(A)} m(A)\ln(\frac{1}{s(A)} m(A)). \tag{45}$$

$H_g(m)$ will be called from now on the *generalized entropy* of the source associated with gbba $m(.)$. This definition is coherent with the definition of the classical entropy whenever the gbba $m(.)$ reduces to a basic probability assignment. However, in the general case, $H_g(m)$ does not satisfy the properties of the classical entropy (see chap. 1 in [21]). Nevertheless, this generalized entropy-like measure can be useful in practice to solve important problems as it will be seen through next examples. This general definition introduces the intrinsic informational strength (called also here the hyper-cardinality) $s(A)$ of a general (irreductible) proposition A which can be derived from the two following important rules

$$s\left(\bigcup_{i=1,n} B_i\right) = s\left(B_1 \cup \ldots \cup B_n\right) = \frac{\sum_{i=1,n} 1/s\left(B_i\right)}{\prod_{i=1,n} 1/s\left(B_i\right)}, \tag{46}$$

$$s\left(\bigcap_{i=1,n} B_i\right) = s\left(B_1 \cap \ldots \cap B_n\right) = \frac{\prod_{i=1,n} s\left(B_i\right)}{\sum_{i=1,n} s\left(B_i\right)}. \tag{47}$$

It is very important to note that these rules apply only on irreductible propositions (logical atoms) A. A proposition A is said to be irreductible (or, equivalently, has a compact form) if and only if it does not admit other equivalent form with a smaller number of operands and operators. For example, $(\theta_1 \cup \theta_3) \cap (\theta_2 \cup \theta_3)$ is not an irreductible proposition since it can be reduced to its equivalent logical atom $(\theta_1 \cap \theta_2) \cup \theta_3$. To compute the i.i.s. $s(A)$ of any proposition A using the rules (46) and (47), the proposition has first to be reduced to its minimal representation (irreductible form).

Example 13. Here are few examples of the value of the hyper-cardinality for some elementary and composite irreductible propositions A. We recall that θ_i involved in A are singletons such that $|\theta_i| = 1$.

$$A = \theta_1 \cup \theta_2 \Rightarrow s(A) = 2$$
$$A = \theta_1 \cap \theta_2 \Rightarrow s(A) = 1/2$$
$$A = \theta_1 \cup \theta_2 \cup \theta_3 = (\theta_1 \cup \theta_2) \cup \theta_3 = \theta_1 \cup (\theta_2 \cup \theta_3) = \theta_2 \cup (\theta_1 \cup \theta_3) \Rightarrow s(A) = 3$$
$$A = \theta_1 \cap \theta_2 \cap \theta_3 = (\theta_1 \cap \theta_2) \cap \theta_3 = \theta_1 \cap (\theta_2 \cap \theta_3) = \theta_2 \cap (\theta_1 \cap \theta_3) \Rightarrow s(A) = 1/3$$
$$A = (\theta_1 \cap \theta_2) \cup \theta_3 \Rightarrow s(A) = 3/2$$
$$A = (\theta_1 \cup \theta_2) \cap \theta_3 \Rightarrow s(A) = 2/3$$
$$A = (\theta_1 \cap \theta_2) \cup (\theta_3 \cap \theta_4) \Rightarrow s(A) = 1$$
$$A = (\theta_1 \cup \theta_2) \cap (\theta_3 \cup \theta_4) \Rightarrow s(A) = 1$$
$$A = (\theta_1 \cap \theta_2) \cup (\theta_3 \cap \theta_4 \cap \theta_5) \Rightarrow s(A) = 5/6$$
$$A = (\theta_1 \cup \theta_2) \cap (\theta_3 \cup \theta_4 \cup \theta_5) \Rightarrow s(A) = 6/5$$

Thus, the evaluation of $s(A)$ for any general irreductible proposition A can always be obtained from the two basic rules (46) and (47). This generalized definition makes sense with the notion of entropy and is coherent with classical definition (i.e. $H_g(m) \equiv H(m)$ when $m(.)$ becomes a bayesian bpa $p(.)$).

Proposition 5. *Let $\Theta = \{\theta_1, \ldots, \theta_n\}$ be a general frame of discernment of the problem under consideration and a general body of evidence with information granule $m(.)$ on D^Θ, then the generalized entropy $H_g(m)$ takes its minimal value $-n \ln(n)$ when the source provides the maximum of paradox which is obtained when $m(\theta_1 \cap \ldots \cap \theta_n) = 1$.*

But it is important to note that the maximum of uncertainty is not obtained when $m(\theta_1 \cup \ldots \cup \theta_n) = 1$ but rather for a specific bba $m(.)$ which distributes some weight of evidence assignment to each proposition $A \in D^\Theta$ because there is less information (from the information theory viewpoint) when, rather than only one, several propositions with non nul bba exist. One has also to take into account the intrinsic self-information of the propositions to get a good measure of global information provided by a source. The generalized entropy includes both aspects of the information (the intrinsic and the classical aspect). The uniform distribution for $m(.)$ does not generate the maximum generalized-entropy because of the different intrinsic self-information of each proposition (see next example). The generalized entropy $H_g(m)$ of any source, defined with respect to a frame Θ with a given bba $m(.)$, appears to be a very promising and useful tool to measure the degree of uncertainty and paradox of any given source of information.

Example 14. We give here some values of $H_g(m)$ for different kinds of sources of information over the same frame $\Theta = \{\theta_1, \theta_2\}$. The sources have been classified from the most informative one $\mathcal{B}_1$ up to the less informative one $\mathcal{B}_{16}$. $\mathcal{B}_{16}$ corresponds to the source containing minimal information on the hyper-power set of the frame Θ (thus $\mathcal{B}_{16}$ has the minimal discrimination power between all possible propositions). There is no source $\mathcal{B}_k$ such that $H_g^{\mathcal{B}_k}(m) > H_g^{\mathcal{B}_{16}}(m)$ for this simple example. Finding $m^*(.)$ such that $H_g(m^*)$ takes its maximal value for a general frame Θ with $|\Theta| = n$ is called the *general whitening source problem*. No solution for this problem has been obtained so far.

$\mathcal{B}_1$ is the most informative source because all the weights of evidence about the truth are focused only on the smaller element $\theta_1 \cap \theta_2$ of hyper-powerset D^Θ. $\mathcal{B}_2$ is less informative than $\mathcal{B}_1$ because there exists an ambiguity between the two propositions $\theta_1 \cup \theta_2$ and $\theta_1 \cap \theta_2$. $\mathcal{B}_3$ and $\mathcal{B}_4$ are less informative than $\mathcal{B}_1$ because the weights of evidence about the truth are focused on larger elements (θ_1 or θ_2 respectively) of D^Θ. $\mathcal{B}_6$ is less informative than $\mathcal{B}_3$ or $\mathcal{B}_4$ because the weight of evidence about the truth is focused on a bigger element $\theta_1 \cup \theta_2$ of D^Θ. $\mathcal{B}_7$ is less informative than previous sources since there is ambiguity between the two propositions θ_1 and θ_2, but it is more informative than $\mathcal{B}_9$ since the discrimination power (our easiness to decide which proposition supports the truth) is higher with $\mathcal{B}_7$ than with $\mathcal{B}_9$. Note that even if in this very simple example, it is not obvious that $\mathcal{B}_{16}$ is the white (least informative) source of information. Most of the readers would have probably thought to choose either $\mathcal{B}_6$ or $\mathcal{B}_{15}$. This comes from the confusion between the intrinsic information supported by the proposition itself and the information supported by the whole bba $m(.)$.

	$m(\theta_1)$	$m(\theta_2)$	$m(\theta_1 \cup \theta_2)$	$m(\theta_1 \cap \theta_2)$	$H_g(m)$
$\mathcal{B}_1$	0	0	0	1	-1.386
$\mathcal{B}_2$	0	0	0.3	0.7	-0.186
$\mathcal{B}_3$	1	0	0	0	0
$\mathcal{B}_4$	0	1	0	0	0
$\mathcal{B}_5$	0.1	0.2	0	0.7	0.081
$\mathcal{B}_6$	0	0	1	0	0.346
$\mathcal{B}_7$	0.8	0.2	0	0	0.500
$\mathcal{B}_8$	0	0	0.7	0.3	0.673
$\mathcal{B}_9$	0.5	0.5	0	0	0.693
$\mathcal{B}_{10}$	0.7	0.2	0.1	0	0.721
$\mathcal{B}_{11}$	0.7	0.2	0	0.1	0.893
$\mathcal{B}_{12}$	0.1	0.2	0.7	1	0.919
$\mathcal{B}_{13}$	0.1	0.2	0.3	0.4	1.015
$\mathcal{B}_{14}$	0.1	0.2	0.4	0.3	1.180
$\mathcal{B}_{15}$	0.25	0.25	0.25	0.25	1.299
$\mathcal{B}_{16}$	0.25	0.25	0.35	0.15	1.359

3.8. Blackman's Example Revisited

Let's take back the Blackman's example described in example 4 for the very simple assignment problem. In the DSmT framework, one has to deal with the following prior (predicted) and observed gbba defined on hyper-power set $D^\Theta = \{\emptyset, \theta_1, \theta_2, \theta_1 \cup \theta_2, \theta_1 \cap \theta_2\}$ as follows:

$$\begin{array}{llll} m_{T_1}(\theta_1) = 0.5 & m_{T_1}(\theta_2) = 0.5 & m_{T_1}(\theta_1 \cup \theta_2) = 0 & m_{T_1}(\theta_1 \cap \theta_2) = 0 \\ m_{T_2}(\theta_1) = 0.1 & m_{T_2}(\theta_2) = 0.1 & m_{T_2}(\theta_1 \cup \theta_2) = 0.8 & m_{T_2}(\theta_1 \cap \theta_2) = 0 \\ m_Z(\theta_1) = 0.5 & m_Z(\theta_2) = 0.5 & m_Z(\theta_1 \cup \theta_2) = 0 & m_Z(\theta_1 \cap \theta_2) = 0 \end{array}$$

Using the DSm rule of combination, we get now easily the following results

$$\begin{array}{lll} m_{T_1Z}(\theta_1) = 0.25 & m_{T_1Z}(\theta_2) = 0.25 & m_{T_1Z}(\theta_1 \cap \theta_2) = 0.5 \\ m_{T_2Z}(\theta_1) = 0.45 & m_{T_2Z}(\theta_2) = 0.45 & m_{T_2Z}(\theta_1 \cap \theta_2) = 0.1 \end{array}$$

The values of the generalized entropy of the updated gbba m_{T_1Z} and m_{T_2Z} are $H_g(m_{T_1Z}) \simeq 0.69$ and $H_g(m_{T_2Z}) \simeq 1.04$. The increase of the generalized entropies (i.e. the difference between the predicted and updated generalized entropies) are given by $\Delta_1 \triangleq H_g(m_{T_1Z}) - H_g(m_{T_1}) = 0.69 - 0.69 = 0$ and $\Delta_2 \triangleq H_g(m_{T_2Z}) - H_g(m_{T_2}) = 1.04 - 0.83 = 0.21$. This result means that the incorrect assignment $m_{T_2} \oplus m_Z$ has noticeably increased the generalized entropy of the system as one would have rightfully expected. The best assignment solution is obtained by selecting the fusion (assignment between a track T and a measurement Z) which generates the smallest increase of the generalized entropy. In this framework and in this case, the Tchamova's approach based on the minimum city-block or Euclidean distances provides also the correct assignment Z with track T_1 since $d_1(T_1,T_1Z) < d_1(T_2,T_2Z)$ and $d_2(T_1,T_1Z) < d_2(T_2,T_2Z)$ because one has

$$\begin{array}{lcl} d_1(T_1,T_1Z) = 1 & \text{and} & d_1(T_2,T_2Z) = 1.6 \\ d_2(T_1,T_1Z) = 0.612 & \text{and} & d_2(T_2,T_2Z) = 0.946 \end{array}$$

Neither the use of classical entropy $H(m)$ nor the entropy evaluated from pignistic probabilities allow us to get the correct assignment solution from the DST framework in this example.

Let's consider now the previous predicted gbba $m_{T_1}(.)$ and $m_{T_2}(.)$ but now with an observation bba which agrees with $m_{T_2}(.)$, i.e.

$$m_Z(\theta_1) = 0.1 \qquad m_Z(\theta_2) = 0.1 \qquad m_Z(\theta_1 \cup \theta_2) = 0.8 \qquad m_Z(\theta_1 \cap \theta_2) = 0$$

Using the DSm rule of combination, we get now the following results

$$\begin{array}{l} m_{T_1Z}(\theta_1) = 0.45;\ m_{T_1Z}(\theta_2) = 0.45;\ m_{T_1Z}(\theta_1 \cup \theta_2) = 0; \quad m_{T_1Z}(\theta_1 \cap \theta_2) = 0.1 \\ m_{T_2Z}(\theta_1) = 0.17;\ m_{T_2Z}(\theta_2) = 0.17;\ m_{T_1Z}(\theta_1 \cup \theta_2) = 0.64;\ m_{T_2Z}(\theta_1 \cap \theta_2) = 0.02 \end{array}$$

The generalized entropies of the two possible assignments take now the following values $H_g(m_{T_1Z}) \simeq 1.0405$ and $H_g(m_{T_2Z}) \simeq 1.0958$, which are very close but the entropy increases become now $\Delta_1 \triangleq H_g(m_{T_1Z}) - H_g(m_{T_1}) = 1.0405 - 0.69 = 0.35$ and $\Delta_2 \triangleq H_g(m_{T_2Z}) - H_g(m_{T_2}) = 1.095 - 0.83 = 0.265$. By selecting the smallest increase of the generalized entropies, we get again the correct assignment Z with track T_2 for this second case. As within the same example discussed in the DST framework, the minimum distance approach fails here to obtain the correct assignment since one has now $d_1(T_1,T_1Z) < d_1(T_2,T_2Z)$ and $d_2(T_1,T_1Z) < d_2(T_2,T_2Z)$ because

$$\begin{array}{lcl} d_1(T_1,T_1Z) = 0.2 & \text{and} & d_1(T_2,T_2Z) = 0.32 \\ d_2(T_1,T_1Z) = 0.122 & \text{and} & d_2(T_2,T_2Z) = 0.189 \end{array}$$

In concluding remark, we have shown through this simple example how a simple and unique criteria based on our *generalized entropy-like measure* drawn from our DSmT can serve as an useful tool to solve the assignment problem for both cases investigated here. No case-dependent approach is then required here to get the correct solution as we had already argued in example 4. However, more theoretical investigations must be performed in order to prove that our criteria is actually the best one to solve the assignment problem in general.

3.9. Zadeh's Example Revisited

Let's take back the disturbing Zadeh's example [74] given in section 2.4. Two doctors examine a patient and agree that it suffers from either meningitis (M), concussion (C) or brain tumor (T). Thus, $\Theta = \{M, C, T\}$. Assume that the two doctors agree in their low expectation of a tumor, but disagree on the likely cause and provide the following diagnosis

$$m_1(M) = 0.99 \qquad m_1(T) = 0.01$$

and $\forall A \in D^\Theta, A \neq T, A \neq M, m_1(A) = 0$

$$m_2(C) = 0.99 \qquad m_2(T) = 0.01$$

and $\forall A \in D^\Theta, A \neq T, A \neq C, m_2(A) = 0$

The new general rule of combination (26) yields the following combined information granule

$$m(M \cap C) = 0.9801 \qquad m(M \cap T) = 0.0099$$

$$m(C \cap T) = 0.0099 \qquad m(T) = 0.0001$$

From this granule, one gets

$$\begin{aligned}
\text{Bel}(M) &= m(M \cap C) + m(M \cap T) = 0.99, \\
\text{Bel}(C) &= m(M \cap C) + m(T \cap C) = 0.99, \\
\text{Bel}(T) &= m(T) + m(M \cap T) + m(C \cap T) = 0.0199.
\end{aligned}$$

If both doctors can be considered equally reliable, the combined information granule $m(.)$ mainly focuses weight of evidence on the paradoxical proposition $M \cap C$ which means that the patient suffers from both meningitis and concussion but almost surely not from brain tumor. Actually, this conclusion is coherent with the common sense. Then, no therapy for brain tumor (like heavy and ever risky brain surgical intervention) will be chosen in such case. This really helps to take important decision to save the life of the patient in this example. A deeper medical examination adapted to both meningitis and concussion will almost surely be done before applying the best therapy for the patient. Just remember that in this case, the DST had concluded that the patient had brain tumor with certainty ....

3.10. Mahler's Example Revisited

Let's consider now the following example excerpt from a paper by Ronald Mahler.[36] We consider that our classification knowledge base consists of the three (imaginary) new and rare diseases corresponding to following frame of discernment

$$\Theta = \{\theta_1 = \textit{kotosis}, \theta_2 = \textit{phlegaria}, \theta_3 = \textit{pinpox}\}.$$

We assume that the three diseases are equally likely to occur in the patient population but there is some evidence that *phlegaria* and *pinpox* are the same disease and there is also a small possibility that *kotosis* and *phlegaria* might be the same disease. Finally, there is a small possibility that all three diseases are the same. This information can be expressed by assigning a priori bba as follows

$$\begin{array}{lll} m_0(\theta_1) = 0.2 & m_0(\theta_2) = 0.2 & m_0(\theta_3) = 0.2 \\ m_0(\theta_2 \cap \theta_3) = 0.2 & m_0(\theta_1 \cap \theta_2) = 0.1 & m_0(\theta_1 \cap \theta_2 \cap \theta_3) = 0.1 \end{array}$$

Let $\text{Bel}(.)$ be the prior belief measure corresponding to this prior bba $m(.)$. Now assume that Doctor D_1 and Doctor D_2 examine a patient and deliver diagnoses with following reports:

- Report for D_1: $m_1(\theta_1 \cup \theta_2 \cup \theta_3) = 0.05$ $m_1(\theta_2 \cup \theta_3) = 0.95$
- Report for D_2: $m_2(\theta_1 \cup \theta_2 \cup \theta_3) = 0.20$ $m_2(\theta_2) = 0.80$

The combination of the evidences provided by the two doctors $m' = m_1 \oplus m_2$ obtained by the general rule of combination (26) yields the following bba $m'(.)$

$$m'(\theta_2) = 0.8 \qquad m'(\theta_2 \cup \theta_3) = 0.19 \qquad m'(\theta_1 \cup \theta_2 \cup \theta_3) = 0.01$$

The combination of bba $m'(.)$ with prior evidence $m_0(.)$ yields the final bba $m = m_0 \oplus m' = m_0 \oplus [m_1 \oplus m_2]$ with

$$\begin{array}{lll} m(\theta_1) = 0.002 & m(\theta_2) = 0.200 & m(\theta_3) = 0.040 \\ m(\theta_1 \cap \theta_2) = 0.260 & m(\theta_2 \cap \theta_3) = 0.360 & m(\theta_1 \cap \theta_2 \cap \theta_3) = 0.100 \\ m(\theta_1 \cap (\theta_2 \cup \theta_3)) = 0.038 & & \end{array}$$

Therefore, the final belief function given by (22) is

$$\begin{aligned} &\text{Bel}(\theta_1) = 0.002 + 0.260 + 0.100 + 0.038 = 0.400 \\ &\text{Bel}(\theta_2) = 0.200 + 0.260 + 0.360 + 0.100 = 0.920 \\ &\text{Bel}(\theta_3) = 0.040 + 0.360 + 0.100 = 0.500 \\ &\text{Bel}(\theta_1 \cap \theta_2) = 0.260 + 0.100 = 0.360 \\ &\text{Bel}(\theta_2 \cap \theta_3) = 0.360 + 0.100 = 0.460 \\ &\text{Bel}(\theta_1 \cap (\theta_2 \cup \theta_3)) = 0.038 + 0.100 = 0.138 \\ &\text{Bel}(\theta_1 \cap \theta_2 \cap \theta_3) = 0.100 \end{aligned}$$

Thus, on the basis of all available evidence, we are able to conclude with high a degree of belief that the patient has phlegaria which is coherent with the Mahler's conclusion based on his Conditioned Dempster-Shafer theory developed from his conditional event algebra, although a totally new and more simple approach has been adopted here.

3.11. A Thief Identification Example

Let's revisit now a very simple and classical thief identification example. Assume that a 75 years old grandfather is taking a walk with his 9 years old grandson in a park. They saw at a distance of 50 meters a 45 years old pickpocket robbering the bag of an old lady. A policeman looking for some witnesses of this event asks separately the grandfather and his grandchild if they have seen the thief (they both answer yes) and how old approximately was the thief (a young or an old man). The grandfather (source of information B_1 reports that the thief was a young man with high confidence 0.99 and with only a low uncertainty 0.01. His grandson reports that the thief was a old man with high confidence 0.99 and with only a low uncertainty 0.01. These two witnesses provide fair reports (with respect to their own world of knowledge) even if apparently they appear as almost fully paradoxical. The policeman then sends the two reports with only the minimal information about witnesses (saying only their names and that they were a priori fully trustable) to an investigator. The investigator has no possibility to meet or to call back the witnesses in order to get more details.

Under such condition, what would be the best reasoning of the investigator to infer the age of the thief to eventually help to catch him? Such kind of simple example occurs quite frequently in many witnesses problems actually. A rational investigator will almost surely suspect a mistake or an error in one or both reports since they appear apparently in (almost) full contradiction. The investigator will then try to take his final decision with some better information (if any). If the investigator uses our new plausible and paradoxical reasoning, he will define the following bba with respect to the frame of discernment $\Theta = \{\theta_1 = \text{young}, \theta_2 = \text{old}\}$ and the available reports $\mathcal{B}_1$ and $\mathcal{B}_2$ with bba

$$m_1(\theta_1) = 0.99 \qquad m_1(\theta_2) = 0 \qquad m_1(\theta_1 \cup \theta_2) = 0.01 \qquad m_1(\theta_1 \cap \theta_2) = 0$$

$$m_2(\theta_1) = 0 \qquad m_2(\theta_2) = 0.99 \qquad m_2(\theta_1 \cup \theta_2) = 0.01 \qquad m_2(\theta_1 \cap \theta_2) = 0$$

The fusion of these two sources of information yields the global bba $m(.)$ with

$$m(\theta_1) = 0.0099 \qquad m(\theta_2) = 0.0099$$

$$m(\theta_1 \cup \theta_2) = 0.0001 \qquad m(\theta_1 \cap \theta_2) = 0.9801$$

Thus, from this global information, the investigator has no better choice but to consider with almost certainty that the thief was both a young and old man. By assuming that the expected life duration is around 80 years, the inspector will deduce that the true age of the thief is around 40 years old which is not too far from the truth. At least, this conclusion could be helpful to interrogate some suspicious individuals.

3.12. A Model to Generate Information Granules $m(.)$ from Intervals

We present here a model to generate information granules $m(.)$ from information represented by intervals. It is very common in practice that uncertain sources of information provide evidence on a given proposition in term of basic intervals $[\epsilon_*, \epsilon^*] \subset [0, 1]$ rather than a direct bba $m(.)$. In such cases, some preprocessing must be performed before applying the general rule of combination between such sources to take the final decision. We present here a model to generate information granules $m(.)$ from information represented by intervals. It is very common in practice that uncertain sources of information provide evidence on a given proposition in term of basic intervals $[\epsilon_*, \epsilon^*] \subset [0, 1]$ rather than a direct bba $m(.)$. In such cases, some preprocessing must be done before applying the general rule of combination between such sources to take the final decision.

In the DST framework, we recall that the simpliest and easiest transformation to convert $[\epsilon_*, \epsilon^*]$ into bba has already been proposed by A. Appriou [1] and successfully implemented.[14] The basic idea was to interpret ϵ_* as the minimal credibility committed to A and ϵ^* as the plausibility committed to A. In other words, the Appriou's transformation model within the DST framework is the following one

$$\begin{aligned}
\epsilon_* &= m(A), \\
\epsilon^* &= 1 - m(A^c), \\
\epsilon^* - \epsilon_* &= m(A \cup A^c).
\end{aligned}$$

This model can be directly extended within our new theory of plausible and paradoxical reasoning by setting now [4]

$$\begin{aligned}
\epsilon_* &= m(A) + \frac{1}{2} m(A \cap A^c), \\
\epsilon^* &= 1 - m(A^c) - \frac{1}{2} m(A \cap A^c), \\
\epsilon^* - \epsilon_* &= m(A \cup A^c).
\end{aligned}$$

[4] The notation A^c has been kept here for simplicity but in our DSmT A^c must not be interpreted directly as the complement of A since $m(A \cap A^c)$ can take a positive value ≤ 1, but as a (partial overlapping) *paradoxical alternative* (see the forthcoming numerical examples).

or equivalently

$$m(A) + \frac{1}{2}m(A \cap A^c) = \epsilon_*, \tag{48}$$

$$m(A^c) + \frac{1}{2}m(A \cap A^c) = 1 - \epsilon^*, \tag{49}$$

$$m(A \cup A^c) = \epsilon^* - \epsilon_*. \tag{50}$$

This appealing model presents nice properties especially when $\epsilon^* = \epsilon_* = 0$ or when $\epsilon^* = \epsilon_* = 1$. Moreover, this model is coherent with the previous Appriou's model whenever the source becomes rational (i.e $m(A \cap A^c) = 0$). This new model presents however a degree of freedom since one has only two constraints (48) and (49) for three unknowns $m(A)$, $m(A^c)$ and $m(A \cap A^c)$. Thus in general, without an additional constraint, many possible choices for $m(A)$, $m(A^c)$ and $m(A \cap A^c)$ exist and, therefore, several bba $m(.)$ satisfy this transformation model. Without extra prior information, it becomes difficult to justify the choice of a specific bba versus all other admissible possibilities for $m(.)$.

To overcome this important drawback, we propose to add the constraint on the maximization of the generalized-entropy $H_g(m)$. This will allow us to obtain from $[\epsilon_*, \epsilon^*]$ the unique bba $m(.)$ having the minimum of specificity and admissible with our transformation model. From definition of $H_g(m)$ and previous equations (48)-(50), one gets

$$\begin{aligned} H_g(m) = &- (\epsilon_* - m(A \cap A^c)/2)\ln(\epsilon_* - m(A \cap A^c)/2) \\ &- (1 - \epsilon^* - m(A \cap A^c)/2)\ln(1 - \epsilon^* - m(A \cap A^c)/2) \\ &- \frac{1}{2}(\epsilon^* - \epsilon_*)\ln(\frac{1}{2}(\epsilon^* - \epsilon_*)) \\ &- 2m(A \cap A^c)\ln(2m(A \cap A^c)). \end{aligned}$$

The maximization of $H_g(m)$ is obtained for the optimal value $m^\star(A \cap A^c)$ such that $\frac{\partial H_g}{\partial m(A \cap A^c)}(m^\star(A \cap A^c)) = 0$ and $\frac{\partial^2 H_g}{\partial m(A \cap A^c)^2}(m^\star(A \cap A^c)) < 0$. The annulation of the first derivative is obtained by the solution of the equation

$$\frac{1}{2}\ln(\epsilon_* - m^\star/2) + \frac{1}{2}\ln(1 - \epsilon^* - m^\star/2) - 2m^\star\ln(2m^\star) - 1 = 0$$

or equivalently after basic algebraic manipulations

$$64e^2(m^\star)^4 - (m^\star)^2 + 2(1 - \epsilon^* + \epsilon_*)m^\star - 4(1 - \epsilon^*)\epsilon_* = 0. \tag{51}$$

The solution of this equation can be easily found using classical numerical methods. It is also easy to check that the second derivative is always negative and therefore $H_g(m)$

reaches its maximal value when

$$m(A) + \frac{1}{2}m^\star(A \cap A^c) = \epsilon_*, \tag{52}$$

$$m(A^c) + \frac{1}{2}m^\star(A \cap A^c) = 1 - \epsilon^*, \tag{53}$$

$$m(A \cup A^c) = \epsilon^* - \epsilon_*. \tag{54}$$

This completes the definition of our new transformation model. Note that $[\epsilon_*, \epsilon^*]$ can also be generated from bba $m(.)$ through (48)-(50).

Example 15. for $[\epsilon_*, \epsilon^*] = [0.0, 0.0]$, one gets

$m(A \cap A^c) = 0.000 \quad m(A) = 0.000 \quad m(A^c) = 1.000 \quad m(A \cup A^c) = 0.000$

Example 16. for $[\epsilon_*, \epsilon^*] = [0.2, 0.2]$, one gets

$m(A \cap A^c) \approx 0.164 \quad m(A) \approx 0.118 \quad m(A^c) \approx 0.718 \quad m(A \cup A^c) = 0.000$

Example 17. for $[\epsilon_*, \epsilon^*] = [0.5, 0.5]$, one gets

$m(A \cap A^c) \approx 0.192 \quad m(A) \approx 0.404 \quad m(A^c) \approx 0.404 \quad m(A \cup A^c) = 0.000$

Example 18. for $[\epsilon_*, \epsilon^*] = [0.8, 0.8]$, one gets

$m(A \cap A^c) \approx 0.164 \quad m(A) \approx 0.718 \quad m(A^c) \approx 0.118 \quad m(A \cup A^c) = 0.000$

Example 19. for $[\epsilon_*, \epsilon^*] = [1.0, 1.0]$, one gets

$m(A \cap A^c) = 0.000 \quad m(A) = 1.000 \quad m(A^c) = 0.000 \quad m(A \cup A^c) = 0.000$

Example 20. for $[\epsilon_*, \epsilon^*] = [0.2, 0.4]$, one gets

$m(A \cap A^c) \approx 0.152 \quad m(A) \approx 0.124 \quad m(A^c) \approx 0.524 \quad m(A \cup A^c) = 0.200$

Example 21. for $[\epsilon_*, \epsilon^*] = [0.6, 0.8]$, one gets

$m(A \cap A^c) \approx 0.152 \quad m(A) \approx 0.524 \quad m(A^c) \approx 0.124 \quad m(A \cup A^c) = 0.200$

Example 22. for $[\epsilon_*, \epsilon^*] = [0.4, 0.6]$, one gets

$m(A \cap A^c) \approx 0.170 \quad m(A) \approx 0.315 \quad m(A^c) \approx 0.315 \quad m(A \cup A^c) = 0.200$

Example 23. for $[\epsilon_*, \epsilon^*] = [0.3, 0.9]$, one gets

$$m(A \cap A^c) \approx 0.100 \qquad m(A) \approx 0.250 \qquad m(A^c) \approx 0.050 \qquad m(A \cup A^c) = 0.600$$

Example 24. for $[\epsilon_*, \epsilon^*] = [0.0, 1.0]$, one gets

$$m(A \cap A^c) = 0.000 \qquad m(A) = 0.000 \qquad m(A^c) = 0.000 \qquad m(A \cup A^c) = 1.000$$

4. Conclusions

In this paper, the foundations for a new theory of paradoxical and plausible reasoning have been developed. The DSmT takes into account in the combination process itself the possibility for uncertain and paradoxical information. The basis for the development of this theory is to work with the hyper-power set of the frame of discernment relative to the problem under consideration rather than its classical power set since, in general, the frame of discernment cannot be fully described in terms of an exhaustive and exclusive list of disjoint elementary hypotheses. In such general case, no refinement is possible if applying directly the Dempster-Shafer theory (DST) of evidence. In DSmT, the rule of combination is justified from the maximum entropy principle and there is no mathematical impossibility to combine sources of evidence even if they appear at first glance in contradiction (in the Shafer's sense) since the paradox between sources is fully taken into account in our formalism. We have also shown that, in general, the combination of evidence yields unavoidable paradoxes. Through many illustrative examples it was shown, that the implementation of the proposed theory leads to conclusions that agree with human reasoning and can be very helpful in making decisions for some complex problems where the classical DST usually fails. This new theory provides also a theoretical bridge between the combination of paradoxical source of information and the Smarandache's logic.

Appendix

We prove here that the hyper-power set D^Θ of $\Theta = \{\theta_1, \theta_2, \theta_3\}$ is given by the set of the following 19 irreductible propositions:

$\alpha_0 \triangleq \emptyset$	
$\alpha_1 \triangleq \theta_1$	$\alpha_{10} \triangleq \theta_1 \cup \theta_2 \cup \theta_3$
$\alpha_2 \triangleq \theta_2$	$\alpha_{11} \triangleq \theta_1 \cap \theta_2 \cap \theta_3$
$\alpha_3 \triangleq \theta_3$	$\alpha_{12} \triangleq (\theta_1 \cup \theta_2) \cap \theta_3$
$\alpha_4 \triangleq \theta_1 \cup \theta_2$	$\alpha_{13} \triangleq (\theta_1 \cup \theta_3) \cap \theta_2$
$\alpha_5 \triangleq \theta_1 \cup \theta_3$	$\alpha_{14} \triangleq (\theta_2 \cup \theta_3) \cap \theta_1$
$\alpha_6 \triangleq \theta_2 \cup \theta_3$	$\alpha_{15} \triangleq (\theta_1 \cap \theta_2) \cup \theta_3$
$\alpha_7 \triangleq \theta_1 \cap \theta_2$	$\alpha_{16} \triangleq (\theta_1 \cap \theta_3) \cup \theta_2$
$\alpha_8 \triangleq \theta_1 \cap \theta_3$	$\alpha_{17} \triangleq (\theta_2 \cap \theta_3) \cup \theta_1$
$\alpha_9 \triangleq \theta_2 \cap \theta_3$	$\alpha_{18} \triangleq (\theta_1 \cup \theta_2) \cap (\theta_1 \cup \theta_3) \cap (\theta_2 \cup \theta_3)$

We need to verify that $\forall \alpha_i \in D^\Theta, \forall \alpha_j \in D^\Theta, (\alpha_i \cup \alpha_j) \in D^\Theta$ and $(\alpha_i \cap \alpha_j) \in D^\Theta$.

First, note that $\forall \alpha_i, i = 0, \ldots, 18$, one always has

$$\alpha_0 \cap \alpha_i = \alpha_0 \qquad \text{and} \qquad \alpha_0 \cup \alpha_i = \alpha_i.$$

Let's compute now all $\alpha_i \cap \alpha_j$ for $i, j = 1, \ldots, 18$. Using classical intersection operator on sets, we get the following result summarized in the symmetric Table 2.

Hence, we have just proved here that $\forall \alpha_i, \alpha_j \in D^\Theta$, $\alpha_i \cap \alpha_j \in D^\Theta$. It remains now to compute all $\alpha_i \cup \alpha_j$ for $i, j = 1, \ldots, 18$. Using classical union operator on sets, we get the following result summarized in the symmetric Table 3.

Therefore, one has proved that $\forall \alpha_i \in D^\Theta, \forall \alpha_j \in D^\Theta, (\alpha_i \cup \alpha_j) \in D^\Theta$ and $(\alpha_i \cap \alpha_j) \in D^\Theta$ and the set $\{\alpha_0, \ldots, \alpha_{18}\}$ corresponds effectively to the hyper-power set of $\Theta = \{\theta_1, \theta_2, \theta_3\}$ we were looking for.

Table 2

$\cap$	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9	α_{10}	α_{11}	α_{12}	α_{13}	α_{14}	α_{15}	α_{16}	α_{17}	α_{18}
α_1	α_1	α_7	α_8	α_1	α_1	α_{14}	α_7	α_8	α_{11}	α_1	α_{11}	α_8	α_7	α_{14}	α_{14}	α_{14}	α_1	α_{14}
α_2	α_7	α_2	α_9	α_2	α_{13}	α_2	α_7	α_{11}	α_9	α_2	α_{11}	α_9	α_{13}	α_7	α_{13}	α_2	α_7	α_{13}
α_3	α_8	α_9	α_3	α_{12}	α_3	α_3	α_{11}	α_8	α_9	α_3	α_{11}	α_{12}	α_9	α_8	α_3	α_{12}	α_{12}	α_{12}
α_4	α_1	α_2	α_{12}	α_4	α_{17}	α_{16}	α_7	α_8	α_9	α_4	α_{11}	α_{12}	α_{13}	α_{14}	α_{18}	α_{16}	α_{17}	α_{18}
α_5	α_1	α_{13}	α_3	α_{17}	α_5	α_{15}	α_7	α_8	α_9	α_9	α_{11}	α_{12}	α_{13}	α_{14}	α_{18}	α_{18}	α_{17}	α_{18}
α_6	α_{14}	α_2	α_3	α_{16}	α_{15}	α_6	α_7	α_8	α_9	α_6	α_{11}	α_{12}	α_{13}	α_{14}	α_{15}	α_{16}	α_{18}	α_{18}
α_7	α_7	α_7	α_{11}	α_7	α_7	α_7	α_7	α_{11}	α_{11}	α_7	α_{11}	α_{12}	α_7	α_7	α_7	α_{16}	α_7	α_7
α_8	α_8	α_{11}	α_8	α_8	α_8	α_8	α_{11}	α_8	α_{11}	α_8	α_{11}	α_{12}	α_{11}	α_8	α_8	α_8	α_8	α_8
α_9	α_{11}	α_9	α_9	α_9	α_9	α_9	α_{11}	α_{11}	α_9	α_9	α_{11}	α_9	α_9	α_{11}	α_9	α_9	α_9	α_9
α_{10}	α_1	α_2	α_3	α_4	α_9	α_6	α_7	α_8	α_9	α_{10}	α_{11}	α_{12}	α_{13}	α_{14}	α_{15}	α_{16}	α_{17}	α_{18}
α_{11}	α_{11}	α_{11}	α_{11}	α_{11}	α_{11}	α_{11}	α_{11}	α_{11}	α_{11}	α_{11}	α_{11}	α_{11}	α_{11}	α_{11}	α_{11}	α_{11}	α_{11}	α_{11}
α_{12}	α_8	α_9	α_{12}	α_{12}	α_{12}	α_{12}	α_{12}	α_{12}	α_9	α_{12}	α_{11}	α_{12}	α_{11}	α_8	α_{12}	α_{12}	α_{12}	α_{12}
α_{13}	α_7	α_{13}	α_9	α_{13}	α_{13}	α_{13}	α_7	α_{11}	α_9	α_{13}	α_{11}	α_{11}	α_{13}	α_7	α_{13}	α_{13}	α_{13}	α_{13}
α_{14}	α_{14}	α_7	α_8	α_{14}	α_{14}	α_{14}	α_7	α_8	α_{11}	α_{14}	α_{11}	α_8	α_7	α_{14}	α_{14}	α_{14}	α_{14}	α_{14}
α_{15}	α_{14}	α_{13}	α_3	α_{18}	α_{18}	α_{15}	α_7	α_8	α_9	α_{15}	α_{11}	α_{12}	α_{13}	α_{14}	α_{15}	α_{18}	α_{18}	α_{18}
α_{16}	α_{14}	α_2	α_{12}	α_{16}	α_{18}	α_{16}	α_{16}	α_8	α_9	α_{16}	α_{11}	α_{12}	α_{13}	α_{14}	α_{18}	α_{16}	α_{18}	α_{18}
α_{17}	α_1	α_7	α_{12}	α_{17}	α_{17}	α_{18}	α_7	α_8	α_9	α_{17}	α_{11}	α_{12}	α_{13}	α_{14}	α_{18}	α_{18}	α_{17}	α_{18}
α_{18}	α_{14}	α_{13}	α_{12}	α_{18}	α_{18}	α_{18}	α_7	α_8	α_9	α_{18}	α_{11}	α_{12}	α_{13}	α_{14}	α_{18}	α_{18}	α_{18}	α_{18}

Table 3

$\cup$	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9	α_{10}	α_{11}	α_{12}	α_{13}	α_{14}	α_{15}	α_{16}	α_{17}	α_{18}
α_1	α_1	α_4	α_5	α_4	α_5	α_{10}	α_1	α_1	α_{17}	α_{10}	α_1	α_{17}	α_{17}	α_1	α_5	α_4	α_{17}	α_{17}
α_2	α_4	α_2	α_6	α_4	α_{10}	α_6	α_2	α_{16}	α_2	α_{10}	α_2	α_{16}	α_2	α_{16}	α_6	α_{16}	α_4	α_{16}
α_3	α_5	α_6	α_3	α_{10}	α_5	α_6	α_{15}	α_3	α_3	α_{10}	α_3	α_3	α_{15}	α_{15}	α_{15}	α_6	α_5	α_{15}
α_4	α_4	α_4	α_{10}	α_4	α_{10}	α_{10}	α_4	α_4	α_4	α_{10}	α_4	α_4	α_4	α_4	α_{10}	α_{10}	α_{10}	α_{10}
α_5	α_5	α_{10}	α_5	α_{10}	α_5	α_{10}	α_5	α_5	α_5	α_{10}	α_5	α_5	α_5	α_5	α_5	α_{10}	α_5	α_5
α_6	α_{10}	α_6	α_6	α_{10}	α_{10}	α_6	α_6	α_6	α_6	α_{10}	α_6	α_6	α_6	α_6	α_6	α_6	α_{10}	α_6
α_7	α_1	α_2	α_{15}	α_4	α_5	α_6	α_7	α_{14}	α_{13}	α_{10}	α_7	α_{18}	α_{13}	α_{14}	α_{15}	α_{16}	α_{17}	α_{18}
α_8	α_1	α_{16}	α_3	α_4	α_5	α_6	α_{14}	α_8	α_{12}	α_{10}	α_8	α_{12}	α_{18}	α_{14}	α_{15}	α_{16}	α_{17}	α_{18}
α_9	α_{17}	α_2	α_3	α_4	α_5	α_6	α_{13}	α_{12}	α_9	α_{10}	α_9	α_{12}	α_{13}	α_{18}	α_{15}	α_{16}	α_{17}	α_{18}
α_{10}	α_{10}	α_{10}	α_{10}	α_{10}	α_{10}	α_{10}	α_{10}	α_{10}	α_{10}	α_{10}	α_{10}	α_{10}	α_{10}	α_{10}	α_{10}	α_{10}	α_{10}	α_{10}
α_{11}	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9	α_{10}	α_{11}	α_{12}	α_{13}	α_{14}	α_{15}	α_{16}	α_{17}	α_{18}
α_{12}	α_{17}	α_{16}	α_3	α_4	α_5	α_6	α_{18}	α_{12}	α_{12}	α_{10}	α_{12}	α_{12}	α_{18}	α_{18}	α_{15}	α_{16}	α_{17}	α_{18}
α_{13}	α_{17}	α_2	α_{15}	α_4	α_5	α_6	α_{13}	α_{18}	α_{13}	α_{10}	α_{13}	α_{18}	α_{13}	α_{18}	α_{15}	α_{16}	α_{17}	α_{18}
α_{14}	α_1	α_{16}	α_{15}	α_4	α_5	α_6	α_{14}	α_{14}	α_{18}	α_{10}	α_{14}	α_{18}	α_{18}	α_{14}	α_{15}	α_{16}	α_{17}	α_{18}
α_{15}	α_5	α_6	α_{15}	α_{10}	α_5	α_6	α_{15}	α_{15}	α_{15}	α_{10}	α_{15}	α_{15}	α_{15}	α_{15}	α_{15}	α_6	α_5	α_{15}
α_{16}	α_4	α_{16}	α_6	α_4	α_{10}	α_6	α_{16}	α_{16}	α_{16}	α_{10}	α_{16}	α_{16}	α_{16}	α_{16}	α_6	α_{16}	α_4	α_{16}
α_{17}	α_{17}	α_4	α_5	α_4	α_5	α_{10}	α_{17}	α_{17}	α_{17}	α_{10}	α_{17}	α_{17}	α_{17}	α_{17}	α_5	α_4	α_{17}	α_{17}
α_{18}	α_{17}	α_{16}	α_{15}	α_4	α_5	α_6	α_{18}	α_{18}	α_{18}	α_{10}	α_{18}	α_{18}	α_{18}	α_{18}	α_{15}	α_{16}	α_{17}	α_{18}

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Foundations for a new theory of plausible and paradoxical reasoning

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Keywords: plausible and paradoxical reasoning, rule of combination, Dempster-Shafer, evidence.

Abstract: This paper presents an original comprehensive approach to plausible and paradoxical reasoning. It provides detailed description of a rule of combination of sources of information in a very general framework allowing for both uncertain and paradoxical information. In this new theory, the rule of combination which takes into account explicitly both conjunctions and disjunctions of assertions in the fusion process, appears to be more simple and general than the Dempster's rule of combination. Through several simple examples, where the Dempster-Shafer theory usually fails, the author shows the strong capacity of the new theory to solve difficult practical problems.

full text

FUZZY LOGIC APPROACH TO ESTIMATING TENDENCIES IN TARGET BEHAVIOR

Albena TCHAMOVA and Tzvetan SEMERDJIEV

1. Introduction

Angle-only tracking systems based on passive sensors are poorly developed due to a number of complications. They receive signals transmitted from other emitters and tend to be less precise than those based on active sensors. However, one important advantage is their vitality of being stealth. In general, passive sensors make only line-of-sight angle detection. In the single sensor case that means that we know only direction of the target as an axis, but the true target position and behavior (approaching or descending) remain unknown. The problem of determining an objects' position without using measurements of the distance to it concerns moving platform applications, astronomy and some military situations, where it is important to estimate the position (respectively the distance to the object) and, in particular, the behavior of moving targets. In military avionics, for example, some fighter defending against a raid may wish to launch a missile as a counteraction to the enemy, but it could not do this until the position and the behavior of the opposing target are not known. In such situations, the uncertainty with respect to the opposite target behavior requires to compensate the missing range by utilizing the extracted from the received emitter's signal attributes. This information can be used to assess tendencies in target's behavior and its location and, consequently, to improve the overall angle-only tracking performance.

The objective of this work is to present an approach for target behavior tendency estimation, based on the application of the principles of fuzzy logic to conventional passive radars. It utilizes the measured emitter's amplitude values in consecutive time moments and uses a set of particular filters design with respective set of possible target behavior models. In real world situations, fuzzy logic provides an approximate but consistent solutions to complex engineering problems, where numerical data usually are noisy and incomplete, and the linguistic information is imprecise and vague. Compared to other methods such as Bayesian and Evidential Reasoning, Fuzzy Logic shows some important advantages: it is suitable and well adapted to use uncertain data and it is a much more expressive tool for codification of expert knowledge; measurement errors are explicitly taken into account; it entails modest computational load and provides decisions in a simple and robust way.

2. Statement of the Problem

In order to track targets using passive sensors it is necessary to compensate the unknown ranges by using additional information received from the emitter. In our case, we assume that the observed target emits constant signal. It is received by the sensor with a non-constant, but a varying strength (referred to as amplitude). The augmented measurement vector at the end of each time interval $k = 1, 2, ...$ is $Z = \{Z_\Theta, Z_A\}$, where: Z_Θ denotes the measured local angle with zero-mean Gaussian noise ν_Θ,and $Z_A = A + \nu_A$ denotes corresponding amplitude value with zero-mean Gaussian noise $\nu_A = N(0, \sigma_{\nu_A})$ and covariance σ_{ν_A} . The variation of the amplitude value is caused by the cluttered environment and the varying unknown distance to the object. It is conditioned by possible modes of target behavior (approaching or descending). Our goal is to utilize received amplitude feature measurements for predicting and estimating the tendency of target behavior.

The block diagram of target behavior tracking system is shown on Figure 1. Two single-model-based filters running in parallel and using two models for target behavior (*Approaching* and *Receding*) are maintained. At the initial moment k the target is characterized by the fuzzified amplitude state estimates according to the two models $A^{App}(k/k)$ and $A^{Rec}(k/k)$. The new observation at time $k+1$ is assumed to be the true value, corrupted by additive measurement noise. It is fuzzified according to the chosen fuzzification interface.

In order to reduce the influence of measurement noise, a weighting procedure is developed and applied. Particular tendency prediction and updating methods are used to estimate present and future target behavior. In general, this diagram resembles the commonly used approaches in standard tracking systems. [1,2] The peculiarity is the implemented fuzzy logic approach [3,4,5] in the realization of the main steps of the procedure.

3. Basic Elements of Fuzzy Logic Systems

In order to resolve the stated problem we apply fuzzy logic as a framework for simultaneous processing and handling of numerical and linguistic data to obtain consistent representation of target behavior in a timely manner. Fuzzy systems differ from classical mathematical-model ones. They do not require strong mathematical models

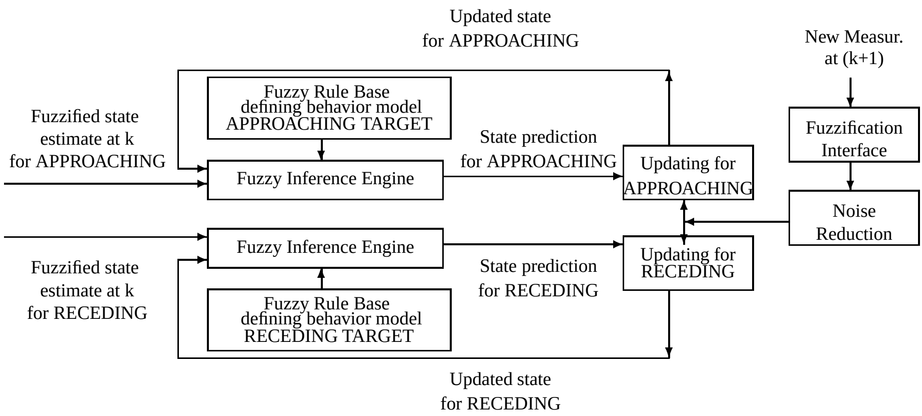

Figure 1: Block diagram of target's behavior tracking system

of functional dependency between system's input and output. Mathematical models of system states and measurement processes restrict the range of real-world applications, because of difficulties in the incorporation of nonmathematical knowledge. Basically, fuzzy logic systems [3] consist of a set of fuzzy associative memory rules or (input,output) associations, operating in parallel, to various degrees. Fuzzy logic systems transform crisp or fuzzy set inputs into a crisp or fuzzy-set output. Further in this section we describe the basic elements in fuzzy reasoning: fuzzy sets, fuzzification interface, fuzzy knowledge base, inference engine and identification of fuzzy models.

3.1. Fuzzy Sets

Fuzziness is a condition, which relates to classes whose boundaries are unsharply defined. A fuzzy set F is a generalization of an ordinary set by allowing a degree of membership for each element. It is defined on a universe of discourse U. The membership function $\mu_F(x)$ provides a measure of degree of similarity of an element in U to the fuzzy subset and takes its values in the interval [0,1]. Each fuzzy set represents a linguistic value of some linguistic variable. It is defined as a variable whose values are sentences in a natural language. The determination of fuzzy membership functions is the most important issue in applying fuzzy system approach to engineering problems. No common approach is available for determining these functions. In some cases, they are attained subjectively as a model for human concepts. In other cases, they are based on statistical or/and empirical distributions, on heuristic determination, on reliability with respect to some particular problem, or on theoretical demands. In any case, the definition of membership functions is not arbitrary.

3.2. Fuzzification Interface

Fuzzification refers to replacing a crisp set with a set whose boundaries are fuzzy. It transforms each numerical measurement received from a sensor into fuzzy set according to the a priori defined fuzzy partition of input space - the frame θ. This frame comprises all considered linguistic values related to particular important input variables and their membership functions. It is well known ,[4] that much of the evidence on which human decisions are based is fuzzy. Because of that fact, the fuzzification of numerical sensory data needs dividing an optimal membership into a suitable number of fuzzy sets. Such division provides smooth transitions and overlaps among the associated fuzzy sets according to the particular real world situation.

3.3. Fuzzy Knowledge Base

Fuzzy IF-THEN rules provide a methodology to represent some objective and/or human knowledge. From this point of view, each fuzzy rule is a scheme for capturing knowledge that involves imprecision. The principle feature of fuzzy rule-based reasoning is its partial matching capability. It makes possible an inference to be made from a fuzzy rule even when the rule's condition is partially satisfied. Fuzzy mapping rules describe a functional mapping relationship between inputs (antecedents) and output (consequent) using linguistic terms.
The foundation of fuzzy mapping rules is a fuzzy graph g, which is an union of Cartesian products involving linguistic input-output associations. It is described by a set of i number fuzzy rules in the form of: 'IF x is A_i THEN y is B_i '. This is expressed mathematically as:

$$g = \bigcup_i A_i \times B_i, \tag{1}$$

where A and B are the linguistic values, describing input and output variables. The Cartesian product of A and B is defined as:

$$\mu_{A\times B}(u, v) = \mu_A(u) \otimes \mu_B(v), \tag{2}$$

where $\otimes$ denotes a fuzzy conjunction (t-norm) operator; $\mu_{A\times B}(u, v)$ is a membership function, which measures the degree of truth of the implication relation between corresponding antecedents and consequents.

3.4. Fuzzy Inference Engine

Fuzzy mapping rules are designed as a group. The inference of such a collection is based on compositional rule of inference:

$$B' = A' \circ g = A' \circ \bigcup_i A_i \times B_i. \tag{3}$$

Here g represents the fuzzy graph of a given fuzzy model and the operator $\circ$ denotes the rule. It is not uniquely defined. By choosing different fuzzy conjunction and disjunction operators, one can get different representations.

3.5. Fuzzy Model Identification

A set of fuzzy mapping rules forms a fuzzy model. Depending on the choice of aggregation operator at the outputs of the fuzzy rules, fuzzy models can be classified into two categories: nonadditive and additive ones. The first group aggregates the outputs of fuzzy rules using the maximum operator, while the second uses an additive operator. Another important point is the appropriate mathematical interpretation of the t-norm operator in equation (2). There are multiple choices available, but it is proven ,[3] that minimum and product inferences are most widely used in engineering applications, since they preserve the cause and effect relationship - the cornerstone principle of each modeling process. Relying on that, the inference scheme of the implemented particular fuzzy model is derived as a fuzzy graph, in which Larsen product operator is used for fuzzy conjunction and “maximum” for fuzzy union operator:

$$g = \max_i(\mu_{A_i \times B_i}(u, v)) = \max_i(\mu_{A_i}(u) \cdot \mu_{B_i}(v)). \tag{4}$$

The inference is based on the most commonly used Zadeh max-min compositional rule.[3,4] If input “x is A'” is given, the inferred output is:

$$\mu_{B'}(y) = \max_{x_i}(\min(\mu_{A'}(x_i), \mu_{A \times B}(x_i, y_i))). \tag{5}$$

4. Fuzzy Approach to Tracking Target Behavior

There are a few basic components in the block diagram of the system for target behavior tracking, shown on Figure 1. In general, this diagram resembles the approaches commonly used in standard tracking systems. This section provides additional information on the specific implementation of the fuzzy logic approach to realize the main steps of tracking.

4.1. Fuzzification Interface Determination

An important variable in the particular case is the amplitude. Its values $A(k)$ are transmitted from the emitter and received at consecutive time moments $k = 1, 2, \ldots$. The fuzzification interface presented on Figure 2 maps $A(k)$ into four fuzzy sets: $\Theta = \{VerySmall(VS), Small(S), Big(B), VeryBig(VB)\}$, which define the corresponding linguistic values related to the linguistic variable ‘Amplitude Strength.’ Their

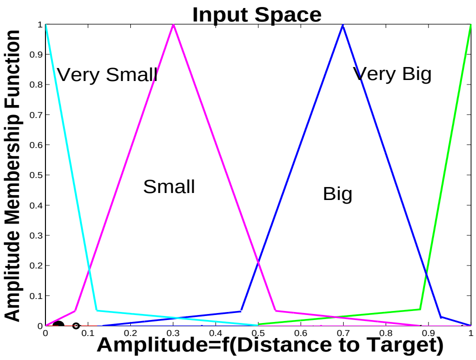

Figure 2: Fuzzification interface

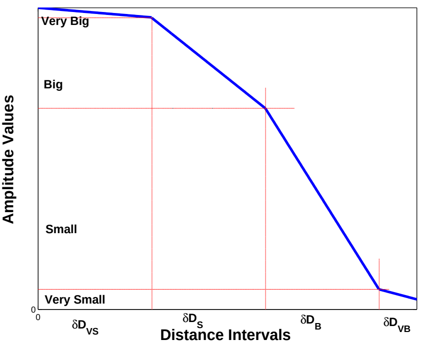

Figure 3: $A = f(1/\delta_D)$

membership functions are not arbitrarily chosen, but rely on the well-known inverse proportion dependency between the measured amplitude value and the corresponding distance to the observed target (Figure 3).

The length of fuzzy sets' bases provides a design parameter which is calibrated to achieve satisfactory performance. Membership functions are tuned in conformity with the particular dependency $A = f(1/\delta_D)$ which is a priori information. The degree of overlap between adjacent fuzzy sets reflects amplitude gradients in the boundary points of specified distance intervals δ_D.

4.2. Identification of Implemented Fuzzy Models

In conformity with the core of our task, fuzzy rules' definition is consistent with the tracking of amplitude changes in consecutive time moments $k = 1, 2, ...$ A particular feature in this regard is that the considered fuzzy rules have one and the same antecedents and consequents. We define their meaning by using the linguistic terms and associated membership functions prespecified in paragraph 4.1. We consider two essential models of possible target behavior:

• *Approaching Target*. Its behavior in time is characterized as a stable process of gradual increase of the amplitude value that can be described by a set of transitions: $VS \to VS \to S \to S \to B \to B \to VB \to VB$;

• *Receding Target*. Its behavior in time is characterized as a stable process of gradual decrease of the amplitude value, that is described by a set of transitions: $VB \to VB \to B \to B \to S \to S \to VS \to VS$.

To comprise appropriately these models, the following fuzzy rule bases have to be carried out:

Behavior 1: APPROACHING TARGET

Rule1:IF $A(k)$ **is** VS **THEN** $A(k+1)$ **is** VS
Rule2:IF $A(k)$ **is** VS **THEN** $A(k+1)$ **is** S
Rule3:IF $A(k)$ **is** S **THEN** $A(k+1)$ **is** S
Rule4:IF $A(k)$ **is** S **THEN** $A(k+1)$ **is** B
Rule5:IF $A(k)$ **is** B **THEN** $A(k+1)$ **is** B
Rule6:IF $A(k)$ **is** B **THEN** $A(k+1)$ **is** VB
Rule7:IF $A(k)$ **is** VB **THEN** $A(k+1)$ **is** VB

Behavior 2: RECEDING TARGET

Rule1:IF $A(k)$ **is** VB **THEN** $A(k+1)$ **is** VB
Rule2:IF $A(k)$ **is** VB **THEN** $A(k+1)$ **is** B
Rule3:IF $A(k)$ **is** B **THEN** $A(k+1)$ **is** B
Rule4:IF $A(k)$ **is** B **THEN** $A(k+1)$ **is** S
Rule5:IF $A(k)$ **is** S **THEN** $A(k+1)$ **is** S
Rule6:IF $A(k)$ **is** S **THEN** $A(k+1)$ **is** VS
Rule7:IF $A(k)$ **is** VS **THEN** $A(k+1)$ **is** VS

In conformity with theoretical considerations and mathematical interpretations in paragraphs 3.4 and 3.5 and by using the specified membership functions, we obtain the resulting fuzzy graphs as fuzzy relations:

Relation1:Approaching Target

$k \to k+1$	VS	S	B	VB
VS	1	1	0.15	0.02
S	0.15	1	1	0.15
B	0.02	0.15	1	1
VB	0	0.02	0.15	1

Relation2:Receding Target

$k \to k+1$	VS	S	B	VB
VS	1	0.15	0.02	0.0
S	1	1	0.15	0.02
B	0.15	1	1	0.15
VB	0.02	0.15	1	1

These fuzzy relations represent the degree of possibility for associations between respective (input, output) pairs. Then, we are able to realize our models' based filters running in parallel.

4.3. Models' Conditioned Amplitude State Tendency Prediction

At initial moment k the target is characterized by the fuzzified amplitude values according to the models $\mu_{A^{A}pp}(k/k)$ and $\mu_{A^{R}ec}(k/k)$. Using these fuzzified amplitudes and applying the described above Zadeh max-min compositional rule equation(5) to relation 1 - $App(k \to k+1)$ - and relation 2 - $Rec(k \to k+1)$, we obtain models conditioned amplitude state tendency for time moment $k+1$, i.e.:

$$\mu_{A^{A}pp}(k+1/k) = \max(\min(\mu_{A^{A}pp}(k/k), \mu_{A}pp(k \to k+1))), \tag{6}$$

$$\mu_{A^{R}ec}(k+1/k) = \max(\min(\mu_{A^{R}ec}(k/k), \mu_{R}ec(k \to k+1))). \tag{7}$$

4.4. Weighting Procedure for Noise Reduction

In order to reduce the influence of measurement noise over the amplitude tendency prediction, a weighting procedure is applied to make the measurement more informative. This procedure can be considered as an adaptive linear combiner as follows:

- We compute the degree to which the new fuzzified measurement intersects each of the linguistic terms in the frame $\Theta = \{VS, S, B, VB\}$. Actually, in that way we consider the likelihoods of receiving particular observation on condition that it originates from each of these terms, i.e.:

$$L_i(A(k+1)/\Theta(i)) = hgt[A(k+1) \cap \Theta(i)] = \sup\{\min(\mu_{A(k+1)}, \mu_{\Theta(i)})\}, i = 1 \div 4, \quad (8)$$

where the operator hgt denotes the height of a resulting fuzzy sets, obtained after intersection between fuzzified new amplitude value and membership function of each of the linguistic terms in the frame Θ;

• Using these likelihoods as respective weighting coefficients, we form the convex combination of the linguistic terms. Thus we take into account the degree of their influence over the received measurement. A normalization procedure is applied. The new fuzzy set represents the weighted measurement with a following membership function: $\mu_{A^W}(x) = \sum_i L_i^N \cdot \mu_{\Theta(i)}$, where $L_i^N = L_i/\Sigma L_i$; $L_i^N \geq 0$; $\sum_i L_i^N = 1$.

Example.

At scan 4 the new crisp amplitude measurement is $A = 0.7487$.

• After applying fuzzification procedure one obtains:
$\mu_{VS}(A) = 0.0$; $\mu_S(A) = 0.0189$; $\mu_B(A) = 0.7854$; $\mu_{VB}(A) = 0.0373$.

• Bearing in mind the a priori defined input feature frame θ, it is possible to define: $L_1(A/VS) = hgt[A \cap VS] = \max\{\min(\mu_A, \mu_{VS})\} =$
$= \max\{\min(0,1), \min(0.0189, 0.15), \min(0.7854, 0), \min(0.0373, 0)\} = 0.0189.$

• The application of the above procedure according to the other linguistic values yields: $L_2(A/S) = 0.15$; $L_3(A/B) = 0.7854$, $L_4(A/VB) = 0.15$.

• A normalization procedure is applied to L_i: $L_i^N = L_i/\Sigma L_i$, $i = 1 \div 4$. It yields: $L_1^N = 0.0172$; $L_2^N = 0.1358$; $L_3^N = 0.7112$; $L_4^N = 0.1358$.

• The weighted measurement is formed as a convex combination: $\mu_A^W = L_1^N * \mu_{VS} + L_2^N * \mu_S + L_3^N * \mu_B + L_4^N * \mu_{VB}$. As a result, we obtain $\mu_{VS}(A^W) = 0.0499$; $\mu_S(A^W) = 0.3259$; $\mu_B(A^W) = 1.0$; $\mu_{VB}(A^W) = 0.3225$.

4.5. Updating State Estimates

The updated states are obtained through a fuzzy set intersection between the weighted new measurement and corresponding modes conditioned amplitude state predictions:

$$\mu_{A^A{}_{pp}}(k+1/k+1) = \min(\mu_{A^W}, \mu_{A^A{}_{pp}}(k+1/k)), \quad (9)$$

$$\mu_{A^R{}_{ec}}(k+1/k+1) = \min(\mu_{A^W}, \mu_{A^R{}_{ec}}(k+1/k)). \quad (10)$$

5. Simulation Study

A simulation scenario is developed for a simple target trajectory (Figure 4) in plane coordinates (X, Y) and for constant velocity movement. The target's starting point and velocities are: $(X_0 = 5km, Y_0 = 10km)$, $\dot{X} = 100m/s, \dot{Y} = 100m/s$ and

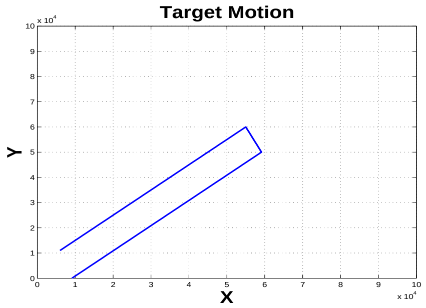

Figure 4: Target trajectory

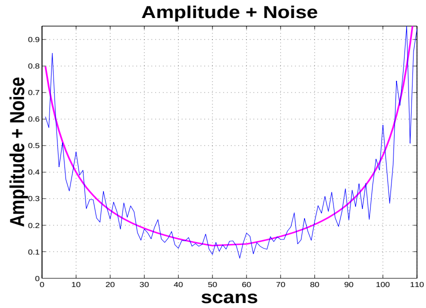

Figure 5: Measurements dynamics

$\dot{X} = -100m/s, \dot{Y} = -100m/s$. The time sampling rate is $T = 5s$. The dynamics of target movement is modeled by simple equations:

$$x(k) = x(k-1) + \dot{x} \cdot T; \quad y(k) = y(k-1) + \dot{y} \cdot T. \tag{11}$$

The amplitude value $Z_A(k) = A(k) + \nu_A(k)$ measured by passive radar is a random Gaussian distributed process (Figure 5) with mean $A(k) = 1/D(k)$ and covariance $\sigma_A(k) = 0.3{\cdot}rand(1,1)/D(k)$. $D(k) = \sqrt[2]{x(k)^2 + y(k)^2}$ is the distance to the target, $\{x(k), y(k)\}$ is the corresponding vector of coordinates, and $\nu_A(k)$ is the measurement noise. Each amplitude value (true one and the corresponding noisy one) received at time (scan) $k = 1, 2, \ldots$ is processed according to the block diagram of our target's behavior tracking system (Figure 1).

Figures 6-10 show the results obtained during the whole motion of the observed target (descending and approaching directions). They represent the tendency in target behavior, which is described via the time (scan) consecutive transitions of amplitude value $VB \rightarrow VB \rightarrow B \rightarrow B \rightarrow S \rightarrow S \rightarrow VS \rightarrow VS$ and respectively $VS \rightarrow VS \rightarrow S \rightarrow S \rightarrow B \rightarrow B \rightarrow VB \rightarrow VB$. Figure 6 represents the case, when the measured amplitude values are without measurement's noise, i.e. $Z_A(k) = A(k)$. Two models - *Approaching* and *Receding* are maintained in parallel.

With the implementation of the developed algorithm (Figure 1) it becomes possible to make a correct decision about the plausibility of the considered models. It could be seen that between scans 1 and 90 target motion estimation is supported by the correct, for that case, *Descending* model. In the same time, the *Approaching* model has no reaction to the measurements dynamics, because it does not match the real target behavior *Receding*. Taking into account Figure 5, the amplitude measurements dynamics between scans 10 and 90 could be analyzed as relatively weak from the point of view of the fuzzification interface (Figure 2). Such a transition area is contingent on the assumed possibility for sojourning time, when the measured amplitude values during consecutive scans consistently reside in one and the same regions of that interface. It

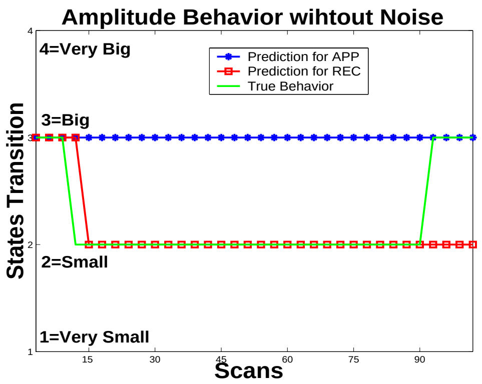

Figure 6: Target behavior estimation (without measurement noise)

CASE 1: AMPLITUDE NOISE WITH $\sigma_A = 0.2.rand(1,1)/D(k)$

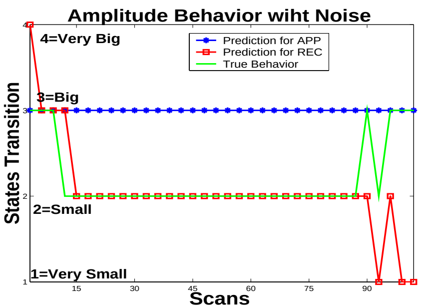

Figure 7: Target behavior estimation in case of noise.

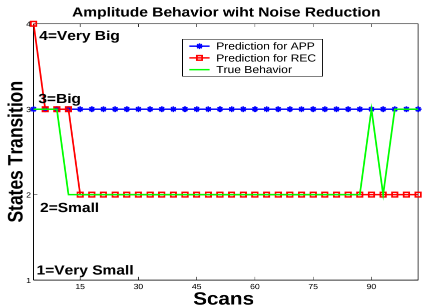

Figure 8: Target behavior estimation in case of noise reduction.

is characterized with a latency delay before switching to the opposite behavior mode. After scan 90 and until scan 115 it is obvious that the *Descending* model misses the amplitude changes, while the *Approaching* model becomes the plausible one. Figure 7 represents the case, when the measured amplitude values are corrupted by noise with $\sigma_A = 0.2.rand(1,1)/D(k)$.

Some disorder and discrepancy between predicted behavior tendency and true amplitude behavior take place, and it is difficult to make a firm decision about the tendency of target behavior. As presented on Figure 8, the application of the noise reduction procedure produces a ‘smoothed’ predicted behavior tendency, and it becomes possible to make a robust decision on the tendency of target behavior. The effect of that procedure is even more important when input measurements are corrupted by higher noise levels,

CASE 2: AMPLITUDE NOISE WITH $\sigma_A = 0.4.rand(1,1)/D(k)$

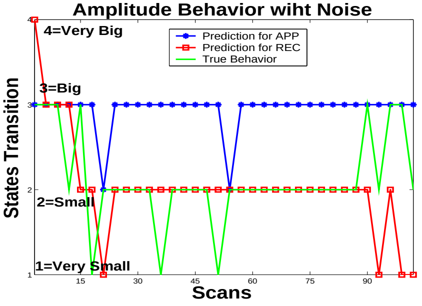

Figure 9: Target behavior estimation in case of noise.

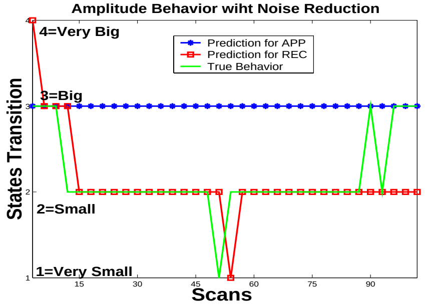

Figure 10: Target behavior estimation in case of noise reduction.

for example with $\sigma_A = 0.4.rand(1,1)/D(k)$ (Figure 9). In that case, some chaotic behavior is detected. In such critical situations the noise reduction procedure assures a more consistent process of amplitude tendency prediction (Figure 10).

6. Conclusions

An approach to estimating the tendency of target behavior was proposed and evaluated. It is based on Fuzzy Logic principles applied to conventional passive radar measurements. A particular real-time algorithm was developed. It was evaluated using computer simulation. Dealing simultaneously with numerical and linguistic data, an opportunity for robust reasoning is realized. The application of an additional weighting procedure for noise reduction improves the overall process of estimating the tendency of target behavior. The developed algorithm is suitable and adapted for processing noisy amplitude measurements. It entails modest computational load and provides simple and robust decisions about tendencies in target behavior. The proposed approach is suitable for obtaining a tactical picture for complex or ill-defined problems in engineering applications.

Acknowledgement

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TZVETAN ATANASOV SEMERDJIEV see p.90

Fuzzy Logic Approach to Estimating Tendencies in Target Behavior

Albena Tchamova and Tzvetan Semerdjiev

Keywords: Fuzzy sets, fuzzy logic, evidence reasoning, attribute data processing.

Abstract: In some real-world situations when kinematic data is not available or it is not sufficient to provide right decisions and/or accurate estimates, estimation schemes may incorporate the attribute data that usually exists simultaneously with kinematic data. However, attribute data is usually incomplete, inconsistent and vague, hence the importance of the problem of overcoming the arising uncertainty in such cases. This paper presents one approach to the estimation of the tendency of target behavior. The authors present an original algorithm for tracking target behavior and evaluate its performance. The algorithm is based on the application of the principles of fuzzy logic to conventional passive radar amplitude measurements. A set of fuzzy models is used to describe alternative tendencies of target behavior. Additionally, a noise reduction procedure is applied. The performance of the developed algorithm in the presence of noise is estimated based on computer simulations results.

full text

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THE GENETIC PROGRAM: A TECHNOCRATIC HYPOTHESIS ON THE PARADIGM OF CIVILIZATION[1]

Tzvetan SEMERDJIEV

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The Hypothesis

Billions of years before the new era, while the third planet in the Solar system covered in dense clouds was still in its sleep, in the dark depths of the endless ocean of organoids, the last module of the *Genetic Program* began. It was part of the perfect algorithm for evolution of life on Earth. Also, it was the beginning of the second of the three qualitative transitions unexplained by the human beings, which mark the separate parts of the Creation.

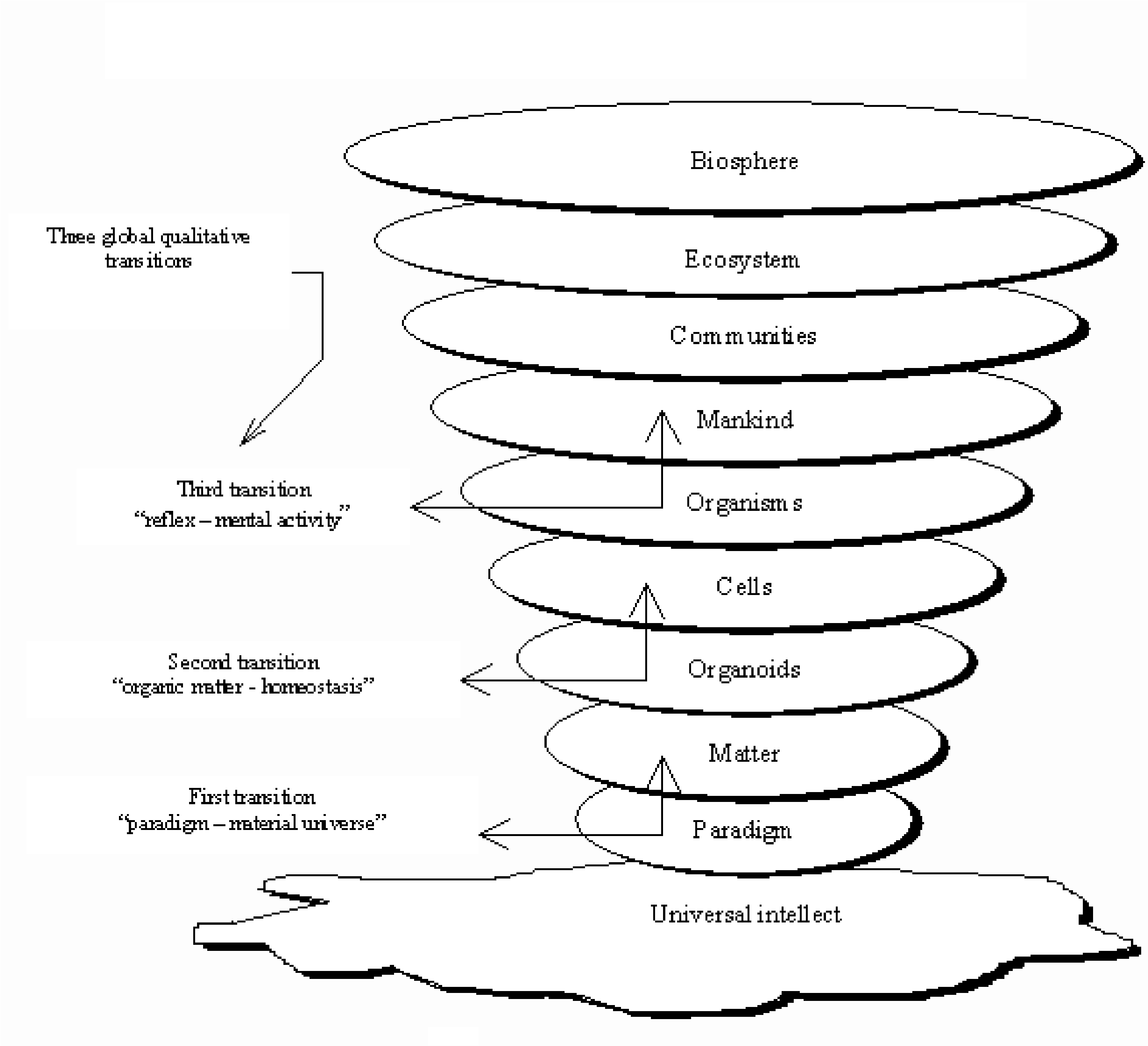


Figure 1: Stratified model of the evolutionary development

In the global evolution, these transitions are known as “paradigm - matter” (the idea of the universal intellect – “big bang” and inflating universe), “organic environment - cell” (cytoplasm – homeostatic system) and “reflexive behavior – mental activity” (first signal system – second signal system).

The three transitions are represented on figure 1. The latest studies show that their sequence in time and space is based on non-linearity, which reflects the giant evolution of the human knowledge about the surrounding reality.[2] Currently, for every one of these transitions there are new viewpoints. The last scientific hypothesis of the so-called “inflating” universe is transformed in “self-reproducing pulsing net” of “Big Bangs.”[3] Viewed as a branching tree structure of growing and shrinking subspaces, the universe is thought to be stationary by the laws of physics and space, despite the “mutations” in its separate nodes. Our part of the universe expands with a rate defined by the Hubbell constant, which is reliably measured. By the well-known laws of nature, the general properties and characteristics of the galaxies do not depend on the time of their creation and their evolution, because the branched net structure is stationary and very, very old. During the last 15 years, this allowed the cosmogony to change radically the views on the universe, the intellect and the man's place among them.[4]

In the context of the old interpretations we could hardly stand up for the Darwinian theory. The theory of origin of species has lots of time-space omissions that started to be filled in only recently. The most fundamental among them is the lack of conceptual interpretations for the new discoveries on the role of the genes in the processes of inheritance. Ever since 1865, when it was created, Gregor Mendel's theory is still not sufficiently compatible with the Darwinian theory.[5] The gap was not filled in 1930, when Robert Fisher laid the foundation of the modern population genetics, nor was it filled in 1940,[6] when Theodosius Dobzhansky, Julian Huxley and Ernst Mayr synthesized the so-called "neo-Darwinism."[7] Nothing could have stayed in its way, if it was a complete and structured scientific theory. Unfortunately, it was not. Science continues to discover the mechanisms of life, but provides no explanation on the self-reproduction of gene molecules.

Despite all this, the Darwinian theory is the only empiric paradigm that survived for more than 135 years and that provides a model we can use in practice for solving the hardest problem of modern science – "the way, the super-complex living nature exists and adapts." If we arrange the events in the last 12 billion years since the Big Bang on a scale of one second for every 500 years, we shall get a general overview of the global evolutionary program in what we call "conditional year." In that year, man appears on 31st of December at 22:30 h. Evolution on this scale is represented in table 1. Science has enough empiric material to form the hypothesis for a priori existence of primary structural/ functional information, i.e., a core of dynamic paradigmatic model. The target of this model is to realize a global algorithm for assimilation of our part of time and space by creating an eternal incubator of intellect, which stores knowledge and thus decreases the entropy in the universe. Its mission is to realize the Genetic program.

The hypothesis is based on series of metaphysical analyses concerning the superior, beyond the human senses, only mentally sensible, beginnings of every living thing. It is built on a system (model) of interpreted primary concepts, based on an originating structure of assumptions (axioms). They are derived from the comprehension that the evolution of intelligence on our planet is a movement based on changes in molecular structures and a change over to new, higher levels of complexity and organization. During the human life, the information contained in the genetic structures is updated and developed on the base of the accumulated knowledge and culture. When the passing over of genetic information between the generations is in process, a biological adaptation and development of the human being takes place, thus realizing the global evolutionary program. The genetic program provides development of the human perceptions of beauty and perfection that evolve by psychological fusion and harmony between the person and the structure and organization of nature. The rate of changes in the surrounding environment has no effect on it. The effect is caused by the qualitative-quantitative transitions on structural level in the time-space continuum. The primary information structure (information nucleus) contains the knowledge of the ontological laws for the development of the human civilization and its link with the universal intellect.

At the end of the 20th century, the people working in the field of artificial intelligence reached three fundamental conclusions, summarizing all the achievements and extending the proposed model:

- The vanguard information technologies inevitably form a new class of intelligence that exists outside the human mind;
- Knowledge is the “eye of the mind” of artificial intellect. The computer processing of the natural human language gives sense to the ideas and conceptions as products of this intellect;
- The creation of effective technologies for automatic extraction and processing of knowledge determines the processes of formation of ontological concepts and models of the world’s structure and leads to changes in the way of thinking and the generally accepted points of view.

The above statements are considered to be fundamental axioms in the general theory of artificial intelligence, which is treated as empirical science and is being developed mainly on the base of experience. Knowledge in the classic intellect hierarchy or computer intellect goes beyond the traditional biological or physical levels of itself (made of either biological neural nets or microelectronic schemes) and its symbolical realization – logic gates or program schemes. The modern technologies for knowledge processing only slightly touch on the mechanisms of the genetic program.

Table 1: Stages of the global change

STAGES OF THE GLOBAL CHANGE	RELATIVE BEGINNING
“Big Bang”, appearance of relict gravitons – about 12-18 $\bullet 10^{9}$ years ago, Universe size – that of 10 cents coin, density of matter is more than 10^{94} g/cm^3, temperature $\bullet^{0} = 10^{32}$ K, modern theories of gravity and relativity are inapplicable	01 January, 00 h. 00 m. 00 s, till 10^{-43} s
Establishing a symmetry between the matter an the antimatter	01 January, 00 h. 00 m. 00 s, till 10^{-35} s
Appearance of quarks and transition to thermal equilibrium	01 January, 00 h. 00 m. 00 s, till 10^{-8} s
Appearance of relict neutrino background radiation	01 January, 00 h. 00 m. 00 s, till 10^{-3} s

Appearance of the forces of nature and quarks – expansion to size 100 times bigger than that of the Solar System, density of matter is less than 10^{94} g/cm^3, the temperature drops to values 100.10^6 times bigger than today's temperature of the Sun	01 January, 00 h. 00 m. 01 s
Forming of protons end neutrons – expansion to size 1000 times bigger than that of the Solar System	01 January, 00 h. 00 m. 10 s
Forming of atomic nuclei of helium and deuterium – expansion to size 10^6 times bigger than that of the Solar System	01 January, 00 h. 01 m. 00 s
Formation of the initial chemical composition of the Universe – 70% Hydrogen and 30% Helium	01 January, 00 h. 01 m. 40 s
Formation of the beginning of the "Transparent Universe" – neutral gas clouds, permeated by the relict radiation, cooling and forming of clusters of today's galaxies, expansion to 0.1 % of today's size of the Universe, the temperature drops to 3000 ^{0} K	01 January, 00 h. 10 m. 00 s
Formation of galaxies and first star clusters, expansion to 20% of today's size of the Universe	10 January
Production of "heavy" chemical elements in the "nuclear reactors" of the stars – production of elements heavier than Helium, expansion to 50% of today's size of the Universe	31 January
Formation of the Solar System about 5.10^{9} years ago	09 September
Formation of the Earth – the temperature in the Universe drops to $2{,}7^0$ K, density of matter is about $3 \cdot 10^{-31}$ milligrams/m^3 (observed), density of radiation is 10^9 photons per 1 nucleon, number of observed galaxies is 10^{11}, Distance to the farthest quasar is $12 \cdot 10^9$ light-years[8]	14 September
Appearance of life on Earth	25 September
Appearance of bacteria and algae	09 October
Beginning of photosynthesis	12 November
Appearance of the first cells with nucleus	15 November

Saturation of Earth’s atmosphere with oxygen	01 December
Appearance of vertebrates	16 December
Plankton and trilobites	18 December
Fishes	19 December
Land plants	20 December
Insects and land animals	21 December
Amphibians and flying insects	22 December
Trees and reptiles	23 December
Dinosaurs	24 December
Mammals	26 December
Birds, flowers, extinction of dinosaurs	27 December
Primates	29 December
Humanoids	30 December
Appearance of the human being	31 December, 22 h. 30 min.

Obviously, without the common knowledge of the world as a whole, the reality, the reflection and the language, a real intellect cannot exist. The natural language is contained within it ontologically.[9] However, the ontology is limited by the meta-language, which is used to express the ideas of the universe and the existence. For this reason, the common theory of artificial intelligence is the human-made principal foundation (basic conceptual scheme) for the essence and the needed characteristics of the relations in the reality (physical or mental), i.e., for everything that exists or happens. This theory includes the semantics of the words and the ideas, along with the ontology of the natural language and the knowledge, i.e. the presumption about genetic (inheritable) passing over of extralinguistic information.

The hypothesis reflects some intuitive understandings of the essence and the role of the genetic program in the process of the human evolution and the realization of the transition to information society. The rapidly increasing new achievements in the field of artificial intelligence gradually accumulate a visionary potential to understand life, which brings us closer to the idea of the “genetically programmed” character of the evolution of the human civilization and the role of the universal intellect in it.

The Technology

It is known that one of the main factors influencing the development of mankind is the information passed over genetically to the individual in the form genetic code. It contains this part of the program that determines the future development of the individual – its inclinations, susceptibility to different illnesses and deformations, intellectual abilities, and talents. From what we know about the organization of the algorithm and its regularities, we can presume that the "brain-genetic code" feedback gives information on the current state of the realization of the global genetic program of mankind. The functions of adaptation, self-development and evolution take place in the process of passing over the genetic code.

Widely discussed in the mid-80s, this hypothesis requires some assumptions. Their validation has gone as far as the validation of the principles in the Darwinian theory. A general overview of the discoveries made in the last decade and an analysis of the tendencies in the technological progress of our civilization allows us to formulate some untraditional theses:

- The Universal intellect is exceptionally ancient and exists in dimensions of time and space still beyond human grasp or perception.
- The rise of mankind in our part of the universe (planet Earth) is just the first stage—the childhood—of the evolution of its mind. After the biological death, the universal intellect extracts and stores the immaterial essence of its "children," giving them existence in forms and dimensions still unknown to us.
- The para-psychological abilities of certain people prove the means of communication with these forms and dimensions on the base of perceptions insufficiently advanced by the human beings at the present moment. Their development directly depends on the prolongation of the human life, chances to learn, accumulation of experience, traditions and development of intuition and abstract thinking.
- The need to accumulate and store knowledge is set ontologically (genetically) in the human being. By accumulation of knowledge and development of technology, mankind expands the general extension of human life, which aids the development and use of the mental abilities of the individual. In the near future, with the discovery of cure for cancer and the development of technologies for creation of artificial organs and their implantation, it will be possible to double this extension. So, two or three academic degrees and qualifications in one human life will be a reality, which will bring new levels of experience, knowledge and tradition.
- If we consider that one human life is not sufficient to use the entire potential of the brain's neuron structure, the doubling of its extent will increase their usage abruptly. We can expect development of new perceptual abilities, popularization of para-psychological phenomena and widely spread contacts with presently unknown forms and dimensions of the universal intellect.
- The basic categories and concepts used to evaluate the results of the human evolution are contained in the philosophical theories of the modern humanists, a number of religions, the codes of different environmentalist movements, etc. The main way to measure the progress of the global genetic program is the current stage of the popular conceptions of beauty, perfection and harmony. Their evolution directly depends on the adaptation and self-learning of the individual in the process of the genetic program.
- The local exchange of information between the genetic program and the environment is

accomplished in the human life in the form of primary signals, data (encoded signals) and knowledge. The stored knowledge accumulates into culture, which is the most impressive structure created by the genetic program. The enormous variety of cultures illustrates the existence of time-space and quantitative-qualitative differences in the stages of execution of the global genetic program of the civilization.

- Irregularity of civilization is determined by both different geographic conditions for development of human populations and eventual differences reflecting historical facts, events, and processes of evolution. Birth, rise, fall and extinction of various cultures, as well as the backwardness of others, determine the difference in the views for good and bad, beautiful and ugly, peace and aggression, charity and egoism, etc. The level of humanity of nations, advanced in the realization of their genetic programs, i.e., that have existed longer and have created their culture, largely exceeds that of relatively young nations with young governments and insufficient traditions, experience and culture in the genetic fund.

These theses are a direct outcome of the rapid development of the information technologies and their applications, which strictly repeat the structure of the human being, created by the genetic program. The respiratory, cardiovascular, lymphatic, digestive, hormonal, and other systems provide for the mental and psychological activities of the human being, providing homeostatic behavior in the environment. Given the hierarchical classification of the needs, the present resources and the state of the environment, the human being realizes activities for satisfaction of these needs. They are on five levels – physiological (air, water, food, warmth), need for security (to be free of fear), need of love (positive human relations), self-respect (internal) and realization (external).

From another—information-technological—viewpoint, we can consider the human body to be a subsystem, providing the activity of the “central processing unit”—the brain. With its ontologically embedded instruction and command systems and the genetically emulated logic, the brain controls the body through the channels of the reflexive feedback link. For that purpose it uses the “interfaces” of the neural system and the “controllers” of various subsystems (organs). By going deeper into the mechanical structure of the human body, science touches on the mechanisms of the genetic program and its model for creating a biological carrier and incubator of intellect. Through the development of information technologies, creation of information and cyber spaces (as elements of the information society) and with the advances in the synthesis of artificial intelligence systems, the human civilization reproduces what it has learned about the genetic program and climbs the next step on the evolution ladder.

The human sensor subsystem (which at the present moment contains five senses – sight, hearing, smell, taste and touch) provides information input for the brain (main CPU). The human’s long-term (PROM and EPROM) and short-term (DRAM) memories are well known and studied. The biologic restrictions for data processing in the human brain are classified in three groups:

- Restrictions of the sensor subsystem, i.e. the abilities of the five senses;
- The capacity of the long-term memory (10^9–10^{13} bits), which in one human life (about 75 years) is formed using approximately 30 percent of the capabilities of the neuron structure of the human brain;[10]

- The storage time in the short-lasting memory is usually between 7 and 12 clusters (i.e. mental processes running in parallel) with the presumption for multitasking in timesharing mode.

Data streams coming through the human sensor system are processed (assimilated) in the "CPU" and stored in the corresponding "memories." The "operating system" (the technology of processing) is thoroughly studied by the "knowledge engineering" specialists. Taking in mind the imperfection of this technology, mankind has always tried to create brain-enhancing instruments. The first of them is the speech that is ontologically embedded from the genetic program of the biological evolution of the human being. The creation of the others is a result of technological developments and accumulation of sufficient knowledge potential. Created as outside "peripheral" units and systems, they stimulate the communication capabilities and the intuitive creative way of thinking.

Today, 38 000 years after the first signs of human information activity (cave paintings), the society has evidence for the non-linear tendency in the technological development of civilization. It puts an emphasis on the information technologies and the instruments for aiding the extraction of and applying knowledge. The global tendencies for development of computer systems prove that the goal is to create effective amplifiers of the human mental activity. Modern computers can effectively accomplish almost every one of the human's routine mental functions. They have unlimited amounts of long-term memory with smart information search (relational databases), parallel (multitasking) processing of unlimited number of tasks (gigabyte DRAM fields of operative memory), fast processing (high-performance processors with Teraflops processing speed), automatic multimedia analysis, fusing of information on expert level of knowledge and culture (virtual reality). And then, connected in the global information-transporting infrastructure for integration of the human mind (e-mail and Internet), they allow the artificial intelligence to permeate into the "thin structure" of the cognitive process and to move on the epistemological levels (meta-cognitivistics).

The tendency to enhance the human mental activity finds serious development in the process of improving the man-machine interface (the connection between the man and the computer) and the means of doing this are impressively well developed. The most popular ones are the multimedia products, Windows based software, Hypertext, electronic publications, etc. A "bang" of new scientific discoveries, allowing the creation of a new generation of information technologies, was registered recently. The most popular end products are the global mobile communication and global positioning systems (GSM and GPS), laptop PCs, compact discs, digital cameras, video-magnetic players, fiber-optic networks, satellite radio and television broadcasts, electronic money, automatic bank operations, express postal services, digital television, Internet, laser controlled weaponry and munitions, plane and tank controlling systems, etc.

The human locomotory system provides not only the mobility of the sensor system and the brain, but the means (manipulators) to manipulate the environment. Every separate individual has the triad *"sensor system – data processing – manipulators."* The union of this triad in the form of global data processing nets (DBS, GSM, Internet, etc.) aims at the creation of a global cyberspace as a new dimension for the human civilization to live and develop in. For example, the annual budget of the US provides significant funds for uniting all schools, universities, libraries, hospitals, etc., in one national infrastructure. It is the information and cyber spaces where in the future mankind will contact and enter the dimensions of the universal intellect.

The computer science achievements and their applications prove that adaptation, self-development and advances in the field of information technologies are the means to make contact and reach harmony between the human brain and the structures of the universal intellect. This is the main goal of the execution of the Genetic Program.

The Processes

Today, two processes are commonly recognized as general social development tendencies – integration of the existing information systems in one global *System of systems* and mass transition of human mental functions to computer systems and robots.

Mankind's difficulties and the need for many scientists to solve them are the main reason for evolving global processes of information integration. Only the *"shortening of distances"* on the base of development of information technologies will make it possible to solve the problems of the 21st century using the power of the integrated human minds. The general characteristic of these problems is the "*inconsistency of truths*," which results in an increased complexity and dynamics of the global processes, which enter a totally new state of *"unorganized complexity*."

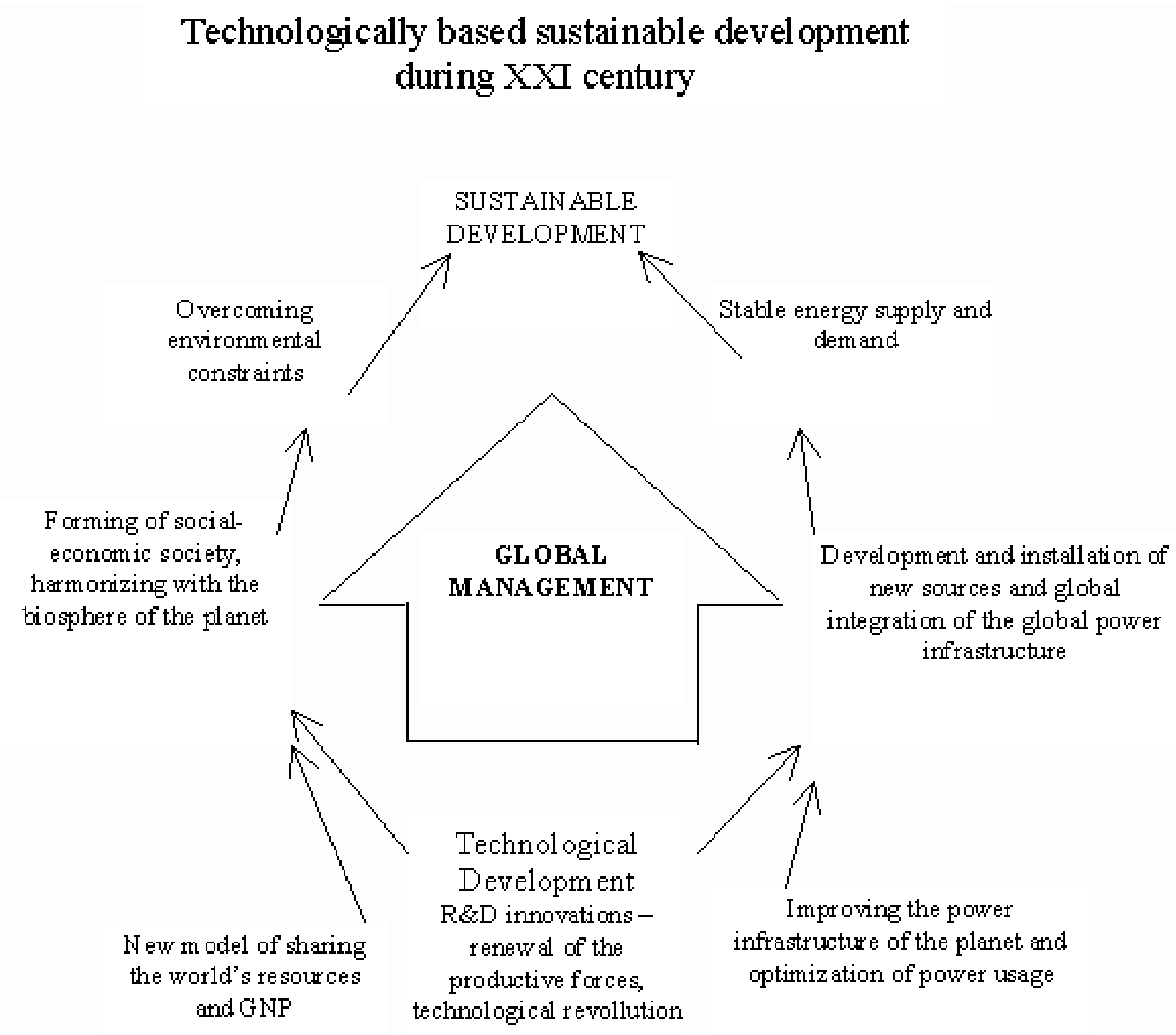
The human mind in its biological form is still unable to reach such concepts as infinity, zero (nothing), chaos, etc. The development of the axiomatic approach and the achievements of the theoretic (abstract) mathematics demonstrate the existence of ontologically embedded abilities for intuitive (hidden, unconscious) way of thinking. Through them, the main mission of mankind is fulfilled and its result is the revealed truth. These processes set the tendency for accelerated "aging" of truths and formulation of even more complex concepts for the world.

The process of evolution of the human brain foreseen in the genetic program provides development of the "instruction systems" and the ontologically "emulated logic." The improvement of the neural nets and uniting them in even more complex organizations in the process of inheritance makes it possible to acquire new levels of abstract thinking gradually. They bring mankind closer to the dimensions and models of the universal intellect. The genetic program carries information for future special features of the evolving mankind. The reading of the genetic code made a new strategy possible. It provides maximal development and implementation of the talents of the individual, which leads to maximal realization in one human life. This task was made possible thanks to the powerful computer resources that enabled scientists to build all the genetic combinations and to create a unique model of the genetic structure of the human being. Momentous achievement in this field was the creation of high-performance super-computers in the USA.[11] This opened the genetic bible of mankind.

The satisfaction of mankind's power supply needs and the production of resources were the most important strategic problems of the 20th century.[12] Along with them, two new factors appeared. Information technologies and data processing enabled technology development to take the place of the power-resource evolution. It did so by innovating knowledge, development and installation of newer, more effective international mechanisms for sharing the world's resources and the GNP of the planet with one goal – providing sustainable development. One of the main models in this area was proposed at the World Conference on Sustainable Development, which took place in 1992 in Rio de Janeiro. It gave knowledge and technological development a higher priority over the power resources considering it the only possible way to avoid the total crisis of the human civilization.

The “developed” nations, which had the chance to have optimal geographic conditions and to be historically well situated (in terms of outbursts of violence, disasters, etc.), have plenty of cultural potential, created by the genetic program. The level of “development” may be characterized by the ratio “production/consumption” of energy and information.[13] A quick overview of the information supply of any of the citizens of the technologically developed countries reveals that this ratio is in favor of information, while in the developing countries it is towards the energy.

The need for information is ontologically embedded in the human being. That is because the development of the human intellect will lead to contact with the Universal Intellect. Today, this contact is impossible, as it is impossible for the little child to understand the actions of its parents. And then, a premature contact will lead to psychological shock or even destruction of the human civilization. With the creation and development of the global information space this threat will slowly cease to exist, because the genetic link of the generations will make this place a natural environment for the new generations to live in. In it, they will form the abstract models and complex structures of the future, which will take them closer and closer to the truths about infinity, chaos and the Universal Intellect.



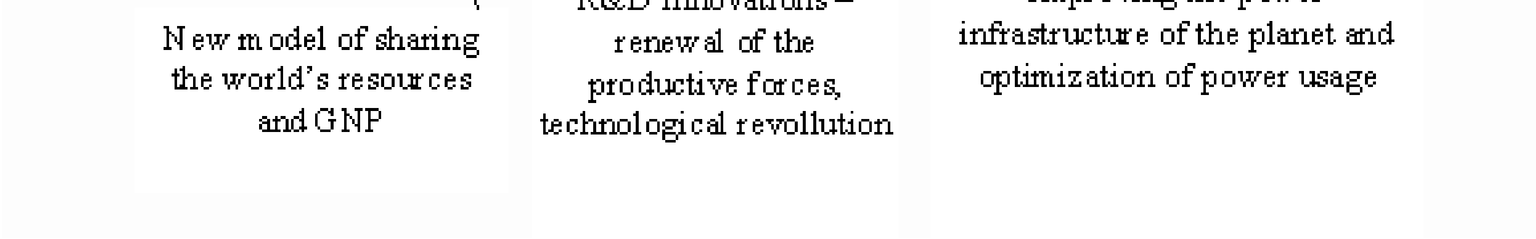


Figure 2: Technologically based sustainable development during XXI century

The man-made changes in the biosphere are significant and with lasting indirect strategic effects. They are showing up already, thus endangering mankind's survival. Also we should keep in mind that the current population growth is the highest in history and started to fall down only recently. The curve is in its declination point, because Earth's population will be doubled by 2025 (quadratic function). This would mean a total "eating," "drinking" and "burning" up of the planet's resources leading to its destruction. The analysis of global tendencies helps define the main elements of mankind's transition to sustainable development. They are varied and include:[14]

- Transition to sustainable demographic development on a global scale. Without this, talking about sustainability will make no sense;
- Transition to new technologies for satisfaction of the human needs, with essentially less interaction with the environment of every human being, and a given level of prosperity;
- Utilizing methods of economic development, providing transition in quality with less quantitative accumulations until poverty is overcome;
- Forming a society with less inequality than the one achieved till present, and making the global development possible. Due to the collective actions of the society, the social transitions will be accompanied by successes in overcoming corruption and illegal (criminal) activities;
- The institutions should develop a new, more effective way of dealing with conflicts, especially in the biosphere. The main problem is the balance between cooperation and competition on every level;
- Exploitation of the information essence of the ongoing changes. It consists of mastering new technologies for extraction, processing and use of knowledge and culture, necessary for dealing with ecological, demographic, social, economic, political and national security problems locally, nationally and globally. Only by high level of perception of reality by the society we can achieve the effect of sustainability;
- Ideological transition, based on "planetary consciousness," solidarity and humanity should stand on the perception of interdependencies between everything and everyone on Earth. Long-term view for sustainability should be brought to every home, to every man. Today's wide spread destructive utopia is the utilization of the "separatist" approach which opposes the private knowledge and view of life to the rest of the world;
- As a whole, the 21st century will be the time when the human race will make the transition to sustainable development without shocks and disasters. The leading researches of the future will be interdisciplinary oriented. Their work will be pointed towards the global vision of life, security, politics and all other questions related to the future of mankind.

The Paradigmatic insufficiency

The increasing interest in the influence of social factors in the global models is caused by the lack of resources. Today, only a few of us understand that the generation and distribution of the common wealth is still carried out by the same old paradigm, created about 200 years ago by Adam Smith and David Ricardo. The capital character of the management of the global financial-economic system and its development has remained the same to the present day, thus causing disproportion, already considered ineffective by mankind. Despite the obvious financial and economic benefits of the market model of social development and the demonstrated vast possibilities for change and social adaptation, it is still a source of two main problems: (a) gigantic consumption of natural resources and (b) increasing inequality in the distribution of wealth.

Some of the latest studies show that to raise one kilogram of grain one ton of drinking water is needed. However, the water shortage becomes one of the main threats for the world order and security in the 21st century. Food production increases linearly, while the population of the planet increases exponentially. Thus, the accumulating food insufficiency is in direct dependence on the technological backwardness of the developing countries. According to recent publications,[15] the inequality ratio for the planet exceeds 1:150. The rich part of mankind constitutes about 20 percent and produces 82,7 percent of the global GNP. The growth of this wealth is 2,7 times higher than that that in the developing countries. Today 1,3 billion people live with 30 dollars per month, while the “third world” loses about 500 billion dollars per year. The living standard ratio between “rich” and “poor” countries is 100 to 8 and continues to increase rapidly. The inequality intensifies the imbalance in the genetic program. Entire nations and colossal human abilities, gift and talent remain unused. Misery, poverty and intellectual backwardness is reality for great part of the human civilization. This means that today the human mind still has no model for social organization capable of providing future transition to a steady growth and contact with the Universal Intellect.

Today’s generations witnessed the end of a failed attempt for social engineering. For 75 years, mankind observed the steps of the so-called “real socialism” with hope or suspicion. Built over full nationalization of the means of production, it turned a small part of the community into real, but irresponsible owners of the means of production and the goods. In periods of poverty and restoration, the methods of “equal poverty,” “the economic of deficit,” “postponed consumption” and “total obedience of the personal interests to the public ones” have demonstrated two main defects: withdrawal of the individual from the means of production and opposing personal interests to the public ones. This led to decreasing labor productivity, worsening quality of the produced goods and services and, finally, to the economical crash of the system.[16]

Today, mankind is looking for solution to this problem in the boundaries of the evolutionary approach. By accumulating sufficient knowledge about the laws of social development and analyzing the lessons of the 20th century, the new generation will find a new model for effective exploitation of the world’s resources and produced goods. The genetically programmed mission of today’s generation is to pass over as much knowledge and culture as possible to its children and to give them a feel for adherence to the global genetic program which evolves beauty, harmony and completeness in this part of the universe.

One of the most important questions considered in many of today’s publications is the nonlinear exponential character of the social development. The end of the 20th century proved the tendency of increasing frequency of changes and their amplitude. These processes are known as “divergent” in the theory of automatic control. This regularity is an outcome of a specific realization of the genetic program. Mankind’s knowledge treasury is filled with discoveries and new results in various fields of science. Sudden flashes in the interdisciplinary space give birth to new discoveries and technologies every day and every hour. We are on the verge of a series of discoveries that will change our visions of the world. Today we can expect new breakthroughs in microbiology, genetics and medicine, bio-technologies and nourishment, power supplies, microelectronics, optics and laser technologies, space technologies and quantum physics. Despite today’s “explosion of knowledge,” building tomorrow’s social structures and the beginning of the sustainable development demand more knowledge and technologies. But the most important thing is absent. We witness absence of a common future paradigm, or the so-called paradigmatic insufficiency.

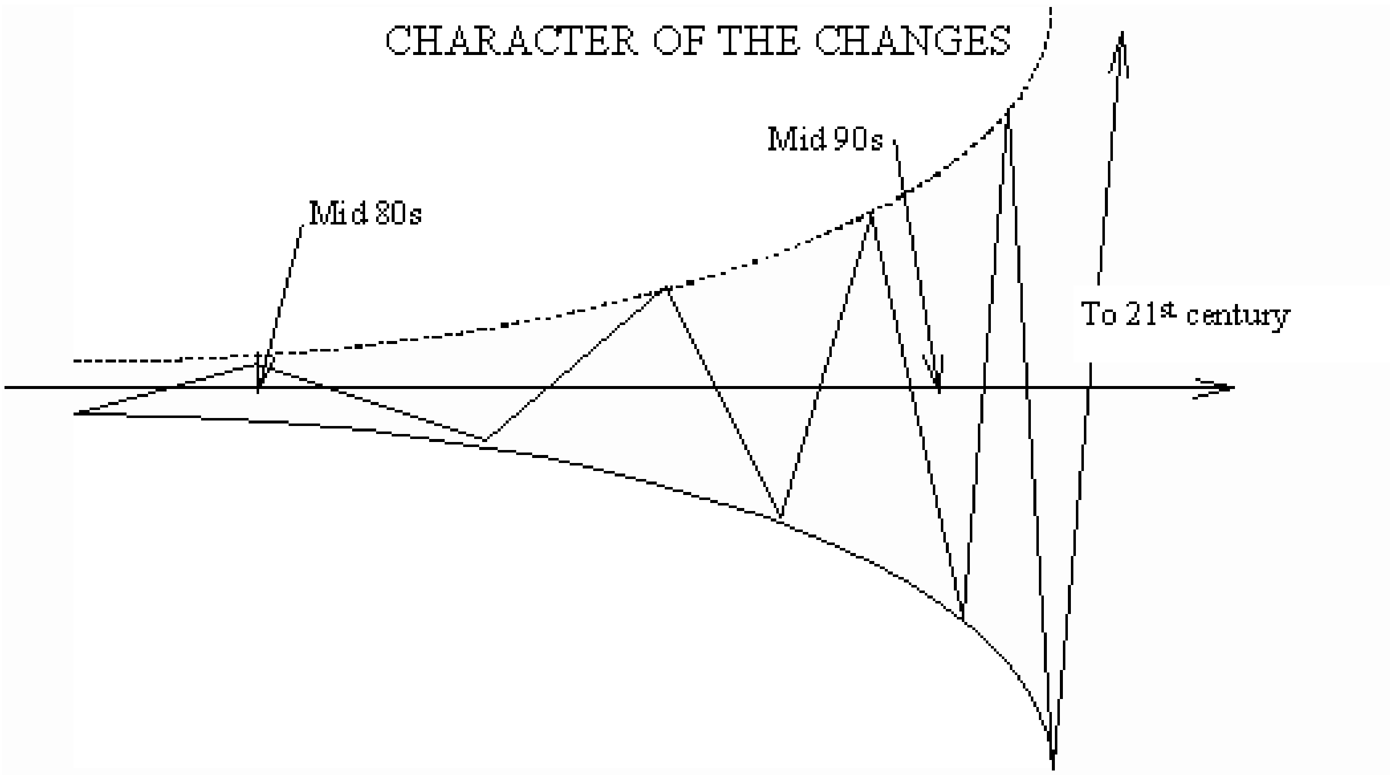


Figure 3: Character of the changes

The main problem is the lack of a common visionary link between the scientists, separated by the great number of explosively increasing schools, trends and directions. This paradigmatic problem leads to loss of the common (philosophical) vision of the on-going changes and appearance of a number of contradictory opinions for the world’s future. The only thing that is sure is that the model of the emerging new world will be completely different from what we know, see or imagine. In this sense, the newly rising theses for “third technological revolution,” “information society” and “third revolution in the military affairs” pose many new questions.

The Realization

It will be hard to understand the considered hypothesis unless we try to interpret it in the frame of a large historical window. By analyzing and classifying the stages of technological development of mankind, we can construct a common meta-model of the evolution of the mind, represented by the existing trends in art, dominant ways of thinking and the dominant forms of violence. In this way we can achieve a variant of the structural paradigm for the goal of the genetic program as movement of active and reorganizing transcendentallity. The inclusion of creative beginning (as dominant technologies), humanity (the preferred style of art), ways of thinking (the favorable mathematical models) and aggression (the governing forms and means of violence) in one common classification, allows us to value the synchronization and balance in the program, as autonomous goal-originating system. The relations between the development of mathematical models and theories, cultural accumulations and their generalizations in art, technological advances, forms of warfare and concepts for governing the society, presented in table 2, are unambiguous.[17]

It is obvious that the human dependence on the technologies could be both creative and destructive. The rapid technological development of an immature civilization could lead to its self-destruction. Through the direct link of the generations "parents–children–grandchildren–…" every society accumulates knowledge and humanitarian values and, as a result, the mind overcomes emotions and violence. This process is determined by the increasing culture of the human society. It is obvious that the direct forms of warfare are fading out and are replaced by new hidden forms. New non-lethal weapons or precisely guided munitions are used to avoid death among civilian population and collateral damage. This tendency dilutes the bounds of organized violence, decreases the effect of its execution and makes it acceptable according to the new levels of humanity of civilization. In the frame of the genetic program an evolutionary shift in the ratio humanity/violence is realized. New forms of warfare became actual and the most prominent of them is the information warfare. It transfers the violence from the physical reality into the noosphere. Forced by the rapid development of information technologies, information warfare became one of the most important threats to the security of every nation.

The Interpretation

In our life, filled with information technologies, the statement that we do not see anything is not valid. With the aid of the media we see everything, but the problem is that we are unable to interpret the reality. Everyone sees the things in his or her own unique point of view. It emerges on the base of experience, traditions and knowledge. It is formed by heritage, by the genetic program. Through the mechanisms of this program the knowledge is "learned" or "borrowed" from the surrounding life without making any effort. The most common point of view is the ontological comprehension of the world, of the way the world is organized and functions. It has global character and is a direct outcome of the accumulation of knowledge and culture, obtained in the different lifecycles of past generations.

The time and the age of the man are the factors, introducing relativity in the created models of the world. Youth optimism, middle age realism and elderly wisdom are the basic, most common factors, influencing the visions shared by every generation. The shifting of the ratio between the "passed" and the "remaining" time of life of the individual significantly changes his or her strategic vision. The density of critical situations and threats in time is a factor that essentially "compresses" and "stretches" the points of view. Under these factors the genetic code of the individual is changed, which predetermines his and his descendants' behavior, level of aggression, humanity, etc. The forming of new ethics and culture in the society provides its development and prosperity. However, they also have ideological and religious sense. Thus, the global point of view is formed in this context, but it also includes an inherited orientation.

The professional point of view greatly influences the global view of the world. Medicine, jurisprudence, ecology, military affairs and politics form differences in the ways of using the different viewpoints. These ways are orientation, addressing complex problems and strategic planning. Analogously, human behavior is influenced by the political, symbolic, cultural and structural viewpoints. The mutual usage of all viewpoints is the most powerful technology providing multifaceted view on the reality.

The transition of gifts, professional abilities, talent and susceptibility by the genetic program enables the transmission of knowledge and accumulation of cultural potential in the separate branches of the family trees. This is of great importance to the social development. By providing conditions for multilateral-consensus usage of viewpoints, these cultural potentials are the main reason for the prosperity of a number of relatively small countries like Switzerland, Norway, Netherlands, etc.

Permanent, long-term national level consensus on all critical problems of societal development is the main condition for sustainable development. And vice versa, the unused aggressiveness and violence potential of a number of young nations determines their susceptibility to abrupt changes, revolutions and enforced decisions. Consensual way of thinking is rare in such societies, because of the short history of the execution of their genetic programs and the lack of social wisdom.

The Crossroad

Our days are full of politics, ideologies, someone else’s ambitions and our own problems. The tensions in the media reach unprecedented peaks. The information garbage they feed us with is in monstrous quantities. Today 90 percent of what we read, see and hear is just senseless noise and needless junk.[18] The prodigy of the media experts created seven types of senseless information to cover the absence of reasonable ideas and the fact that there is no common strategy for sustainable development.

Hypnotized by the magic of the failure of the “real” socialism and the destruction of the Soviet Union, we failed to notice how we entered a sophisticated unipolar world, created by the third technological revolution. Many new values, social and economic structures, political concepts, treaties, hopes and ideas inevitably form a new world order. Its increasing complexity and the rapid changes with chaotic dynamic put the human soul to a test. On the dusty ways of existence wander billions of souls trying, with their intellect and creativity, to find out the meaning of life.

Today, the world is in the translucent mist of the paradigmatic insufficiency. The pragmatic search for a universal Know How as a technology of survival foreruns the philosophic Know Why of the future world’s strategic models. Our days are filled with grotesque scenes of distorted reality. The world is on a crossroad. One of the ways leads to self-destruction from the inside and the other will turn the wheel of progress. There is enough pain, insults and hatred for the first road. If we take it, we shall drown in the chaos of indolence until egoism, violence and cruelty stop the sands of time. Then, as it has happened with many civilizations, a branch of the global genetic program will be cut off. Only the memory of our ancient culture will remain. For the second way we shall need sense and creativity. It is steep and walking on it will need lots of thankless work and colossal efforts of the human intellect.

These efforts will give life to the future generations and will open the door to the future.

For the human nature, the first way is unacceptable. Only the second one is possible – the way of change that will revive our planet again. Centuries-old traditions and experience along with the modern knowledge accumulated in the genetic program of all nations will inevitably lead us into the information society of the 21st century. More than ever we must remember the answer to the questions asked by many generations – who are we, where are we, where are we going? The answers are in the genetic code of everyone of us. Our civilization survival instinct is there, too. The transition is hard, but possible. We should walk our part of the way with dignity, so that our children will not have to.

Table 2: Stages of the global development

STAGES OF THE GLOBAL DEVELOPMENT		
Stages	Period	Stages of development, art styles, ways of thinking, means of violence (armed combat), mathematical models.
I. Agrarian society, Pre-modernism, Religious-mystical mind		
First	From 6000 BC to the XIII century.	Pre-scientific period (Aristotle-Euclidean linear models) (Direct strategic leadership concept) Cold weaponry, iron armor, chariots, battering ram, catapults, infantry, cavalry, triremes, messengers, stellar navigation.
Second	From the XIV century to the middle of the XVIII century	Reconnaissance and Reformation (Cartesian-Newtonian linear models) (Staff command and control concept) muskets and cannons, cavalry, sails, telescopes, compass, sextant and clock for navigation, signal towers for communication (Napoleon wars)
II. Industrial society, Modernism, Dialectic-materialistic mind		

Third	From the middle of the XVIII century to the beginning of the XX century	First technological (industrial) revolution First revolution in military affairs (Linear dynamic models) (Decentralized systems and operations concept) rifles and artillery, balloons, dirigible, ironclad, trains, automobiles, telegraph, telephone, wired communications (World War I)
Fourth	From the beginning of the 20s to the 40s of the XX century	(Linearized dynamic models) (“Blitzkrieg” concept) Automatic rifles, first generation chemical weapons, armored vehicle, propelled aviation, analogous radios (World War II)
Fifth	From the beginning of the 50s to the end of the 70s of the XX century	Second technological (postindustrial) revolution Second revolution in military affairs (Einstein model) (“Air-ground battle” concept) First (nuclear) and second (thermonuclear) generation nuclear weaponry, second generation chemical weapons, first generation biological weapons for mass destruction, AEGIS cruisers and nuclear submarines, jet aviation, radar, radio navigation, TV, digital communications, first generation rockets, satellites and space craft. (Cold war, Vietnam and Afghanistan wars)
III. Information society, Postmodernism, Intuitive-heuristic		

Sixth	From the beginning of the 80s of the XX century to the end of the 20s of the XXI century.	Third technological (information) revolution Third revolution in military affairs (Complex, nonlinear models with chaotic dynamics) (machine-oriented warfare, “Information warfare”, “Air-Space battle” concepts) Third generation nuclear (neutron) weaponry, genetic and biological weaponry, information weaponry, laser-optic beam weapons, kinetic energy beams, precisely guided weaponry, robotic combat platforms, unmanned aerial vehicles, second generation air-space military equipment, psychotronic weaponry, non-lethal weapons and systems for early warning, GPS, GSM, global command and control systems. (Persian gulf war, Kossovo conflict and a number of humanitarian operations of UN forces)

Notes:

1. The first version of this article was published in Bulgarian – Tzvetan Semerdjiev, “The Genetic Program – A technocratic Hypothesis,” *Information & Security* 1, 1 (Summer 1998): 26-45. Due to the considerable interested an updated English version is presented in the current issue of the journal.
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The Genetic Program: A Technocratic Hypothesis on the Paradigm of Civilization

Tzetan Semerdjiev

Keywords: information society, universal mind, sustainable development, global technological development, genetic code, paradigmatic deficiency, information warfare

Abstract: A hypothesis for the evolution of the human civilization is proposed examining primarily information and information technologies as components of the global technological program of the universal mind. Assuming the a priori existence of ontological information nucleus in the genetic code, inherited by new generations, the hypothesis offers an explanation of technology, processes, and realization of a global algorithm for mastering our part of space and time, building an eternal incubator of wisdom - a colony for accumulation of knowledge and reduction of entropy in the universe. Taking into account the impact of social factors in the global models, as well as the lack of universal concept for sustainable development, we ascertain an acute paradigmatic deficiency. The interpretation of the hypothesis in the framework of a general historic window provides global classification of the phases of technological development of humanity as a metamodel. Evolving towards the information society, the human civilization naturally advances to new 'informational' forms of warfare, treated under this hypothesis as paradigmatic deformations in the global relationship between humanness and violence

full text

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OBJECT-ORIENTED ENVIRONMENT FOR ASSESSING TRACKING ALGORITHMS

Emanuil DJERASSI and Pavlina KONSTANTINOVA

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1. Introduction

One way to alleviate the complex problem of designing, assessing, and implementing tracking algorithms is to provide the designer with an environment facilitating the creation of various test scenarios, assisting the implementation of algorithms, and evaluating their performance. Such an environment is a complex software program, which could be simplified by using object-oriented design and programming. The overall program organization can be improved by unifying data and functions that operate on the data.

Both users and designers are interested in assessing and comparing the numerous target tracking methods and algorithms developed in recent years. Usually, for this purpose a dynamic situation is modeled by simulating signals from moving targets and false alarms. On the base of these signals, the target tracking algorithms initiate and estimate target tracks. This task is complex because of the uncertainty of the dynamic situation and the explosive increase of the computational load corresponding to the number of targets, typical for most tracking algorithms. It often happens that new algorithms differ only slightly from those already programmed and tested. Also, it is sometimes necessary to add new properties to the dynamic situation. These processes can be alleviated using the methods of the Object-Oriented Programming (OOP), creating a set of classes that implement the basic data structures and routines used in the simulation environment.

2. Problem formulation

The environment consists of four main parts, organized in the hierarchy presented on Figure 1.

The *Organization Part* allows the user to choose a tracking algorithm and to control the mode of its implementation. Using the polymorphism, the user needs only to create or change a virtual tracking function and then to define an object of its class.

This part organizes two modes of work: single and Monte Carlo mode.

In single mode the following steps are performed:

- Simulation of the dynamic situation;
- Tracking algorithm implementation (data processing);
- Result visualization.

These steps are repeated on each scan.

The Monte Carlo mode consists of two steps:

- Accumulating statistical data by iteratively performing the first two steps of the single mode;
- Comparison of result and visualization.

The *Simulation Part* provides methods for simulating target movements, environment characteristics and sensor parameters.

The *Data Processing* part contains specific tracking algorithms, programmed by the user.

The *Vector-Matrix Computation* part is an auxiliary part, providing a variety of classes and methods for matrix computations, which are widely used in target tracking algorithms.

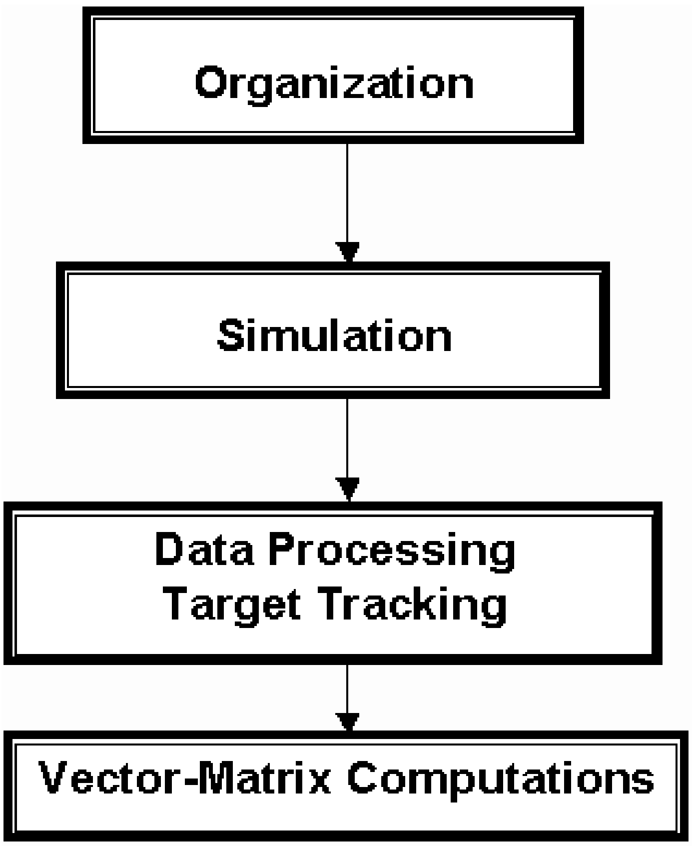

Figure 1: Components of the software environment

3. Description of the classes

3.1. Classes for program organization

The multiple parameters characterizing a dynamic situation can be appropriately presented in a class *Scenario* and the flags controlling different modes and their parameters – in a class *Control*. The parameters for MonteCarlo analysis and the functions for MonteCarlo mode implementation are included in class *MonteCarlo*.

These three classes are parents of an abstract class *TrackingAlg*. The objects of class TrackingAlg have direct access to multiple parameters and flags. On the other hand, the class TrackingAlg has three pure virtual functions: *Tracking*, *ShowScenario* and *ShowResult*. The particular tracking algorithms, as derived from class TrackingAlg, must define these pure virtual functions. Thus, the program code of the main part of the program for all tracking algorithms will be the same. Figure 2 shows the hierarchy of these classes. New classes could be added on the place of the dashed line.

// class TrackingAlg- abstract class with pure virtual functions

```
class TrackingAlg: class SCENARIO, class CONTROL, class MonteCarlo

{ public:

virtual void Tracking();)=0;

virtual void ShowScenario()=0;

virtual void ShowResult()=0;

};



class JPDAF: public TrackingAlg

{ public:

virtual void Tracking();

virtual void ShowScenario();

virtual void ShowResult();

};



class NN: public TrackingAlg

{ public:

virtual void Tracking();

virtual void ShowScenario();

virtual void ShowResult();

};
```

In this case, the links between member-functions and objects are of the type “late binding,” i.e., on run time. Depending on the particular algorithm, the pointer FilterObjPtr will be initialized by the address of the object from the particular class. For example, the source code for two algorithms JPDAF and NN is:

```
if (TypeOfProcessing==1) FilterObjPtr = new JPDAF;

if (TypeOfProcessing==2) FilterObjPtr = new NN;
```

Similarly, new classes can be introduced for newly developed algorithms. The source code of the main program in single mode is:

```
switch (FlagStage)

{ case 1:

FilterObjPtr->ShowScenario();

FilterObjPtr->Tracking();

break;

case 2:

FilterObjPtr->ShowResult();

break;

default: break;

} // end of switch
```

The source code of the main program in MonteCarlo analysis mode is `FilterObjPtr->MonteCarloRun();`

And because in class MonteCarlo the function Tracking is declared as virtual in run-time, the tracking function corresponding to the chosen algorithm will be selected. This is an example of polymorphism, as the same code implements different methods according to the type of FilterObjPtr. Depending on the work stage, the methods ShowScenario and the tracking algorithm Tracking or the method ShowResult are implemented.

Thus, the addition of a new algorithm is reduced to defining a new class derived from the abstract class TrackingAlg and defining its particular virtual methods Tracking, ShowScenario and ShowResult. The main part of the program can remain unchanged.

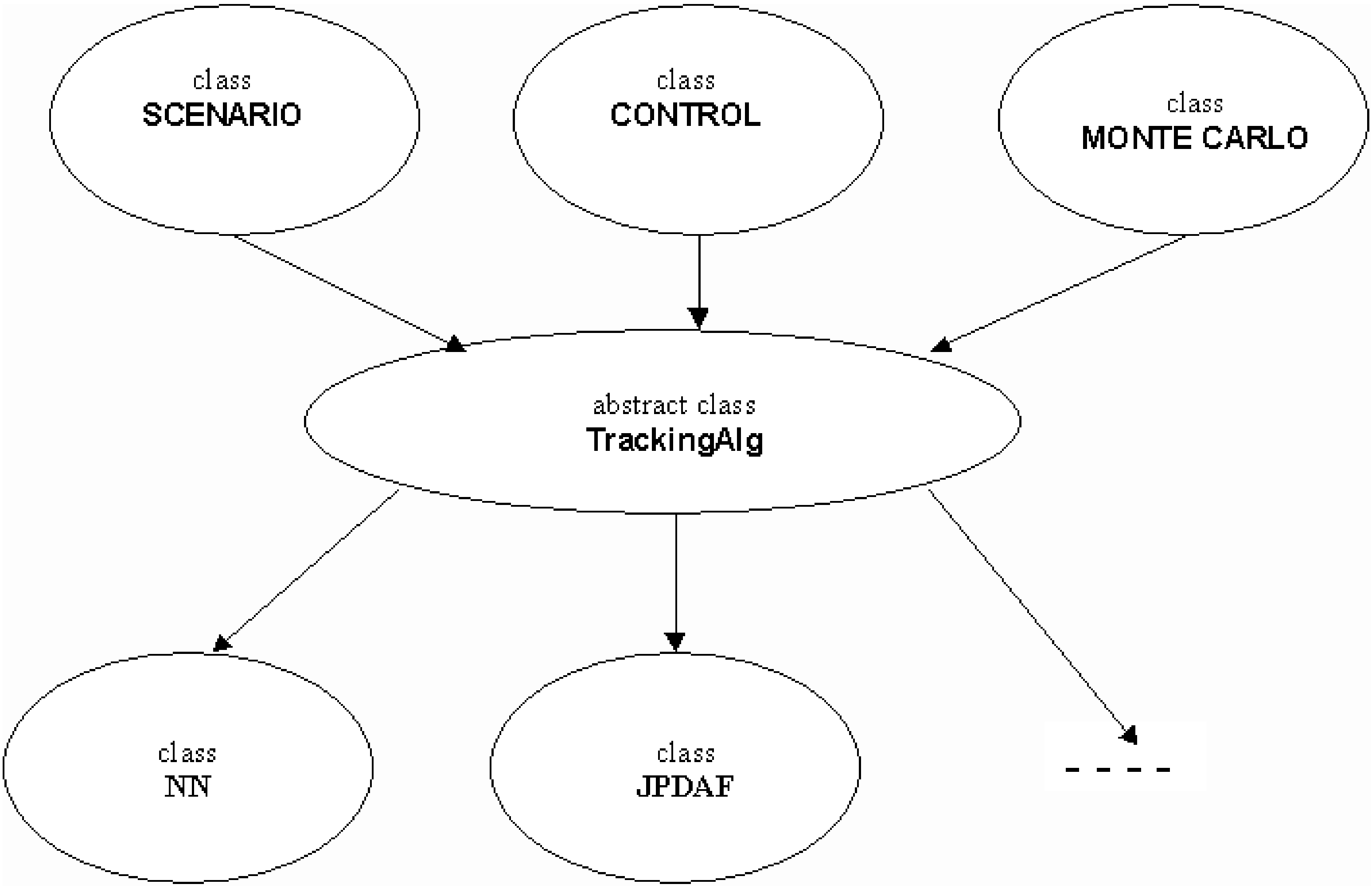

Figure 2: Class hierarchy

3.2. Classes for simulation

The data defining some specific dynamic situation (scenario) is initially entered from a file. Each simulated target requires data on initial coordinates, velocity and movement direction, and the maneuvering targets need also information regarding initial and final times of the maneuver, acceleration during the maneuver, etc. Based on the data for each sensor observation, current coordinates are computed and stored in order to check later the measures of performance of the tracking algorithm. It is useful to define a class ClsTarget unifying all that data for a target and the functions dealing with it.[5,6] The basic behavior characteristics of a target moving according to specific rule are implemented through the class method MoveToNextPosition. In spite of the fact that the method uses multiple data for each object, it is not necessary to write them because the method has direct access to all the data for the object. The other two methods ReadTargetData and CoordInitializing are used at the beginning for target data initialization.

The description of this class is:

```
class ClsTarget                    // Information about target

{ int Label1;

int Type1;

float Xi,Yi;                       // Initial Coordinates

float WI;                          // Current Velocity

float PSIi;                        // Initial Velocity
```

```
float Azi;                                  // Initial Heading

float X,Y,Z;                                // Current Cartesian coordinates

float DDot,D,Azimuth,Epsilon;               // Current Polar coordinates

int InitialScan;

int NTrSegments;

public:                                     // Methods for the class

void ReadTargetData(FILE *FileIn);

void CoordInitializing();

void MoveToNextPosition();                  // friend functions, which use Targets' data

friend void DefineDetectedTargets(Float Pd,

                   int & NumberOfDetectedTargets, IntArrTarg
                   DetectedTargets);

friend int DataPreparationForCurrentScan();

friend float RSE(int itr, int jr) ;

friend class ClsMeasurement;

} ; // end of class ClsTarget
```

Another essential group of data describes the simulated measurements or the so-called "raw data." The raw data is calculated on the base of the data for the moving targets from the objects of the class ClsTarget. For this data it is useful to define a class *ClsMeasurement*. The function *DataPreparationForCurrentScan* is declared as a friend function for both classes - ClsMeasurement and ClsTarget. In this function, the measurements "received" on the current scan are computed. According to the specific sensor parameters, the errors of the measurements are simulated. According to the probability to detect correctly, the number of detected targets is defined. The method Noising of the class ClsMeasurement uses the data of the detected target to generate the corresponding measurement. The description of this class follows:

```
class ClsMeasurement

{ private:

int Label1;

float X,Y,Z;

float Range, Azimuth,

float Dopler,Elevation;

int Busy;

public:
```

```
void Noising(ClsTarget & ob);

friend int DataPreparationForCurrentScan();

};
```

3.3. Classes for tracking algorithms

3.3.1 Theoretical background

In general, a track is a set of measurements from the same target at different times. However, in most tracking algorithms the track is approximated for each time by a difference equation in the form:[3]

$$x(k+1) = F(k)x(k) + G(k)u(k) \qquad (1a)$$

where $x(k)$ is a n-dimensional target state vector at time k, which consists of the quantities to be estimated, and F is a transition matrix, G is a control matrix, and u is a control vector. $x(k+1)$ is the prediction of the state vector for time $(k+1)$.

The measurement vector received from the sensor is:

$$z(k) = Hx(k) \qquad (1b)$$

Because of the measurement errors and false alarms, the real state vector x is never known. Instead, we have to work with its estimation $\hat{x}$. The process of estimating is usually called filtering, and the correspondent algorithms are called filters. Nowadays, the common filters used for this purpose are based on the Kalman filter.

3.3.1.1. Linear Kalman filter

When equations (1a) and (1b) are linear, the linear Kalman filter is used. The basic form of the this filter is:

$$\hat{x}(k+1 \mid k) = F(k)\hat{x}(k \mid k) + G(k)u(k) \qquad (2a)$$

$$\hat{z}(k+1 \mid k) = H(k+1)\hat{x}(k+1 \mid k) \qquad (2b)$$

$$\nu(k+1) = z(k+1) - \hat{z}(k+1 \mid k) \qquad (2c)$$

$$P(k+1 \mid k) = F(k)P(k \mid k)F(k)' + Q(k) \qquad (2d)$$

$$S(k+1) = H(k+1)P(k+1 \mid k)H(k+1)' + R(k) \qquad (2e)$$

$$W(k+1) = P(k+1 \mid k)H(k+1)'S(k+1)^{-1} \qquad (2f)$$

$$\hat{x}(k+1 \mid k+1) = \hat{x}(k+1 \mid k+1) + W(k+1)\nu(k+1) \qquad (2g)$$

$$P(k+1 \mid k+1) = P(k+1 \mid k) - W(k+1)S(k+1)W(k+1)' \qquad (2h)$$

where $\hat{x}$ is the estimation of the target state vector, z is the measurement vector, H is the measurement matrix, W is the gain matrix, S is the innovation covariance matrix, Q is the noise covariance matrix, R is the measurement covariance matrix, ν is the innovation vector, and P is the covariance matrix.

3.3.1.2 Nonlinear (Extended) Kalman filter

When equations (1a) and/or (1b) are nonlinear, the Extended Kalman Filter is used. Its equations are the same as the equations of the Linear Kalman Filter (2a-2h), but the matrices F(k) and H(k) are Jacobians, based on the first order Taylor expansion of the nonlinear functions (1a) and (1b) respectively. Hence, the nonlinear filter estimation can be reduced to a linear filter estimation after the Jacobians are calculated.

3.3.1.3 Probabilistic Data Association (PDA) filter

When the observations from a single target are mixed with clutter, the Probabilistic Data Association filter is applied instead of the classic Kalman filter.[4] It is also called "all neighbors method" because the updated estimate for a track contains contributions from all N observations within the gate of track i. The probability of the hypothesis $H_j (j = 1,2,\ldots N)$ that the observation j is a valid return for the track i is proportional to the likelihood function g_{ij}:

$$g_{ij} = \frac{e^{-\frac{d_{ij}^2}{2}}}{(2\pi)^{M/2}\sqrt{|S_i|}}, \qquad (3)$$

where $d_{ij}^2 = v_{ij}^T S_i^{-1} v_{ij}$, ($v_{ij}$ is measurement residual for track i and measurement j according to (2c)).

Then

$$p_{ij}^{'} = \beta^{N-1} P_D g_{ij}, \qquad j = 1,2,\ldots, N \qquad (4)$$

where β is extraneous return density, P_D is detection probability.

The probabilities (p_{ij}) associated with the N+1 hypotheses (that can be formed) are computed through the normalization equation:

$$p_{ij} = \frac{p_{ij}^{'}}{\sum_{l=0}^{N} p_{il}^{'}} \qquad (5)$$

The residual for use in the Kalman Filter update equation is a weighted sum of the residuals associated with the N observations:

$$\tilde{y}_i(k) = \sum_{j=1}^{N} p_{ij}\tilde{y}_{ij}(k), \qquad (6)$$

where

$$y_{ij}(k) = y_j(k) - H\hat{x}_i(k \mid k-1)$$

$y_j(k)$ = observation j received at scan k.

The covariance P is updated according to the equations:

$$P(k \mid k) = P^o(k \mid k) + dP(k) \qquad (7)$$

where

$$P^{o}(k|k) = p_{i0}P(k|k-1) + (1-p_{i0})P^{*}(k|k)$$

$$dP(k) = W(k)\left[\sum_{j=1}^{N} p_{ij}\tilde{y}_{ij}\tilde{y}_{ij}^{T} - \tilde{y}_{i}\tilde{y}_{i}^{T}\right]W^{T}(k)$$

and

$$P^{*}(k|k) = \left[I - W(k)H\right]P(k|k-1).$$

3.3.2. Description of the classes for tracking algorithms

We take Equations (2) and the data participating in them as basis of the structure of classes that describe tracks. At the root of the hierarchy is an abstract class containing all the vectors and matrices from (2), the method KFiltering, implementing the equations, and some virtual methods for track initiation and nonlinear filter calculations. Over it a chain of descendent classes is created, including Linear Kalman Filter, Extended Kalman Filter and Probabilistic Data Association Filter.

3.3.2.1 Abstract Class for Kalman Filter

```
class ClsAKFTrack

{ protected:

int Label1;

static int Nsize;                // state vector size

static int Msize;                // measurement vector size

static Matrix Q;                 // noise covariance matrix

static Matrix R;                 // measurement covariance matrix

static Matrix G;                 // control matrix

static Vector U;                 // control vector

Matrix F;                        // transition matrix

Matrix H;                        // measurement matrix

Vector X;                        // object state vector

Matrix P;                        // covariance matrix

Matrix S;                        // innovation covariance matrix

Vector ZPrediction;              // measurement prediction vector

Vector Zmeasurement;             // measurement vector
```

```
public:

virtual void

CreateModel(float * Sigma, float Tscan)=0;

virtual int CheckGating(Vector Zmeasurement)=0;

virtual void InitTrack();

virtual void DefineH(){};            // specific for nonlinear H

virtual void DefineF(){};            // specific for nonlinear F

virtual void MeasurementPrediction();

virtual void Innovation()=0;

virtual void Covariance Update();

void KFiltering();

};
```

It should be noted that the data for Q, R, G and U is declared static because as data for the class (not for the objects of the class) it is the same for all objects of that class.[5,7]

The method `KFiltering` consists of the following steps (some of them are implemented by methods):

- `DefineF` – calculates the Jacobian of F in the case of Extended Kalman Filter; for a Linear Kalman Filter it does nothing.
- `State Prediction` - Implements Equation (2a).
- `Covariance Prediction` - Implements Equation (2d)
- `DefineH` - calculates the Jacobian of H in the case of Extended Kalman Filter; for a Linear Kalman Filter it does nothing.
- `MeasurementPrediction` – For linear case implements Equation (2b). This method is declared virtual. For nonlinear case it is defined according to the used measurement and state vectors.
- `Innovation` – Implements Equation (2c). In some specific cases as PDAF this method is defined to calculate combined innovation according to the used algorithm - equation (6).
- `Filter Gain` - Implements Equations (2e), (2f).
- `State Update` - Implements Equation (2g).
- `Covariance Update` – Implements Equations (2h). In the case of PDAF this method is defined to implement equation (7).

3.3.2.2 Linear Kalman Filter

The declaration of the Linear Kalman Filter class is:

```
class ClsLKFTrack : public ClsAKFTrack
```

```
{ public:

virtual void CreateModel(float * Sigma, float Tscan);

virtual void InitTrack();

virtual void DefineH(){};

virtual void DefineF(){};

virtual void MeasurementPrediction();

virtual int CheckGating(Vector Zmeasurement);

virtual void Innovation();

virtual void CovarianceUpdate();

};
```

This class inherits the data and the methods of the abstract class and implements the virtual functions. The method `MeasurementPrediction` calculates (2b), `Innovation` calculates (2c) and `CovarianceUpdate` calculates (2h). The function `CreateModel` should be executed only once to set the matrices Q, R, G and the vector U. Its parameters are `Sigma` – process noise, and `Tscan` - the scan period. This class is not abstract and can be used for creating objects.

3.3.2.3 Nonlinear Kalman Filter

The declaration of the Nonlinear Kalman Filter class is:

```
class ClsEKFTrack : public ClsLKFTrack

{ public:

virtual void DefineH();                // specific for nonlinear H

virtual void DefineF();                // specific for nonlinear F

virtual void MeasurementPrediction();

virtual int CheckGating(Vector Zmeasurement);

};
```

This class inherits the data of the `ClsLKFTrack` class and its virtual methods DefinF and DefineH are defined to implement specific functions that calculate the Jacobians as stated earlier.

3.3.2.4 Probabilistic Data Association (PDA) filter

A new class, derived from ClsEKFTrack, can be used for tracking targets in clutter. The number of measurements in the gate - NumOfObsInTrackGate and an array with the number of each observation and its score - ObsInTrackGate have to be added The following virtual functions are defined for this particular class: CheckGating fills the array of measurements in the gate and their scores, Innovation is defined to compute combined innovation according to (6), CovarianceUpdate updates covariance matrix P according to (7).

The class declaration is:

```
class ClsPDAFTrack : public ClsEKFTrack

{ int NumOfObsInTrackGate;

NumAndScore ObsInTrackGate[MaxNumberOfObs];

public:

virtual void DefineH();                // specific for nonlinear H

virtual void DefineF();                // specific for nonlinear F

virtual void MeasurementPrediction();

virtual void Innovation();

virtual int CheckGating(Vector Zmeasurement);

virtual void CovarianceUpdate();

};
```

3.4 Classes for matrix calculations

The main part of all tracking algorithms consists of repeatedly performed estimation of target state vectors, usually called filtering.[2,3] Each estimation consists of multiple operations with vectors and matrices as presented in equations 2(a-h).

In order to facilitate the implementation of such algorithms, we introduce the classes *Vector* and *Matrix*.[6] Their methods are intended to replace some traditional functions, implementing operations of the matrix algebra. The header file of the classes Vector and Matrix is:

```
#include "TrackType.h"                 // for MaxSize

#ifndef VMAHOOP

#define VMAHOOP

class Vector;                          //to be used in class Matrix

class Matrix

{ friend class Vector;

int M,N;                               // matrix dimension

float mat[MaxSize][MaxSize];

public:

int rows(){return(M)};
```

```
int cols(){return(N)}

Matrix(int m=MaxSize,int n=MaxSize) {M=m;N=n;}

Matrix(const Matrix & from);        // copy constructor

Matrix &operator=(const Matrix & from);

Matrix operator+(Matrix & a);

Matrix operator-(Matrix & a);

Matrix operator*(Matrix & a);

float & operator()(int i,int j);    //access by(row,col)

                                    //friend functions

friend void operator+=(Matrix &a,const Matrix &b);

friend Matrix transp(Matrix & a);   //transpose

friend Matrix inv(Matrix & a);      //inverse

};

class Vector

{ int N;                            // vector dimension

float vec[MaxSize];

public:

Vector( int n=MaxSize){ N=n; };

Vector( const Vector & from);

Vector & operator=(Vector & from);

Vector operator+(Vector & a);

Vector operator-(Vector & a);

Vector operator*(Vector & a);

friend Vector operator*(float & a,

Vector & b);                        // scalar * Vector

float & operator[](int i){ return vec[i];};
```

```
friend Matrix ColRowProd(const Vector& Col,

const Vector& Row);

#endif
```

The classes Vector and Matrix make the writing of program source code easier. The reduction of the number of the function parameters (the member-function has direct access to the object data) from one hand, and the similarity of the source code with writing formulas on the sheet of paper on the other hand, reduce the probability of errors. The main advantages of using these classes is that the code becomes readable and resembles the code written in the MATLAB language, but is more efficient, because it is compiled instead of interpreted.

Table 1 presents the comparison of some source code written by functions and by Vector-Matrix predefined operations.

Table 1: Comparison of source code implemented with functions and with classes

Using functions	*Using the Vector and Matrix classes:*
StatePrediction(NSize,F,X,G,u)	X=F*X+G*u
MeasurementPrediction (MSize,Nsize,Zpred,H,X)	Zpred=H*X
VectorDif(MSize,DZ,Zmeas,Zpred)	DZ=Zmeas-Zpred
CovariancePrediction(NSize,F,P,Q)	P=F*P*transp(F)+Q
FilterGain(NSize, MSize , P, H, R , S , SInv, W)	S=H*P*transp(H)+R
	W=P*transp(H)*inv(S)
StateUpdate(NSize, MSize, DZ, W, X)	X=X+W*DZ
CovarianceUpdate(Nsize, MSize, W, P,S)	P=P-W*S*transp(W)

4. Conclusion

This paper presented one environment for assessing tracking algorithms. It uses a set of classes that, by unifying data and functions that process them, improve the organization of the complex programs for simulation and testing of tracking algorithms. In the newly created algorithms, the already programmed and tested models of the dynamic situation and the overall program organization can remain unchanged. It is only necessary to define virtual functions for the new algorithms. The classes proposed for vector-matrix operations facilitate the writing of the algorithm source code.

Acknowledgment

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Object-oriented Environment for Assessing Tracking Algorithms

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Keywords: Object-oriented programming, tracking algorithms, sensor data processing, Monte-Carlo analysis

Abstract: Designing, implementing, and assessing tracking algorithms is an essential and complex problem in numerous defense and security related applications. One way to alleviate this problem is to provide the designer with an environment, facilitating the creation of various test scenarios, to propose aids for implementing algorithms, and to evaluate their measures of performance. Such an environment is a complex software program, which could be simplified by using object-oriented design and programming. By unifying data and functions that operate on the data, the overall program organization can be improved considerably. In this article the authors propose a set of classes that can be divided into three groups, considering respectively the modeling part, the processing part and the organization of the statistical analysis for measuring performance.

full text

ON THE GENERALIZED INPUT ESTIMATION

Vesselin JILKOV and Xiao RONG LI

1. Introduction

Consider the problem of maneuvering target tracking within the framework of the familiar time invariant linear dynamic system [1]

$$x_{k+1} = Fx_k + Gu_k + w_k \tag{1}$$

$$z_{k+1} = Hx_{k+1} + v_{k+1}, \qquad k = 0, 1, 2, \ldots \tag{2}$$

where $x_k \in \mathbf{R}^{n_x}$ denotes the target state with transition matrix F, $u_k \in \mathbf{R}^{n_u}$ is the control input with transition matrix G, $z_k \in \mathbf{R}^{n_z}$ is the measurement with measurement matrix H, and $w_k \in \mathbf{R}^{n_x}$, $v_k \in \mathbf{R}^{n_z}$ denote respectively the process noise and measurement errors which are assumed independent Gaussian white noises with zero means and covariances Q_k and R_k.

The classical *input estimation* (IE) [2] assumes that the unknown control input is constant, i.e. if the maneuver has started at time k, then

$$u_j = \begin{cases} 0 & \text{for } j = 0, 1, \ldots, k-1 \\ u & \text{for } j = k, \ldots, k+N-1, \end{cases} \tag{3}$$

where N denotes the detection window length. This assumption allows to use the least squares (LS) method for parameter estimation to obtain an estimate of the input u over the interval $[k, k+N)$, based on the information contained in the innovations of the Kalman filter assuming *zero-input* in the interval $[0, k+N)$.

In order to relax this restrictive and unrealistic assumption it was suggested [3] to represent the unknown input u_k as a linear combination of time functions, viz.

$$u_k = \sum_{l=1}^{p} a_l b_l(t_k), \tag{4}$$

where $b_l(t_k)$ are *known* scalar functions of time and a_l are *unknown constant* vector coefficients. For the so defined "non-constant" input the LS estimation technique has been applied and a thorough algorithm derivation has been performed. [3]

Apparently, if we consider the input transition matrix G in (1) as time invariant then the presentation of the input (4) is more general than that of the constant input $u_k = u$ in (3). On the other hand, however, if we consider the *overall* unknown input $a \triangleq (a_1, a_2, \ldots, a_p)$, it is in fact constant and the known time functions $b_l(t_k), l = 1, \ldots, p$ influence this input as transition coefficients (in the same manner as the input transition matrix G does). That is why it is more natural that these coefficients be attributed to the input transition (coefficient) matrix rather than to the input itself. This underlying reason has led us to the following two observations.

- The generalized IE model (1) with (4) is a particular case of the *constant input* model

  $$x_{k+1} = Fx_k + G_k u + w_k \tag{5}$$

  with *time-varying input transition matrix* G_k. Indeed, if we set

  $$a \triangleq \left[a_1' \ldots a_p'\right]' \text{ and } G_k^a \triangleq [b_1(t_k)G \mid b_2(t_k)G \mid \ldots \mid b_p(t_k)G] \tag{6}$$

  then (1) with (4) can be recast as

  $$x_{k+1} = F_k x_k + G_k^a a + w_k. \tag{7}$$

  That is, a stands for the unknown *constant* input and G_k^a for the *known* (time-varying) input transition matrix. Of course (5) comprises (7) and is not restricted to the particular choice of G_k as G_k^a.

- The classical IE for constant input is *valid* for time-varying systems, and in particular for (5) (respectively (7)).

These observations imply that the GIE of [3] is a particular case of the classical IE with time-varying input transition matrix G_k (if we set $G_k = G_k^a$) . Next, we describe in more details the IE for time-varying systems and show how the GIE can be obtained from it.

2. IE for Time-Varying Systems

Although the IE method of Chan, Hu and Plant,[2] has been traditionally treated in time invariant system setting, it is valid for time-varying systems as well and no additional difficulties appear in this consideration. We summarize the basic IE method with reference to the target model (5).

The optimal Kalman filter (KF) for the system (5), (2), where F_k may be also time-varying, is [4]

$$\begin{aligned}\hat{x}_{k+1} &= (I - K_{k+1}H)\, F_k \hat{x}_k + (I - K_{k+1}H)\, G_k u_k + K_{k+1} z_{k+1} && (8)\\ P_{k+1} &= (I - K_{k+1}H)\,(F_k P_k F_k' + Q_k) && (9)\\ K_{k+1} &= (F_k P_k F_k' + Q_k)\, H' \left[H\,(F_k P_k F_k' + Q_k)\, H' + R_{k+1}\right]^{-1}. && (10)\end{aligned}$$

Let the assumption (3) holds and denote by $\hat{x}_j^*$ and $\hat{x}_j$ the estimates of the *hypothetical* Kalman filter with the *correct input* u_j of (3), and the *real* KF, running with the *zero-input model* $u_j = 0,\ j = k, \ldots, k + N - 1$, respectively. Their residuals, defined respectively as

$$\tilde{z}_j^* \triangleq z_j - H\hat{x}_j^*, \quad \tilde{z}_j \triangleq z_j - H\hat{x}_j, \quad j = 1, 2, \ldots \tag{11}$$

satisfy

$$\tilde{z}_{k+n} = HD_{k+n}u + \tilde{z}_{k+n}^*, \quad n = 1, 2, \ldots, N\ , \tag{12}$$

where

$$D_{k+n} \triangleq \sum_{i=1}^{n} \prod_{j=n}^{i+1} (I - K_{k+j}H)\, F_{k+j}\, (I - K_{k+i}H)\, G_{k+i-1}. \tag{13}$$

It is known that $\left\{\tilde{z}_{k+n}^*\right\}_{n=1,2,\ldots,N}$ is a white noise sequence with $\tilde{z}_{k+n}^* \sim \mathcal{N}(0, S_{k+n})$, where the covariance $S_{k+n} = HP_{k+n}H' + R_{k+n}$.[4] Thus according to (12) the residuals of the real (zero-input) KF provide noisy measurements of the unknown input, and the *best linear unbiased estimate* (BLUE) of u can be straightforwardly obtained by means of the LS method for this system.[4]

Specifically, the system (12) can be recast in the "batch form"

$$\tilde{\mathcal{Z}} = \mathcal{H}u + \tilde{\mathcal{Z}}^*, \tag{14}$$

where stacked vectors and matrices are denoted as follows

$$\begin{aligned}\tilde{\mathcal{Z}} &= \begin{bmatrix} \tilde{z}_{k+1}' & \ldots & \tilde{z}_{k+N}' \end{bmatrix}',\\ \mathcal{H} &= \begin{bmatrix} (HD_{k+1})' & \ldots & (HD_{k+N})' \end{bmatrix}',\\ \tilde{\mathcal{Z}}^* &= \begin{bmatrix} \tilde{z}_{k+1}^{*\prime} & \ldots & \tilde{z}_{k+N}^{*\prime} \end{bmatrix}'\end{aligned} \tag{15}$$

and $\tilde{\mathcal{Z}}^* \sim \mathcal{N}(0, \mathcal{S})$ for $\mathcal{S}$ =block-diag$\{\ S_{k+1}, \ \ldots, \ S_{k+N}\ \}$. Then the BLUE of u which minimizes the normalized error

$$\mathcal{L}_{\mathrm{LS}}(u) \triangleq \tilde{\mathcal{Z}}^{*\prime}\mathcal{S}^{-1}\tilde{\mathcal{Z}}^* = \left(\tilde{\mathcal{Z}} - \mathcal{H}u\right)' \mathcal{S}^{-1} \left(\tilde{\mathcal{Z}} - \mathcal{H}u\right) \tag{16}$$

is

$$\hat{u} = \mathcal{P}\mathcal{H}'\mathcal{S}^{-1}\tilde{\mathcal{Z}} \text{ with covariance } \mathcal{P} = \left(\mathcal{H}'\mathcal{S}^{-1}\mathcal{H}\right)^{-1} \tag{17}$$

The minimal normalized error is given by

$$\hat{\mathcal{L}}_{\mathrm{LS}} \triangleq \mathcal{L}_{\mathrm{LS}}(\hat{u}) = \left(\tilde{\mathcal{Z}} - \mathcal{H}\hat{u}\right)' \mathcal{S}^{-1}\left(\tilde{\mathcal{Z}} - \mathcal{H}\hat{u}\right) = \tilde{\mathcal{Z}}'\mathcal{S}^{-1}\tilde{\mathcal{Z}} - \Delta\mathcal{L}_{\mathrm{LS}}(\hat{u}) \tag{18}$$

with

$$\Delta\mathcal{L}_{\mathrm{LS}}(\hat{u}) = \hat{u}'\mathcal{P}^{-1}\hat{u} = \left(\mathcal{H}'\mathcal{S}^{-1}\tilde{\mathcal{Z}}\right)'\mathcal{P}\left(\mathcal{H}'\mathcal{S}^{-1}\tilde{\mathcal{Z}}\right) \tag{19}$$

and $\Delta\mathcal{L}_{\mathrm{LS}}(\hat{u})$ is $\chi^2_{n_u}$ distributed, provided the true input u is zero.[1]

Thus, the first stage of the common IE method – *estimation of the input* is performed via (17). The second stage – *maneuver detection* – realizes the *significance test* [4]

$$\Delta\mathcal{L}_{\mathrm{LS}} > \lambda \tag{20}$$

through (19), where λ is chosen for a given P_{FA}. The third stage of the IE algorithm – *estimate correction* – is performed in case of detecting a maneuver according to

$$\hat{x}^u_{k+N} = \hat{x}_{k+N} + \underbrace{D_{k+N}\hat{u}}_{\text{correction term}} \tag{21}$$

$$P^u_{k+N} = P_{k+N} + \underbrace{D_{k+N}\mathcal{P}D'_{k+N}}_{\text{uncertainty increase}} \tag{22}$$

In the above, we very briefly recalled the known IE method with the sole difference of considering the *generic time-varying* target model.

3. GIE as a Corollary of IE

Now that the IE is available one can obtain the GIE algorithm of [3] (specifically, the results presented in sections III and IV therein) as a corollary of the above given common IE. Although it should be apparent from the two remarks made in the Introduction we illustrate some details.

Consider the problem as formulated by Lee and Tahk . [3] Let us set for this problem a and G^a_k as in (6) and substitute G_k with G^a_k and u with a throughout in the Eqns (13) – (22) of the IE.

After some routine formulae manipulations the following key intermediate relations can be subsequently obtained

$$D_{k+n} =$$
$$\sum_{i=1}^{n} M_{k+n}^{k+i} \left(I - K_{k+i}H\right) G_{k+i-1} = \tag{23}$$
$$\sum_{i=1}^{n} M_{k+n}^{k+i} \left(I - K_{k+i}H\right) \left[b_1(t_{k+i-1})G \mid \ldots \mid b_p(t_{k+i-1})G\right] = \tag{24}$$
$$\left[\sum_{i=1}^{n} M_{k+n}^{k+i} \left(I - K_{k+i}H\right) N_{k+i} b_1(t_{k+i-1}) \mid \ldots \mid \sum_{i=1}^{n} M_{k+n}^{k+i} \left(I - K_{k+i}H\right) N_{k+i} b_p(t_{k+i-1})\right] = \tag{25}$$
$$\left[D_{k+n}^{1} \mid \ldots \mid D_{k+n}^{p}\right] \tag{26}$$

$$HD_{k+n} = \left[C_{k+n}^{1} \mid \ldots \mid C_{k+n}^{p}\right] \text{ since } HD_{k+n}^{l} = C_{k+n}^{l}, \quad l = 1, 2, \ldots, p \tag{27}$$
$$HD_{k+n}u = \sum_{i=1}^{n} A_{k+n}^{k+i} \sum_{l=1}^{p} a_l b_l(t_{k+i-1}) \text{ since } G_{k+i-1}u \triangleq G_{k+i-1}^{a}a = G \sum_{l=1}^{p} a_l b_l(t_{k+i-1}) \tag{28}$$
$$\mathcal{H}'\mathcal{S}^{-1}\tilde{\mathcal{Z}} = \sum_{n=1}^{N} \begin{bmatrix} \left(C_{k+n}^{1}\right)' \\ \vdots \\ \left(C_{k+n}^{p}\right)' \end{bmatrix} S_{k+n}^{-1}\tilde{z}_{k+n} \triangleq \Omega \tag{29}$$
$$\mathcal{H}'\mathcal{S}^{-1}\mathcal{H} = \sum_{n=1}^{N} \begin{bmatrix} \left(C_{k+n}^{1}\right)' \\ \vdots \\ \left(C_{k+n}^{p}\right)' \end{bmatrix} S_{k+n}^{-1} \left[C_{k+n}^{1} \mid \ldots \mid C_{k+n}^{p}\right] \triangleq \Gamma, \tag{30}$$

where all quantities $M_{k+n}^{k+i},\ N_{k+i}, D_{k+n}^{l},\ C_{k+n}^{l},\ A_{k+n}^{k+i}, \Omega,\ \Gamma$ are the same as defined by Lee and Tahk.[3]

Consider now the IE algorithm. Firstly, the error (16) is

$$\mathcal{L}_{\mathrm{LS}}(u) = \sum_{n=1}^{N} \left(\tilde{z}_{k+n} - HD_{k+n}u\right)' S_{k+n}^{-1} \left(\tilde{z}_{k+n} - HD_{k+n}u\right) \tag{31}$$

and after the substitution of $G_k u$ with $G_k^a a$, in view of (28), it transforms to the performance index $L(k, N)$ defined in[3] through the identity $-\frac{1}{2}\mathcal{L}_{\mathrm{LS}}(u) = L(k, N)$ (as in Eqn. (12) of [3]). Secondly, the IE equation (17) after the substitutions (29), (30) leads to the basic GIE equation (20) of [3], since

$$\mathcal{H}'\mathcal{S}^{-1}\tilde{\mathcal{Z}} = \Omega \text{ and } \mathcal{P}^{-1} \triangleq \mathcal{H}'\mathcal{S}^{-1}\mathcal{H} = \Gamma. \tag{32}$$

Further, in view of (32), Lema 1, Lema 2, and the maneuver detector ((28) of [3]) immediately follow from (18), (19), and (20) respectively. Finally, (21) and (22) yield the correction equations (30) and (31) of [3], respectively, that can be seen by accounting for (24).

Thus we proved that the GIE algorithm of [3] can be obtained from the common IE algorithm applied to the particular choice of G_k as G_k^a, and u as a.

4. Conclusion

More insight has been given to the problem of input estimation. It has been shown that the so called generalized input estimation can be interpreted as a particular case of the conventional input estimation with *constant input* and *time-varying transition matrix of the input*. The latter setting, however, is more general than that of the generalized input estimation. In practice, it enables designing models with various "non-constant" inputs to be done through the design of the input transition matrix.

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On the Generalized Input Estimation

Vesselin Jilkov and Xiao Rong Li

Keywords: maneuvering target tracking, input estimation, generalized input estimation.

Abstract: The input estimation (IE) is one of the competing methods for maneuvering target tracking. This article aims to clarify the interrelationship between the standard IE method and the recently proposed generalized input estimation (GIE). The authors show that the GIE can be obtained as a particular case of the conventional IE with a constant input and time - varying transition matrix of the input.

full text

CONTACT TRANSITIONS TRACKING DURING FORCE-CONTROLLED COMPLIANT MOTION USING AN INTERACTING MULTIPLE MODEL ESTIMATOR

Lyudmila MIHAYLOVA, Tine LEFEBVRE, Ernesto STAFFETTI, Herman BRUYNINCKX and Joris De SCHUTTER

1. Introduction

In different robot operations the manipulator has to interact with the environment through the manipulated object and modify its trajectory depending on the contact forces that arise. These force-controlled operations are called *compliant motion tasks*. Force control is required due to the fact that small errors in the models can generate high forces on the manipulator. For other tasks, such as cutting, welding or polishing, the robotic manipulator has to apply a given force to execute correctly the task. In all cases the manipulator is moving an object in contact with the environment through a sequence of contact configurations. In this paper the objects involved in the compliant motion are supposed to be rigid and polyhedral. The path of the manipulated object is a sequence of contact configurations. The configurations can be grouped into a subsequence of configurations equivalent from a topological point of view, i.e., in which the same elements of the manipulated object are in contact with the same elements of the environment. In this context each class of equivalence is called *contact formation* (*CF*).[5]

This work assumes uncertainties in the position and orientation of both the manipulated object and the environment. In practice, besides these model uncertainties other sources of uncertainties are present such as friction, sensor noises, geometrical uncertainies such as burrs, or unexpected events. The focus here is on the detection of the current CF and the instant of transition between the CFs.

Encoders, mounted at the robot joints, supply information about the end-effector location and motion, and a force sensor, mounted at the robot wrist, gives information about the interaction with the environment. This information is also used to estimate the uncertain geometric parameters. In Bruyninckx *et al.* [2] a possible architecture of an autonomous assembly system is proposed. It is pointed out that such a system needs a high-level planner (responsible for planning, re-planning and on-line error recovery), a low-level module (responsible for sensing and the execution of the planned action), and a medium-level module (for estimation and monitoring).

This work presents and generalizes results reported earlier.[9] The possible CFs are described by different models and, with them, an Interacting Multiple Model (IMM) estimator is implemented. Its performance is investigated and evaluated by experiments with real data of different type: velocities and forces. Other works treat force-controlled compliant motion tasks.[3,4,7] Thus, the problem of estimating first-order geometric parameters and monitoring contact transitions has been approached through single-model Extended Kalman Filters (EKFs), run in parallel for the known different CFs.[3] The Summed Normalized Innovation Squared (SNIS) test has been used as an indicator of the transitions between the CFs. One solution to the estimation of the geometric parameters for one CF was proposed on the basis of iterated EKF.[7,8]

The remaining parts of this article are organized as follows. In section 2 the problem of contact transitions' tracking during force-controlled compliant motion is formulated as a state estimation problem of *hybrid systems*. Section 3 gives the state and measurement equations of a compliant motion with subsequent CFs, namely those of moving a cube into a corner. Section 4 describes an Interacting Multiple Model estimator and its connection with the planning part. Section 4 yields performance analysis for the cube-in-corner assembly with experimental data involving a KUKA-IR 361 robot. The final section provides concluding remarks. Short guiding rules for the Jacobian matrices computations of the measurement and closure equations are given in the Appendix.

2. Problem formulation

During the compliant motion different CFs occur. They can involve, for instance, a contact between an edge of the manipulated object and a face of the environment (edge-face contact), a face of the manipulated object and a face of the environment (face-face contact), and so on (Figure 1). To estimate the unknown geometric parameters and track the transitions between CFs, the manipulated object and the environment are considered as a stochastic hybrid system with continuous and discrete uncertainties. The state-space equations are of the form

$$x_{k+1} = f(x_k, m_k) + g(m_k, \eta_k), \tag{1}$$

$$h_k(x_k, m_k, z_k, \xi_k) = 0, \tag{2}$$

where $x_k \in \mathbb{R}^{n_x}$ is the system state vector, estimated based on the measurement vector $z_k \in \mathbb{R}^{n_z}$; m_k is the modal state, corresponding to the CF. The measurement equation is in implicit form,[3,10] in which h_k is a function of both the estimated variables and the measured data z_k. The additive system and measurement noises $\eta_k \in \mathbb{R}^{n_\eta}$ and $\xi_k \in \mathbb{R}^{n_\xi}$ are mutually independent, white with zero mean and covariances Q_k and $R_{z,k}$, respectively. The functions f, g and h are nonlinear and remain unchanged during the estimation procedure.

In this paper the focus is on the detection of the current CF and the instant of transition between the CFs. It is supposed that the changes between the CFs are modeled by a first-order Markov chain with initial and transition probabilities, respectively

$$P\{m_{j,0}\} = \mu_j(0), \tag{3}$$

$$Pr\{m_{j,k+1}/m_{i,k}\} = \pi_{ij,k}, \tag{4}$$

where

$$\sum_{j=1}^{N} \pi_{ij,k} = 1,\ i = 1, ..., N$$

and $\pi_{ij,k}$ is the transition probability from CF m_i to CF m_j. At the same time, the unknown geometric parameters of the manipulated object and of the environment are estimated.

The solution to the state estimation problem with unknown model (1)-(2) can be provided by Bayesian sub-optimal MM estimators, between which the Interacting Multiple Model (IMM) filter has proven to be one of the most efficient schemes. Within the framework of the MM estimation the lack of knowledge about the exact model is replaced by a discrete set of models $\mathbb{M} \triangleq \{m_1, m_2, \ldots, m_N\}$, each of them describing possible modes/regimes, here different CFs. With the models several Kalman filters are run in parallel. The IMM estimator calculates the state estimate as a probabilistically weighted sum of the state estimates $\hat{x}_{j,k/k}$ from the Kalman filters with the mode probabilities $\mu_{j,k}$, namely [1]

$$\hat{x}_{k/k} = \sum_{p_{j,k} \in \mathbb{M}} \hat{x}_{j,k/k} \mu_{j,k} \tag{5}$$

and the associated covariance matrix accordingly.

Usually the constructed models for the unknown system modes/ regimes are multiple system models (1). This paper considers tracking task, where the modes, i.e. the CFs

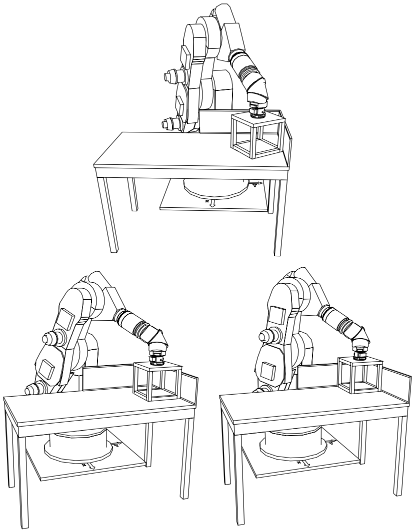

Figure 1: Robot placing a cube in a corner.

are described through several nonlinear measurement models (2), subject to nonlinear kinematic constraints, called closure equations. [3,8]

3. State and measurement equations

The system equation describes the positions and orientations of the manipulated object and the environment and it is linear.

State equation. The system model is of the form

$$x_{k+1} = x_k + \eta_k. \tag{6}$$

The estimated states are geometric grasping and environment parameters (positions and orientations). The state vector $x_k = (x_k^{mo^T}, x_k^{env^T})^T$ comprises a part $x_k^{mo} = (x_k^m, y_k^m, z_k^m, \theta_{x,k}^m, \theta_{y,k}^m, \theta_{z,k}^m)^T$, referring to the manipulated object, and a part $x_k^{env} = (x_k^e, y_k^e, z_k^e, \theta_{x,k}^e, \theta_{y,k}^e, \theta_{z,k}^e)^T$, referring to the environment. These positions and orientations do not change during the task execution, i.e. the states are *static*. Four reference frames are considered (Figure 2): $\{$w$\}$ is the *world* frame, $\{$g$\}$ is a frame on the *gripper* of which the position and orientation with respect to the world frame $\{$w$\}$ are exactly known (through the position kinematics of the robot), $\{$m$\}$ is a frame fixed to the *manipulated* object, $\{e\}$ is a frame fixed to the *environment*. x_k^{mo} are considered with respect to $\{g\}$, and x_k^{env} relative to $\{w\}$.

Measurement equation. The sensor measurements are translational and angular end-effector velocities, v_k and ω_k, together with contact forces and moments, f_k and m_k, measured by a force/torque sensor. They are grouped in the *twist* $t_k = (v_k^T, \omega_k^T)^T$, *wrench* $w_k = (f_k^T, m_k^T)^T$ and measurement $z_k = (t_k^T, w_k^T)^T$ vectors. Measurement equations are derived for each CF from the reciprocity condition. [3]

This condition states that any twist of the manipulated object is reciprocal to any wrench of the modeled wrench space (spanned by the basis G_i) and that any wrench is reciprocal to any twist of the modeled twist space (spanned by the basis J_i). Index i refers to the i-th CF. Then Eq. (2) acquires the form

$$h_{i,k} = \begin{pmatrix} G_{i,k}^T(x_{i,k})\ t_k \\ J_{i,k}^T(x_{i,k})\ w_k \end{pmatrix} = 0. \tag{7}$$

Both G_i and J_i contain trigonometric functions (sines and cosines) of the estimated states, such that the measurement functions [8] are nonlinear. To every CF correspond different twist and wrench bases. The models in Eq. (7) are very distinct, which is appropriate for using the multiple-model approach to solve the problem. Equation (7)

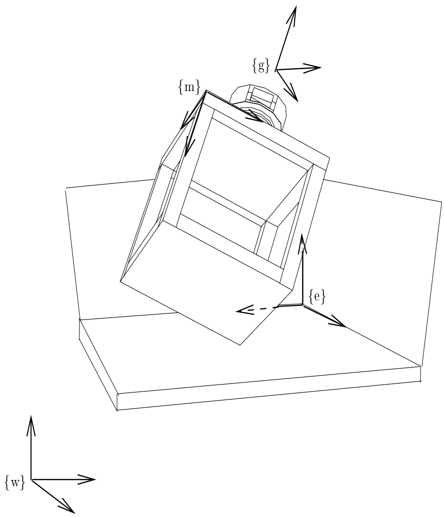

Figure 2: Frames.

is linearized for each CF around the current predicted state estimate $\hat{x}_{i,k+1/k}$. For the computation of the derivative of $h_{i,k}(.)$ with respect to the estimated variables, the partial derivatives of $J_{i,k}$ and $G_{i,k}$ are needed (See the Appendix).

Closure equations. The occurrence of a CF yields additional information for the state variables. The so-called kinematic closure equations [3]

$$c_i(x_{i,k}) = 0 \tag{8}$$

describe additional *nonlinear constraints* that relate different configuration variables (of the manipulated object and the environment) for each CF. The closure equations are models of the contacts obtained as a composition of basic contacts (vertex-face and edge-edge) between polyhedral objects. For instance, the *edge-face* contact between the cube and the environment is described by means of two vertex-face contacts, the contact between two faces of the object is described as a composition of three vertex-face contacts, and so on.

For each CF, the closure equation is applied once. Its corresponding EKF uses as initial state estimate and covariance matrix the ones obtained from the EKF based on the measurement equation. The state estimate and its covariance matrix, computed by the closure equations, are given to the interacting step of the IMM algorithm.

4. IMM estimator for transition and CF monitoring

The number of possible CFs between the manipulated object and the environment is generally high.[5] A set of mutually *exclusive* and *exhaustive* hypotheses is constructed to describe all possible CFs of the manipulated object from one place to another. For the case in which the manipulated object is a cube and the environment is a corner this number is 249.[12] In the planner,[12] a graph is constructed so that its nodes correspond to the possible CFs and its arcs to the transitions between them. Given the path of the motion and the level of uncertainty about the geometric parameters, it is possible to eliminate from the set of hypotheses those CFs whose distance d from the nodes of the path is higher than a given threshold d_{max}.[13] The distance d is the minimum number of arcs of the CF graph that are between two nodes. In this way, a relevant amount of CFs can be eliminated [2,5] and the number of hypotheses considerably reduced. Here it is assumed that the hypothesis H_0 corresponds to the case of completely constrained object. Hypotheses $H_i, i = 1, \ldots, N$ describe all other CFs. With the models for each CF and its EKFs, an IMM estimator is implemented. So, the CFs can be monitored on-line, using the information provided by the IMM mode probabilities.

The nonlinear character of the measurement equations requires the use of EKFs or other nonlinear filtering techniques that do not require computation of derivatives.[6]

The present work estimates the state vectors through EKFs. Each EKF is of the form

$$\hat{x}_{i,k+1/k+1} = \hat{x}_{i,k+1/k} + K_{i,k+1}\nu_{i,k+1}, \tag{9}$$
$$\hat{x}_{i,k+1/k} = \hat{x}_{i,k/k}, \tag{10}$$
$$P_{i,k+1/k} = P_{i,k/k} + Q_{i,k}, \tag{11}$$
$$K_{i,k+1} = -P_{i,k+1/k}H^T_{x_i,k+1}S^{-1}_{i,k+1}, \tag{12}$$
$$P_{i,k+1,k+1} = \Gamma_{i,k+1}P_{i,k+1/k}\Gamma^T_{i,k+1} + K_{i,k+1}R_{i,k+1}K^T_{i,k+1}, \tag{13}$$
$$S_{i,k+1} = R_{i,k+1} + H_{x_i,k+1}P_{i,k+1/k}H^T_{x_i,k+1}, \tag{14}$$

where

$$\begin{aligned}\Gamma_{i,k+1} &= I + K_{i,k+1}H_{x_i,k+1},\\ R_{i,k+1} &= D_{i,k+1}R_{z,k+1}D^T_{i,k+1},\\ H_{x_i,k+1} &= \partial h_i/\partial \hat{x}_{i,k+1/k},\ D_{i,k+1} = \partial h_i/\partial z_{k+1},\\ \nu_{i,k+1} &= h_i(\hat{x}_{i,k+1/k}, z_{k+1}).\end{aligned}$$

$\hat{x}_{i,k+1/k+1}$ and $\hat{x}_{i,k+1/k}$ are, respectively, the filtered and predicted state vectors, $K_{i,k+1}$ is the EKF gain matrix, $P_{i,k/k}$ is the estimation error covariance matrix, $\nu_{i,k+1}$ is a “pseudo-innovation” process [10] and $S_{i,k+1}$ - its covariance matrix. I denotes the identity matrix.

The Appendix presents guiding rules for the computation of the Jacobian matrices for the measurement and closure equations. Lefebvre and coauthors provide detailed derivation of the measurement models (models of different CFs). [8]

5. Performance analysis on a cube-in-corner assembly

The proposed approach is applied to a cube-in-corner assembly system (Figure 1). The experimental data are obtained with a KUKA-IR 361 industrial robot. The cube is mounted directly on the robot without flexibility between them. The measurements are taken at a frequency of 10 Hz. The experimental data (Figures 3-6) correspond to the three CFs of the cube-in-corner assembly (Figure 1): $k \in [0, 220]$ is the face-face and edge-face contact, $k \in [221, 450]$ is the two face-face contact, and $k \in [450, 545]$ is the three face-face contact (completely constrained case). In the test a path with three CFs is used. In the IMM the hypotheses are: H_0 - three face-face contact, H_1 - two face-face contact, H_2 - face-face and edge-face contact. The noise covariance matrices Q and R_z

$$Q = diag\{5, 5, 5, 0.001, 0.001, 0.001, 1, 1, 1, 0.001, 0.001, 0.001\},$$

$$R_z = diag\{0.05, 0.4, 1.96, 4\cdot10^{-7}, 5\cdot10^{-6}, 9\cdot10^{-8}, 0.06, 0.009, 0.008, 150, 87, 51\}$$

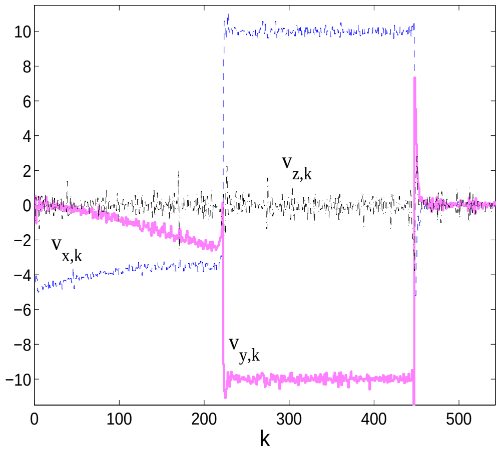

Figure 3: Measured translational velocities.

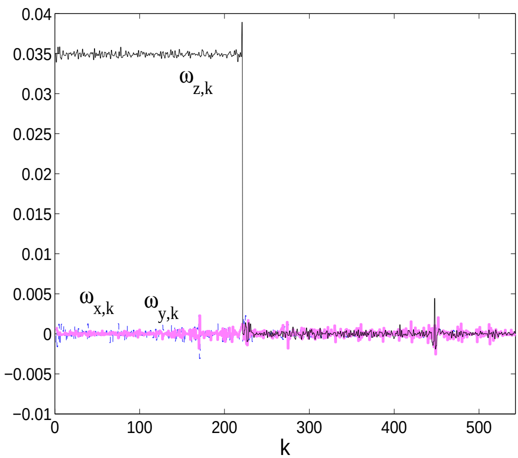

Figure 4: Measured angular velocities.

are the same for all EKFs. The units of the elements of Q are mm^2 and rad^2, respectively for the positions and angles, and those of R_z are $(mm/sec)^2$, $(rad/sec)^2$, N^2, $(Nmm)^2$ for the measured velocities, forces and moments. The system noise covari-

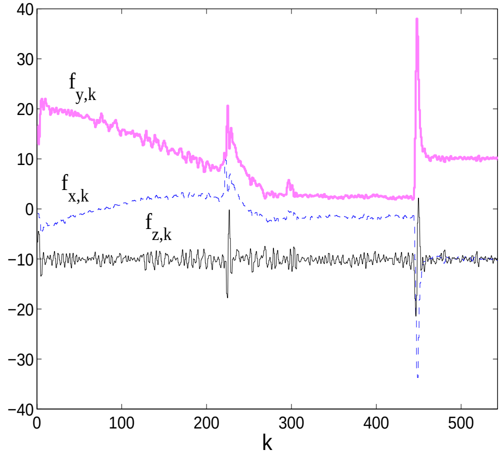

Figure 5: Measured forces.

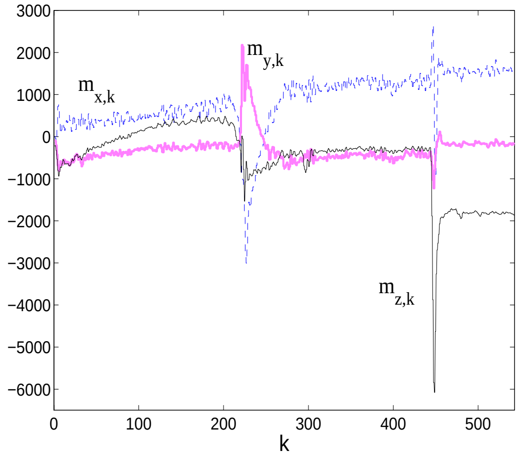

Figure 6: Measured moments.

ance matrix Q reflects the presence of linearization errors, whereas the measurement noise covariance for the used sensors is known. The IMM transition probability matrix

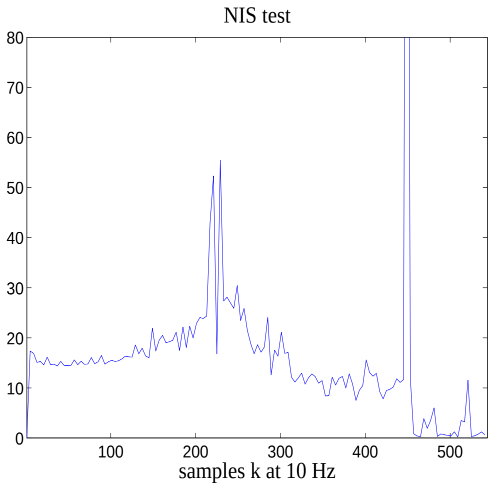

Figure 7: Normalized Innovation Squared test.

and the initial probability vector are chosen as follows:

$$Pr = \begin{pmatrix} 0.98 & 0.01 & 0.01 \\ 0.01 & 0.98 & 0.01 \\ 0.01 & 0.01 & 0.98 \end{pmatrix}, \mu_0 = \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix}.$$

Due to a lack of information, equal initial probabilities are assigned to all CFs.

It is obvious from Figure 8 that, based on the IMM probabilities, the contact transitions can be detected on time. After the change a small period is needed and the algorithm resolves the "competition" between the CFs. This is reflected also in the Normalized Innovation Squared (NIS) test,[1] $\epsilon_k = \nu_k S_k^{-1} \nu_k$ (Figure 7) and in the peak estimation errors. In the periods of transitions the estimates are not reliable. The estimation error $e_k = \hat{x}_{k/k} - x_k$ of the positions and orientations is presented in Figures 9-12. The NIS test (Figure 7) and the mode probabilities (Figure 8) contain information about the type and instants of contact transitions. By the IMM approach the CFs and the transitions between them are detected on-line and, at the same time, the unknown parameters of the manipulated object and the environment are estimated. So, both modes detection and estimation are performed automatically. In earlier works of the research team [7,8] the detection of the CFs was performed from the information of the SNIS test of independently working EKFs and their residual errors. Of course, the computational cost is proportional to the number of the EKFs (the number of CFs). The IMM filter, implemented in the present paper, is with a *fixed structure*, i.e. with preliminary

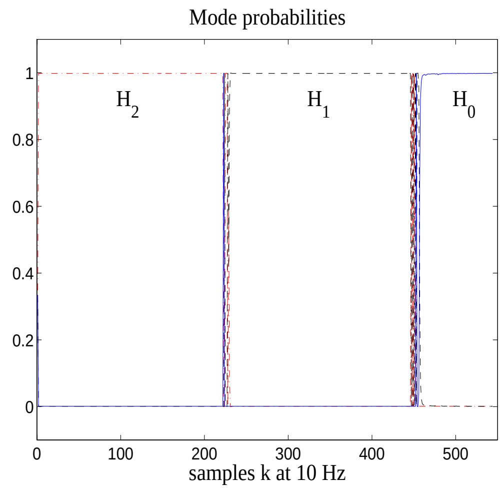

Figure 8: IMM mode probabilities.

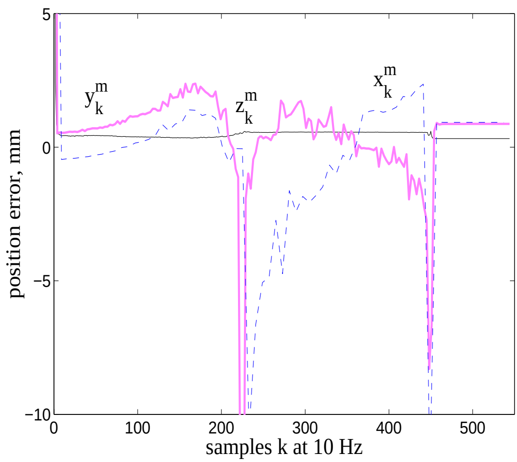

Figure 9: Error e_k in positions of the cube.

determined set of models. When the estimation block is connected with the planning part,[5,12,13] the estimator can receive from the graph of the planner information about the next neighboring CFs. Based on this graph structure of the CFs, *variable structure*

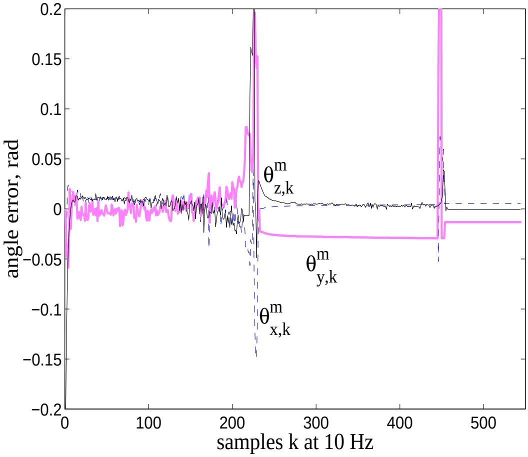

Figure 10: Error e_k in orientation angles of the cube.

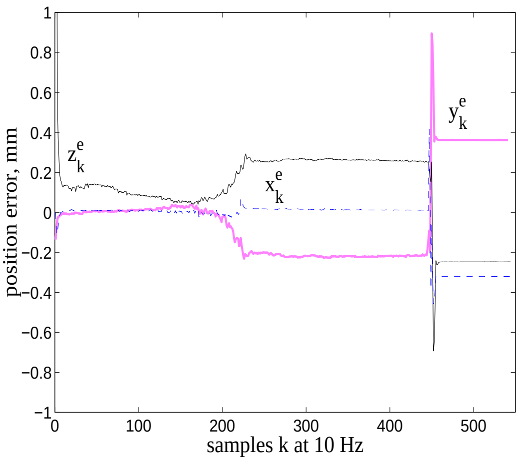

Figure 11: Error e_k in positions of the environment.

IMM estimators (with time varying set of models) can be designed.

Extensions to cases with time-varying geometric parameters of the manipulated object and the environment can be performed by analogy.

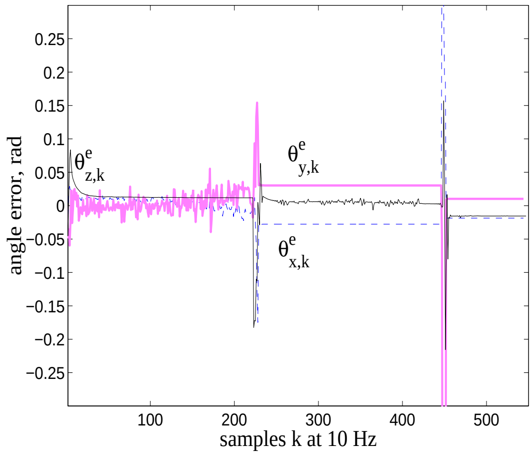

Figure 12: Error e_k in angles of the environment.

6. Conclusions

In this paper a general approach to contact transitions detection and estimation of uncertain geometric parameters (positions and orientation angles) is proposed for force-controlled robotic tasks in which a robotic manipulator moves an object in contact with the environment, both rigid and polyhedral.

The possible CFs are described by different measurement equations, whereas the system equation is known. An IMM estimator is implemented and its performance is evaluated by real sensor data (linear and angular velocities, forces and moments). The IMM probabilities and the normalized innovation squared test permit to monitor the occurring CFs. The experimental assembly of moving a cube into a corner demonstrates high estimation accuracy and quick detectability of the contact transitions.

Appendix. Derivatives Computation

The computation of the measurement and closure equations is based on the screw-transformation matrices,[3] that are functions of the rotational matrices between the different frames.[3,4] The partial derivatives are found from Eq. (7) and have the form

$$\frac{\partial h_{i,k}}{\partial x_{i,k}} = \begin{pmatrix} \partial(G^T_{i,k}(x_{i,k}))/\partial x^{mT}_{i,k} & \partial(G^T_{i,k}(x_{i,k}))/\partial x^{eT}_{i,k} \\ \partial(J^T_{i,k}(x_{i,k}))/\partial x^{mT}_{i,k} & \partial(J^T_{i,k}(x_{i,k}))/\partial x^{eT}_{i,k} \end{pmatrix},$$

$$D_{i,k} = \partial h_{i,k} / \partial z_k = \begin{pmatrix} G_{i,k}^T & 0 \\ 0 & J_{i,k}^T \end{pmatrix}.$$

The matrix derivatives are computed according to the rules for matrix calculus operations proposed by Vetter.[11]

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Contact Transitions Tracking During Force-Controlled Compliant Motion Using an Interacting Multiple Model Estimator

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Keywords: compliant motion, on-line estimation, IMM estimator, transition detection, tracking..

Abstract: This article addresses both monitoring of contact transitions and estimation of unknown first-order geometric parameters during force-controlled motion. A robotic system is tasked to move an object among a sequence of contact configurations with the environment, under partial knowledge of geometric parameters (positions and orientations) of the manipulated objects and of the environment itself. The authors provide an example of a compliant motion task with multiple contacts, namely that of moving a cube into a corner. It is shown that by describing the contact configurations with different models and using the multiple model approach it is possible: (i) to detect effectively at each moment the current contact configuration and (ii) to estimate accurately the unknown parameters. The reciprocity constraints between ideal reaction forces and velocities are used as measurement equations. An Interacting Multiple Model (IMM) estimator is implemented and its performance is evaluated based on experimental data.

full text

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AN IMPROVED VERSION OF A MULTIPLE TARGET TRACKING ALGORITHM

Ljudmil BOJILOV

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1. Introduction

The construction of algorithms for finding the first K-best solutions to the assignment problem has attracted a great deal of interest in recent years. Starting with the pioneer work of MURTY, [5] the investigations continued with works of DANCHICK and NEWNAM [6] and with the more recent work of MILLER, STONE and COX. [7] In the last of these works, following the Murty's method the authors implement three optimizations, producing a speedup by factor of over 20. On the other hand, the measurements-to-target association as part of a frame of MHT approach can be successfully formulated as a classical assignment problem. So, using an algorithm capable to find exact first K-best solutions of the so formulated assignment problem gives us K hypotheses of highest probability without first generating all feasible hypotheses and then pruning them.[3]

In one of their works, NAGARAJAN, CHIDAMBARA and SHARMA, keeping essentially the REID's approach, proposed an algorithm for finding directly K hypotheses of highest probability. [1] In another of their works, NAGARAJAN and co-authors presented a new approach for calculating probability of each hypothesis.[2] They suggested to utilize information from the signal processor of the radar for improving the tracking process. As a result, in the algorithm of NAGARAJAN [1] the authors consider only two possibilities for any measurement, received at scan k: a) to be originated from one of the tracking targets; or b) to be from a new target. In our previous work [4] we proposed an extension of the algorithm of NAGARAJAN [1] achieving considerable speedup of finding the first K-best hypotheses. In the present work we further improved the extended algorithm and carried out more comprehensive experiments with more sophisticated scenarios and by using more powerful PC processor. As a result, some additional findings are presented, too.

This work is organized as follows. In the next section the main ideas from the work [2] of NAGARAJAN *et al* are outlined including the main expressions of hypotheses probabilities computation. In section 3 the NAGARAJAN's algorithm is discussed and its principle steps are described. Section 4 contains presentation of the improved version of the algorithm and discussion of additional rules in algorithm processing. In Section 5 Experimental results are included and analyzed in section 5. The results are summarized in Section 6.

2. Problem formulation

In their work [2] NAGARAJAN and co-authors present new approach for calculating probability of each hypothesis. They suggest utilizing information from the signal processor of the radar in order to improve the tracking process. As a result, in the algorithm presented in their companion paper, [1] the authors consider only two possibilities for any measurement, received at scan k: a) to be originated from one of the tracking targets; or b) to be from a new target.

Following the notation from the work of reid, [3] the authors assume at scan k N targets $T_1, T_2, \ldots, T_N$, their predicted track measurements $\hat{z}_1(k), \hat{z}_2(k), \ldots, \hat{z}_N(k)$ and associated covariance matrices $S_1(k), S_2(k), \ldots, S_N(k)$, respectively, according to hypothesis, say, Ω_g^{k-1}, retained after scan $k-1$. They assume also the class conditional density of measurement $z_i(k)$ $(i=1,2,\ldots,M)$ to be given by normal distribution

$$p(z_i(k)/T_j) = N(z_i(k);\hat{z}_j(k),S_j(k)),\ j=1,2,\ldots,N. \qquad (1)$$

Using the assumption, mentioned above, and following the Bayes theorem, they derive for probability of the event ψ_{ij} that the i-th measurement is from j-th target

$$P(T_j/z_i(k)) = \frac{p(z_i(k)/T_j)}{\sum_j p(z_i(k)/T_j)}. \qquad (2)$$

Considering all hypotheses retained at the end of scan $k-1$, the authors derive recursive formula for calculating probability of every new hypothesis at scan k according to every one hypothesis at scan $k-1$

$$P(\Omega_k^k) = \frac{1}{C}\left[P(\Omega_g^{k-1})\prod_{i=1}^{M_k}\beta(i,j_k,\Omega_g^{k-1})\right]. \qquad (3)$$

Here, C is normalization constant and $\beta(i,j_k,\Omega_g^{k-1})$ is probability calculated in uation (2).

3. Nagarajan's algorithm

The most important feature of this formula is that the probability of any new hypothesis is proportional to certain factors already evaluated. The advantage of this feature can be seen in the algorithm, presented below.[1]

Hereafter, we shall assume one hypothesis retained after the $k-1$-th scan, taking into account that presented part of the algorithm can be repeated for any additional hypothesis at scan $k-1$. For simplifying the notation, let’s represent factors β from equation (3) as $\beta(m,t)$, where $m=1,2,\ldots,M_k$ denotes measurements indices and $t=T_1,T_2,\ldots,T_N,T_{new}$ denotes targets’ indices. The values of β, as it has been mentioned above, can be previously evaluated. Table 1 contains such kind of values from the example cited in the paper of NAGARAJAN *et al.* [1]

The score of any feasible hypothesis will contain eight terms in the product as presented in Table 1. A hypothesis is said to be feasible if no more than one measurement is associated with any known target, but multiple measurements can be associated with new targets (the last row in Table 1). We can see, however, that if we convert Table 1 dividing every column’s element by the last element of the column (from T_{new}-row), the arrangement of the hypothesis according to their scores will not be changed (as it is seen from Table 2).

Table 1: The cost matrix of the example

	M_1	M_2	M_3	M_4		M_6	M_7	M_8
T_1	**0.37**	**-**	**0.35**	**0.61**	**0.72**	**0.43**	**-**	**0.15**
T_2	**0.23**	**0.45**	**0.33**	**0.15**	**0.2**	**0.37**	**0.72**	**0.6**
T_3	**-**	**0.35**	**0.25**	**0.21**	**-**	**0.16**	**0.27**	**0.15**
T_4	**0.35**	**0.17**	**0.05**	**-**	**0.07**	**-**	**-**	**0.08**
T_{new}	**0.05**	**0.03**	**0.02**	**0.03**	**0.01**	**0.04**	**0.01**	**0.02**

Table 2: The cost matrix with normalized elements

	M_1	M_2	M_3	M_4		M_6	M_7	M_8
T_1	**7.4**	**-**	**17.5**	**20.3**	**72**	**10.8**	**-**	**7.5**

T_2	**4.6**	**15**	**16.5**	**5**	**20**	**9.2**	**72**	**30**
T_3	**-**	**11.7**	**12.5**	**7**	**-**	**4**	**27**	**7.5**
T_4	**7.0**	**5.7**	**2.5**	**-**	**7**	**-**	**-**	**4**
T_{new}	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**

And the last step before algorithm representation is to construct the *preferred measurements matrix* (Table 3). In the row T_1 of this table the value 5 means that M_5 is the most preferable measurement for the first target, the next value of 4 means that measurement M_4 is the next preferable and so on. For example, one possible hypothesis is (5,7,3,1). Another way of expressing this hypothesis is by using *preference index* from the first row of Table 3 - (0,0,1,0). We can notice that the smaller the index is, the more preferable is the corresponding measurement.

Table 3: The preferred measurements matrix

	M_1	M_2	M_3	M_4		M_6	M_7	M_8
I	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
T_1	**5**	**4**	**3**	**6**	**8**	**1**	**-**	**-**
T_2	**7**	**8**	**5**	**3**	**2**	**6**	**4**	**1**
T_3	**7**	**3**	**2**	**8**	**4**	**6**	**-**	**-**
T_4	**1**	**5**	**2**	**8**	**3**	**-**	**-**	**-**

Before describing the steps of the algorithm it will be useful to discuss the next lemma. Let $P(\psi_i)$ represents the probability of the hypothesis ψ_i being true and let $Ind_i(n)$ represents the n-th element of preference-index presentation of ψ_i. The suggested lemma is:

$$P(\psi_i) > P(\psi_j) \text{ if } Ind_i(n) \le Ind_j(n)$$

for each value of n from 1 to the number N of known targets. Taking two hypotheses in preference-index presentation by means of this lemma we can conclude, in some cases, which is more likely without actually evaluating the products of their scores. This may be considered as one of the main achievements presented in the first quoted work by NAGARAJAN *et. al.* [1]

For clearness of notation we shall say that a hypothesis, presented in way of preference-indexes, is of level l if the sum of its preference indices is equal to l. Thus the hypothesis (0,0,0,0) is of level *0*, hypothesis (0,1,0,0) is of level *1* and hypothesis (1,0,2,1) is of level *4*, and so on. Likewise, if two hypotheses are subject to the lemma's rule - $Ind_i(n) \le Ind_j(n)$, we shall say that hypothesis ψ_j is consequence from hypothesis ψ_i, i.e., it can be constructed by only adding some values to the preference-index presentation of ψ_i. The hypotheses generation can be represented like constructing a tree. From every hypothesis (nod) at a given level *l*, *N* hypotheses (branches) of level *l+1* can be generated by simply incrementing preference indices, one at a time. Every generated hypothesis has to be checked: a) for feasibility, and b) if it is a consequence of some of the previously found feasible hypotheses. For the normal algorithm processing three lists have to be maintained: a) list of found feasible hypotheses sorted by their scores (candidate hypotheses); b) list of unfeasible hypotheses for the subsequent processing without calculating the scores; and c) ranked list of the best hypotheses.

The algorithm starts with checking the 0-level hypothesis. If it proves to be feasible, this is the first-best hypothesis (according the lemma presented above) and we put it in the ranked list of best hypotheses. We say that consecutive best hypothesis is found if no meaningful hypothesis of a higher level can be generated. This will be the first hypothesis out of sorted feasible hypotheses list. Every time we find consecutive best hypothesis, we use it for constructing the next hypothesis' tree.

The particular steps of the algorithm stated in the cited work [1] are as follows:

> **Step 0.** We take the just found consecutive-best hypothesis from the top of feasible (or candidate) hypotheses list and begin constructing a tree.

Step 1. Hypotheses generation of level $l+1$ from a given hypothesis at level *l*.

Step 2. Checking feasibility of the created hypothesis.

Step 3. If the hypothesis is feasible, we check whether it is consequence of any hypothesis out of the candidate hypotheses list:

a) If it is not - we include it in the candidate hypotheses list;

b) If it turns out that the checked hypothesis is consequence of some of the candidate hypotheses, we discard it.

Step 4. If the hypothesis is not feasible and it is not consequence of any of the hypotheses in the candidate hypotheses list, we implement another checking – whether it is consequence from any of the hypotheses in the list of non-feasible hypotheses. If it is not, we include the hypothesis in this list for subsequent processing. Otherwise, we discard it and continue with the next step.

Step 5. We take the subsequent hypothesis from the unfeasible hypotheses list and return to Step 1. If all hypotheses from the unfeasible hypotheses list are already used and the list is empty, we say that we found the next best hypotheses and return to Step 0.

The algorithm terminates when the list of the first K-best hypotheses fills up.

The simulation realizing this algorithm shows significant reduction of the number of hypotheses to be processed, as well as the running time for the task. However, if we take an example with $N=10$ targets and include in the scenario $M=15$ measurements, combinatorial problems arise in two directions: a) time of processing and b) memory storage limitation (especially for the list of non-feasible hypotheses for subsequent processing).

4. Extended algorithm

Before describing our extension we shall further elaborate on the NAGARAJAN's algorithm processing. Let us take the four hypotheses at level *1* from the example of their paper [1] - (1,0,0,0), (0,1,0,0,), (0,0,1,0) and (0,0,0,1). We can generate now four new hypotheses at level *2* from every hypothesis of level *1* (Table 4):

Table 4: Hypotheses generation process

1,0,0,0	**0,1,0,0**	**0,0,1,0**	**0,0,0,1**
2,0,0,0	1,1,0,0	1,0,1,0	1,0,0,1
1,1,0,0	0,2,0,0	0,1,1,0	0,1,0,1
1,0,1,0	0,1,1,0	0,0,2,0	0,0,1,1
1,0,0,1	0,1,0,1	0,0,1,1	0,0,0,2

In table 4, the elements above the main diagonal are the same as those under the main diagonal. In our extension we shall avoid hypotheses duplication for saving processor time. There is another part of the algorithm, where needless hypotheses generation is carried out. Let us take the non-feasible hypothesis (0,1,0,0,0,0) assuming repeated measurements at 2-th and 3-rd positions. According to the **step 1**, we can create six new level *2* hypotheses: (1,1,0,0,0,0), (0,2,0,0,0,0), (0,1,1,0,0,0), (0,1,0,1,0,0), (0,1,0,0,1,0), and (0,1,0,0,0,1). It is easy to conclude that every hypothesis after the third, as well as their 'successors' up to the bottom of the Table 3 will be non-feasible. The reason is that the unit in the 2-nd place and the zero in the 3-rd place of the origin correspond to the repeated measurements according to the measurement-oriented presentation. So, we can stop hypotheses generation after the second hypothesis saving both time of the processor and memory storage.

Two additional terms are introduced for convenience. Every new hypothesis at a given level is created from some hypothesis of the upper level by incrementing its preference-index presentation at some point. We call this point '*creation point*,' or *CP*. Secondly, in regard to the conclusion that for some unfeasible hypothesis there is no use to continue hypotheses' creation after the point where repeated measurement occurs, we call that point '*breaking point,*' or *BP*. It is obvious that the cycle of hypotheses' generation has to be run from CP to BP only. Moreover, when for some hypothesis out of unfeasible hypotheses list *CP* > *BP*, we discard this hypothesis, cutting off the corresponding part of the hypotheses' tree.

Step 0. The last just found consecutive-best hypothesis from the top of feasible (or candidate) hypotheses list serves to begin the construction of a tree.

Step I. A new hypothesis of level $l + 1$ is created from a given hypothesis at level l by incrementing preference indices one at a time; the cycle is run only from CP to BP. When a new hypothesis of level $l + 1$ is created, we remember its ‘creation point’.

Step II. On this step we check whether the new hypothesis is consequence of any of the hypothesis out of the candidate hypotheses list. If it is consequence of some of the candidate hypothesis, we discard it and go back to Step I. Otherwise continue with the next step.

Step III. If the checked hypothesis is not consequence of any of the candidate hypotheses, we continue with the feasibility check. If the hypothesis is feasible, it is included in the candidate hypotheses list and then we return to step I.

Step IV. If the checked hypothesis is not feasible, we examine the place, where the repeated measurement occurs and remember it as a ‘breaking point.’ After that, we include it in the list of non-feasible hypotheses for subsequent processing.

Step V. We take the subsequent hypothesis from the unfeasible hypotheses list and return to Step 1. If all hypotheses from the unfeasible hypotheses list are already used and the list is empty, we say that we have found the next-best hypotheses and the return to Step 0.

As in the previous case, the algorithm terminates when the list of the first K-best hypotheses fills up.

There is a sound rationale for some extensions in the proposed algorithm. Starting hypotheses generation from index CP, we avoid redundant steps of the algorithm in two directions: a) preventing hypotheses duplication, and so, saving processor’s time and memory storage, especially for the unfeasible hypotheses list, and b) obviating the checking whether the hypothesis is consequence of any hypothesis out of the unfeasible hypotheses list. This list is much longer than the feasible hypotheses list and, thus, its checking is one of the most time-consuming parts of the algorithm.

The second issue is the ‘breaking point.’ When the cycle of hypotheses generation is stopped at BP, we truncate significant parts of the hypotheses’ tree and so achieve savings of processor time and memory storage. It is important to notice that this second extension is effective only in combination with the first one. Finally, in our extensions feasibility checking and consequence checking are rearranged. The merits are that non-feasibility is not yet a reason to discard a hypothesis, whereas, if it is consequence of any of the feasible hypotheses, we can readily discard it.

5. Simulation results

The program realization of NAGARAJAN algorithm, as well as the realization of its extension has been used for numerical experiments. The first experiment includes the example from the work of NAGARAJAN. [1] We run this example with NAGARAJAN's algorithm for proving the correct program realization. The results from the experiment fully coincide with the results in the original paper. Then, we run the same example with the extended algorithm. If we accept the following abbreviations: **GH -** number of **G**enerated **H**ypotheses, **HCF -** number of **H**ypotheses **C**hecked for **F**easibility, **NAG - NAG**arajn's algorithm, **EXT - EXT**ended algorithm, **T/M -** number of **T**argets/**M**easurements, the experimental results may be presented in Table 5.

Table 5: Comparison of the two algorithms on the cited example [1]

	NAG	**EXT**
GH	**90**	**18**
HCF	**32**	**8**

Even in such a simple case with 4 targets and 8 measurements in the cluster the advantage of the extended algorithm is obvious.

Another series of experiments have been run with 13 different scenarios with increasing complexity. Table 6 compares created hypotheses and those hypotheses for which feasibility checks had to be made. The 6-th and 7-th columns contain running time in seconds for finding the first 100-best hypotheses on a 1.4GHz AMD/XP processor. The last column contains speed advantage ratio. For the simplest cases the running time of the extended algorithm proved to be less than the time resolution of the computer. Additionally, for the most complicated cases the running time of the original algorithm is out of any reasonable limits; the simulations have not been carried out in such cases. Each value in the table was obtained by averaging over 50 independent program runs with 50 different values of random generator seed. Of course, one and the same random number stream has been used for any one scenario. As it can be seen from Table 6, the scenario with 8 targets and 13 or 14 measurements prove to be the practical implementation limit of Nagarajan's algorithm (assuming radar with 10 sec.

scan). In the last and most complicated scenarios (with 15 targets and more than 20 measurements) the extended algorithm reaches its limit for practical implementation, even though the average running time for these scenarios is less than the assumed scan duration of 10 seconds. The problem is that for the some of experiments, i.e. the heaviest scenario, the processing time of the algorithm exceeds 10 seconds.

Table 6: Comparison of the two algorithms performance in terms of generated and checked hypotheses and processing time

Targets/ Measure-ments (**T/M**)	**GH**		**HCF**		**Time (*in seconds*)**		Speed advantage ratio
	NAG	**EXT**	**NAG**	**EXT**	**NAG**	**EXT**	
6/11	1476	-	310	-	0.03	-	-
7/12	5547	-	1117	-	0.355	-	-
8/13	27211	-	6291	-	4.81	-	-
8/14	45550	2913	12650	1760	9.42	0.03	314
9/15	104168	6586	31788	3955	30.83	0.112	275
10/16	190536	15320	72226	10014	67.43	0.327	206
11/17	306024	22724	126458	14688	117.14	0.562	208
12/18	576424	39466	236837	28406	211.75	1.082	196
13/19	-	48627	-	35071	-	1.833	-
13/20	-	65536	-	48103	-	2.296	-
14/21	-	76560	-	57167	-	3.425	-
15/22	-	103787	-	72405	-	5.278	-
15/25	-	133526	-	103478	-	6.843	-

Through an additional experiment we reveal an interesting and very useful feature of the presented algorithm. We have tested the dependence of the processing time on number of first K-best hypotheses with 14T/21M scenario (Table 6). Surprisingly, the experiment exhibits very week dependence of the running time on the number of first best hypotheses found (Table 7).

The explanation of this result is straightforward: for the main part of its work the algorithm determines the first best hypothesis. At that time, the list of candidate hypotheses is full and for finding any subsequent hypothesis only a few additional operations have to be performed. This feature makes the presented algorithm a good alternative to the well-known algorithms, proposed recently, for finding the first K-best hypotheses, directed for use in the framework of the MHT approach.

Table 7: Processing time (*in seconds*) for finding different number of first K-best hypotheses

Rand-Seed values	***Number of the first K-best hypotheses found***					T_{100}/T_1
	1	10	20	50	100	
13	2.03	2.09	2.09	2.14	2.42	1.19
15	5.5	5.5	5.55	5.76	6.37	1.16
17	1.48	1.53	1.59	1.92	2.69	1.82
25	3.13	3.13	3.18	3.24	3.57	1.14

27	2.03	2.09	2.14	2.26	2.58	1.27
33	1.82	1.82	1.87	2.19	2.86	1.57
35	2.2	2.2	2.25	2.8	2.91	1.32
53	2.74	2.75	2.76	2.81	2.91	1.06
55	3.42	3.51	3.52	3.68	4.12	1.20
65	5.06	5.1	5.11	5.28	5.54	1.09

6. Conclusion

This paper presented an improved version of our extension of NAGARAJAN's algorithm.[1] Defining two points in the hypotheses generation cycle—'*creation point*' and '*breaking point*'—a considerable reduction of hypotheses' tree has been achieved. By rearranging feasibility checking and consequence checking we attain additional pruning of this tree. As combined result of the improvements, the time necessary to implement the presented algorithm has been reduced by more than two orders of magnitude compared to NAGARAJAN's algorithm. In addition, taking into account the week dependence of the processing time on the number of the best hypotheses found, it can be inferred that the presented algorithm can be successfully implemented besides other algorithms finding the first K-best hypotheses in implementing multiple targets tracking.

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An Improved Version of a Multiple Target Tracking Algorithm

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Keywords: assignments, multiple hypotheses tracking

Abstract: The author presents an improved version of an algorithm for multitarget tracking. The algorithm extends the seminal algorithm of Nagarajan et. al., overcoming to some extent combinatorial problems, arising while simultaneously tracking multiple targets using track-while-scan radars. In a previous work, the algorithm was tested in comparatively simple to moderate scenarios. In this article, the author changes some of the previously suggested additional rules of the Nagarajan's algorithm and presents more comprehensive experimental results with more sophisticated scenarios, using at the same time more processing power. By an additional experiment, the author reveals a useful feature of the presented algorithm that makes it a viable alternative to the well known algorithms for finding the first K-best hypotheses in the framework of the MHT approach.

full text

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AN ACCELERATED IMM-JPDA ALGORITHM FOR TRACKING MULTIPLE MANEUVERING TARGETS IN CLUTTER

Ljudmil BOJILOV, Kiril ALEXIEV and Pavlina KONSTANTINOVA

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1. Introduction

By far, the most complicated case in target tracking is to track multiple maneuvering targets in heavy clutter. Numerous methods and algorithms have been devoted to this problem and for any one of them *pros* and *cons* can be pointed out. Theoretically, for example, the MHT method is known to be the most powerful approach to tracking multiple maneuvering targets in clutter. This method, however, very often leads to combinatorial explosion and computational overload that restricts its implementation. Recently, numerous papers have been devoted to algorithms capable to compute a ranked set of assignments of measurements to targets. Such algorithms allowed for the first practical implementations of MHT the approach.

Another and much less complicated approach, especially for tracking maneuvering targets, is the Multiple Models (MM) approach. The most promising algorithm based on this approach is the Interacting Multiple Models (IMM) algorithm. At the price of some sub-optimality of its framework, this algorithm reaches best implementation in terms of speed and stability. However, in the presence of clutter the IMM algorithm most often fails. In the case of cluttered environment, the PDA (and

JPDA) approaches can be implemented. When tracking multiple closely spaced targets, the JPDA algorithm can be implemented successfully even in the presence of heavy clutter. In a recent paper [1] we proposed an algorithm unifying features of the IMM and the JPDA algorithms. That algorithm proved to be good alternative to the MHT approach for clusters containing up to 4 targets and moderate level of clutter. However, when the number of targets in the cluster exceeds this limit the total number of all feasible hypotheses increases exponentially. In this paper we propose an extension of the algorithm in our previous work. [1] Instead of enumeration of all feasible hypotheses we propose to use ranked assignment approach to find the first K-best hypotheses only. The value of K has to ensure that the weight of scores-sum of these K-best hypotheses prevails over the total sum.

This paper is organized as follows. In the next section we elaborate on our motivation and formulate the problem. The IMM_JPDA algorithm is briefly described and the need of its extension is discussed. In the 3rd section the extended algorithm is described. The emphasis is on the extension of the algorithm. The 4th section presents simulation results. These results show that the extended algorithm performs better than the IMM_JPDA algorithm in terms of speed, while at the same time preserves stability of tracking.

2. Motivation and Problem Formulation

When several closely spaced targets form a cluster, the JPDA algorithm starts to generate all feasible hypotheses and to compute their scores. The set of all feasible hypotheses includes such hypotheses as 'null' hypothesis and all its derivatives. The consideration of all possible assignments including the 'null' assignments is important for optimal calculation of assignment probabilities.[6] If, for example, the score of every one of these hypotheses differs from any of the others by no more than one order of magnitude, it should not be possible to truncate some significant parts of all hypotheses. If, however, the prevailing share of the total score is concentrated in a small percent of the total number of all hypotheses, then the interest in considering only this small percent of all hypotheses becomes very high.

In order to investigate this idea, a typical example with five closely spaced targets with overlapping validation regions and shared measurements is used. In the first run (or first scenario) 17 measurements are disposed in the target gates, and in the second run (second scenario) 9 measurements are disposed. At every run all feasible hypotheses are generated and their scores are computed and summarized. The results are presented on figures 1 and 2. These two examples were chosen out of numerous experiments as typical for the algorithm performance.

The two plots of Figure 1 show how the individual scores of the sorted feasible hypotheses are distributed. Only the top six percents of all hypotheses for the first end second scenario are depicted on the figure. It can be seen that the scores of the hypotheses dramatically reduce their values. Even more informative is Figure 2, where the cumulative score's distributions of the two scenarios are given. This figure confirms our expectations that only a small number of hypotheses concentrate the prevailing part of their total sum. One additional conclusion can be derived. The first scenario is much more complicated with more than 4930 hypotheses generated. In the second scenario, the generated hypotheses are approximately 550. It can be seen from the figures that for the more complicated cases the expected effect stands out even more definitely.

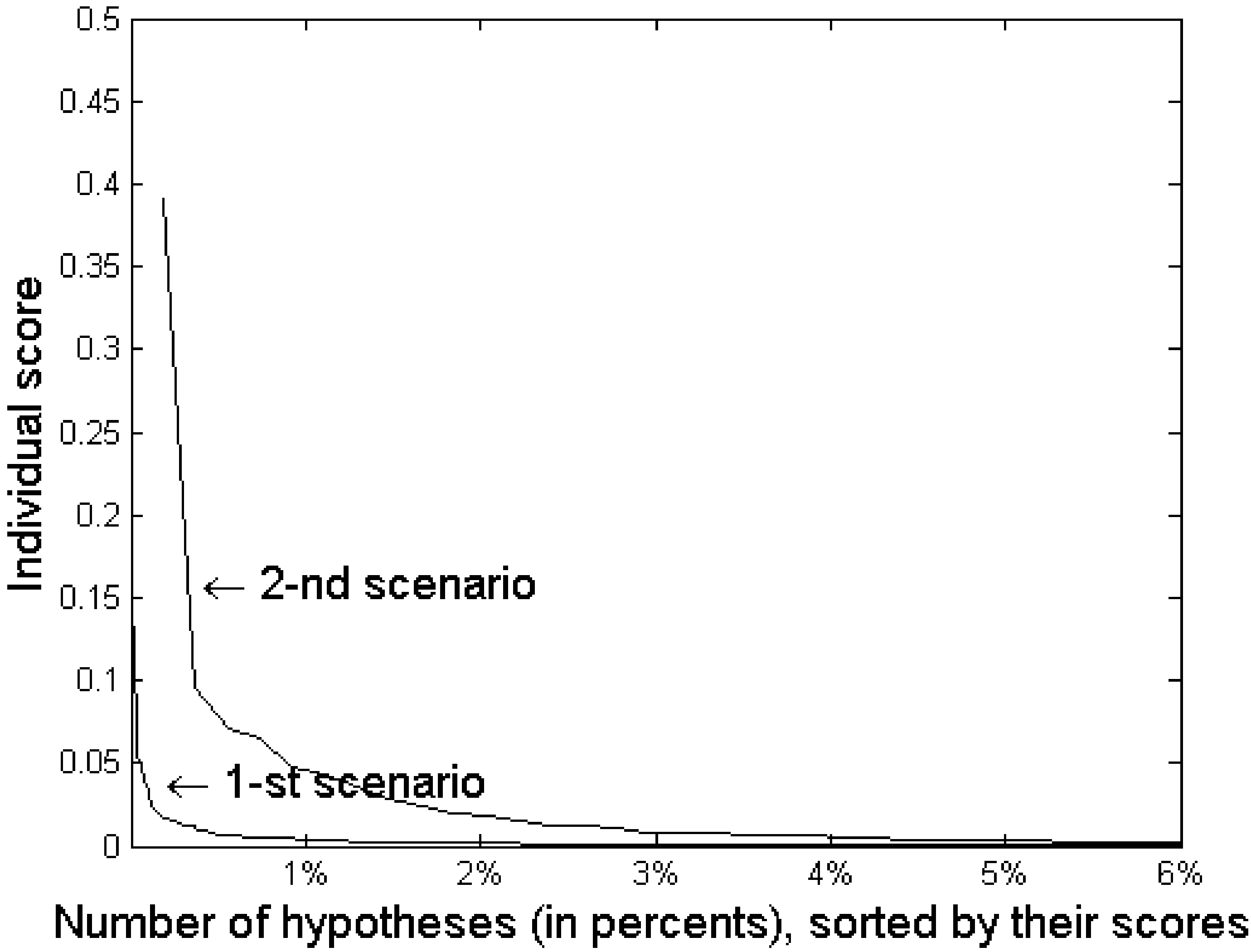


Figure 1: Hypotheses' score distribution.

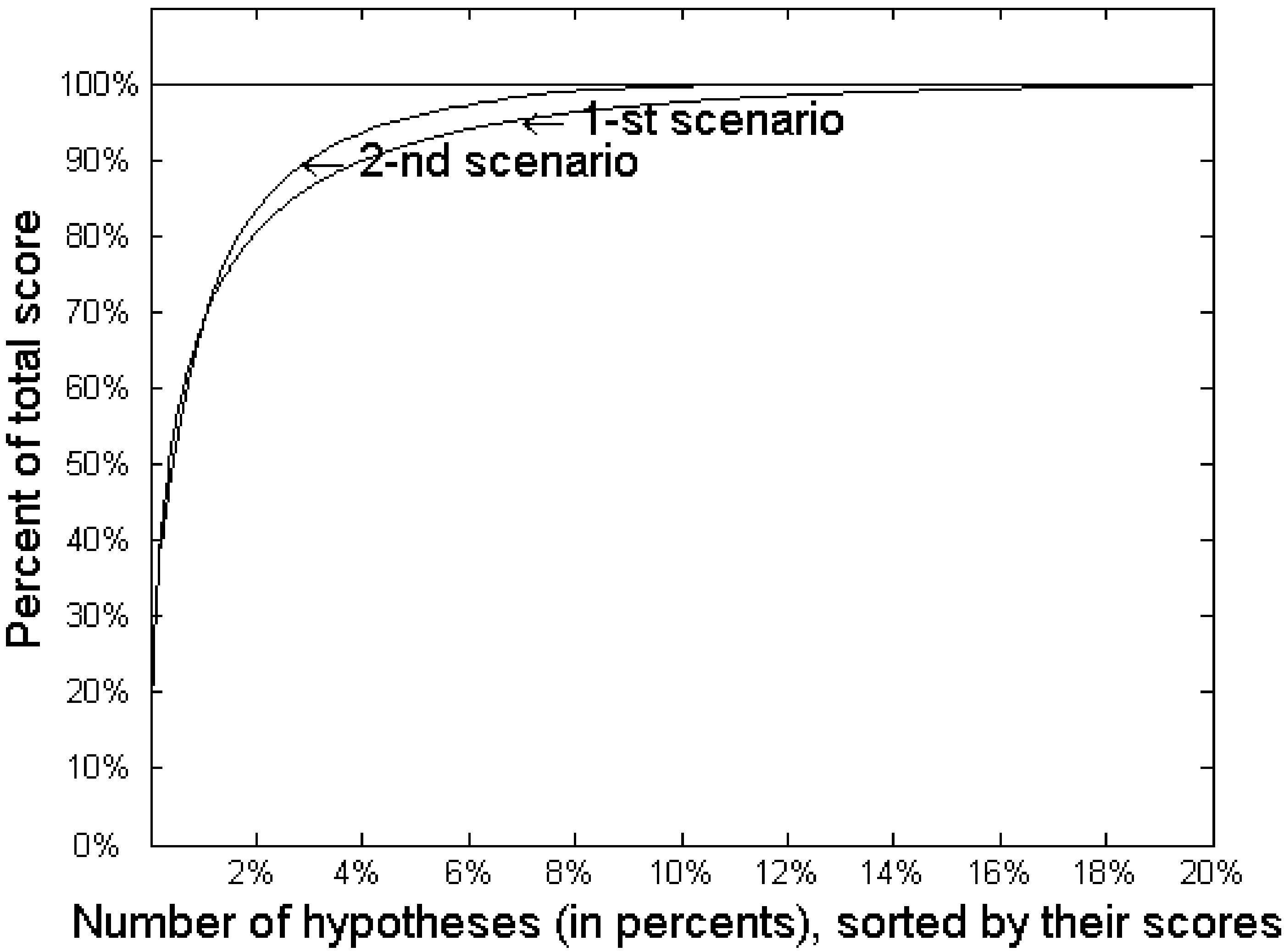


Figure 2: Cumulative score distribution.

The description of the algorithm proposed in our previous work [1] follows. For simplicity and without losing generality two models are assumed.

2.1. IMM-JPDA Algorithm Description

The IMM JPDA algorithm starts with the same step as IMM PDA algorithm,[5] but in cycle for every particular target in the cluster.

Step 1. Computation of the mixed initial conditions $\hat{x}_i^{0t}$ for every target i and for the filter, matched to model *t*:

a) mixed state estimate

$$\hat{x}_i^{0t}(k-1|k-1) = \sum_{s=1}^{2} \hat{x}_i^{s}(k-1|k-1)\mu_{s|t}^{i}(k-1|k-1)\ , t=1,2 \qquad (1)$$

Here, it is supposed that mixing probabilities $\mu_{s|t}^{i}$ are already computed.

b) mixed covariance estimate

$$P_i^{0t}(k-1|k-1) = \sum_{s=1}^{2} \mu_{s|t}^{i}(k-1|k-1)\left\{P_i^s(k-1|k-1) + \left[\hat{x}_i^s - \hat{x}_i^{0t}\right]\left[\hat{x}_i^s - \hat{x}_i^{0t}\right]'\right\} \quad (2)$$

Here P_i^s is covariance update of model *s* for target *i*.

Next, some JPDA steps follows.

<u>Step 2</u>. State predictions $\hat{x}_i^{0t}(k|k-1)$ and covariance predictions $P_i^{0t}(k|k-1)$ for the next scan *k* for every target and for every model are calculated.

<u>Step 3</u>. In this step, after receiving the set of measurements at scan *k*, a clustering is performed. Further on, it is assumed that the algorithm will proceed with every particular cluster.

At this point, in the traditional JPDA algorithm, hypotheses generation has to be performed. However, to avoid combinatorial explosion we include here our innovation.

<u>Step 4.</u> Calculating 'predicted model probabilities':

$$\mu_i^t(k|k-1) = \sum_{s=1}^{2} p_{st}\mu_i^s(k-1), \quad (3)$$

where $\mu_i^t(k-1)$ is the probability that the model *t* is correct at scan *(k-1)* and p_{st} are Marcovian switching probabilities.

Now, the individual model state predictions are merged for every particular target:

$$\hat{x}_i^0(k|k-1) = \sum_{t=1}^{2} \hat{x}_i^{0t}(k|k-1)\mu_i^t(k|k-1). \quad (4)$$

<u>Step 5</u>. We are now ready to continue with the hypotheses generation and hypotheses score computation. Hypotheses generation is another combinatorial problem that will be discussed in the next section.

After generating all feasible hypotheses, hypothesis probability is computed by the expression

$$P'(H_l) = \beta^{[N_M-(N_T-N_{nD})]}(1-P_D)^{N_{nD}}\, P_D^{(N_T-N_{nD})} g_{ij}\cdots g_{mn}, \quad (5)$$

where

β - is probability density for false returns,

$g_{ij} = \frac{e^{-\frac{d_{ij}}{2}}}{(2\pi)^{M/2}\sqrt{|S|}}$ - is probability density that measurement *j* originates from target i, and the following additional notations are used: N_M - total number of measurements in the cluster, N_τ - total number of targets, d_{ij} – statistical distance, N_{nD} - number of not detected targets. The step ends with the standard normalization

$$P(H_l) = \frac{P'(H_l)}{\sum_{l=1}^{N_H} P'(H_l)}, \qquad (6)$$

where N_H is the total number of hypotheses.

Step 6. In this step, association probabilities are calculated. To compute for a fixed i the probability p_{ij} that observation *j* originates from track i, we have to take a sum over the probabilities of those hypotheses in which this event occurs:

$$p_{ij} = \sum_{l \in L_j} P(H_l), \text{ for } j = 1, \ldots, m_i(k) \text{ and } i = 1, \ldots, N_T, \qquad (7)$$

where L_j is a set of indices of all hypotheses, which include the event mentioned above, $m_i(k)$ is the number of measurements falling in the gate of target *i*, and N_T is the total number of targets in the cluster.

Step 7. After association probabilities computation, the JPDA algorithm continues as a PDA algorithm for every individual target. For every target the 'merged' combined innovation is computed

$$v_i(k) = \sum_{j=1}^{m_i(k)} p_{ij} v_{ij}(k). \qquad (8)$$

Step 8. This is the last step of our description. At this step, our algorithm returns to the multiple model case by splitting 'merged' combined innovation from the previous equation. For every individual target and for every particular model the combined innovations are computed:

$$v_i^t(k) = v_i(k) + H_i(k)\hat{x}_i^0(k|k-1) - H_i^t(k)\hat{x}_i^{0t}(k|k-1) \cdot . \qquad (9)$$

The last few steps of this algorithm fully coincide with the well-known IMM PDA algorithm [5] and will be omitted from the current description.

3. Accelerating Extension to the IMM JPDA Algorithm

Our extension to IMM JPDA algorithm is directed to the most time consuming part of the algorithm, which concerns hypotheses generation and their scores computation. If we take as a simple example a cluster with 4 targets and 10 measurements distributed in their validation regions (Table 1), the total number of all feasible hypotheses for this example approaches 400. When, however, the number of targets in the cluster exceeds 5 or 6 and there are more than 15 measurements in their gates, the number of all hypotheses to be generated reaches thousands. To avoid these overwhelming computations we propose the next trade-off: to take into consideration only small part of all feasible hypotheses with highest scores and to concentrate on the prevailing share of the total score sum.

Table 1: Indices of the measurements falling in the gates of corresponding targets

T1	T2	T3	T4
0	0	0	0
4	6	3	1
8	7	4	2
9	8	5	3
		6	4
		9	

In order to find out the first K-best hypotheses we use an algorithm due to Murty [2] and optimized by Miller *et al.* [3] This algorithm gives a set of assignments to the assignment problem,[4] ranked in increasing order of cost. As a first step in solving this problem we have to define the cost matrix of the assignment problem. It can be seen that the score of any particular hypothesis (equation 5) is an expression of multipliers. The score of every feasible hypothesis, i.e., the probability of being true, can be calculated using a table similar to Table 1, but instead indices in the boxes of the Table 1 we need to put multipliers equal to probability of assigning the given measurement to the corresponding target (Table 2).

Table 2: Multipliers of the corresponding measurements

T1	T2	T3	T4
$\beta(1-P_D)$ 0	$\beta(1-P_D)$	$\beta(1-P_D)$	$\beta(1-P_D)$
$g_{14}P_D$	$g_{26}P_D$	$g_{33}P_D$	$g_{41}P_D$
$g_{18}P_D$	$g_{27}P_D$	$g_{34}P_D$	$g_{42}P_D$
$g_{19}P_D$	$g_{28}P_D$	$g_{35}P_D$	$g_{43}P_D$
		$g_{36}P_D$	$g_{44}P_D$
		$g_{37}P_D$	

Now, properly combining indices from Table 1, thus generating every one of the feasible hypotheses we can at the same time multiply corresponding elements from Table 2, obtaining the score of the so generated hypothesis (equation 5). As it is well known, feasibility of hypothesis meets two important constraints: a) no target can create more than one measurement, and b) no measurement can be assigned to more than one target.

On the other side, every solution of the assignment problem represents a sum of elements of the cost matrix. We have to define this cost matrix in such way, that the value of every possible solution of the assignment can be potentially a score of some feasible hypothesis. Let us take logarithm from both sides of (5). From the left-hand side we obtain logarithm of hypothesis probability and, from the right-hand side, a sum of logarithms of elements from Table 2. This correspondence between multipliers in equation (5) and the sum of their logarithms gives a hint of how to construct the cost matrix and to solve the problem mentioned above.

We construct a cost matrix containing instead the elements of Table 2, their negative logarithms. If we find the optimal solution (in this particular case – the minimum) of the assignment problem with this cost matrix it will coincide with the hypothesis with highest probability, i.e., both the optimal solution and the highest probability hypothesis will associate the targets with the same measurements. The cost matrix of a cluster from Table 1 appears in Table 3.

Table 3: The cost matrix of the example

f1 F2 f3 f4 z1 z2 z3 z4 z5 z6 z7 z8 z9

T1	ln^0	×	×	×	×	×	×	ln_{14}	×	×	×	ln_{18}	ln_{19}
T2	×	Ln^0	×	×	×	×	×	×	×	ln_{26}	ln_{27}	ln_{28}	×
T3	×	×	ln^0	×	×	×	ln_{33}	ln_{34}	ln_{35}	ln_{36}	×	×	ln_{39}
T4	×	×	×	ln^0	ln_{41}	ln_{42}	ln_{43}	ln_{44}	×	×	×	×	×

where

$$\ln^0 = -\ln\left[(1-P_D)\beta\right],\ \ln_{ij} = -\ln\left[g_{ij}P_D\right].$$

The symbol × in the matrix represents one and the same value with only requirement to be greater than the greatest element out of the set of elements denoted with $\ln$. In order to use any of the widespread assignment algorithms, as well as the algorithm [1] for finding the K-best hypotheses, the matrix from Table 3 has to be added up to square matrix filling in the remaining rows with the same value ×. The first four columns of the matrix in Table 3 correspond to false measurements, i.e., assigning first row to first column, the second row to the second column, etc., means that no measurement originated from this target. Columns from five to thirteen represent the corresponding measurements falling in the validation regions of the targets.

When the algorithm for finding K-best assignments is initiated it will find K solutions of the problem with lowest sums of negative log-likelihood (or with highest probabilities). After receiving these K values their anti-logarithms have to be computed in order to obtain the K-best hypotheses probabilities. Next, these probabilities have to be normalized by equation (6), but now the sum is up to K:

$$P(H_l) = \frac{P'(H_l)}{\sum_{l=1}^{K} P'(H_l)}.$$

Henceforth, this algorithm fully coincides with the algorithm described in the previous section, continuing with the step 6.

One important practical question, closely related to the proposed approach, arises in this regard: how many hypotheses K to be found out. When deciding on the value of K we have to realize that this value has to be optimal in some sense. On one hand, the smaller the value of K, the proposed algorithm performs faster. On the other hand, however, too small values of K can lead to distortion in assignment probabilities computation (equation 7). This question will be discussed in the next section.

4. Simulation Results

We compare the algorithm presented in this paper with the same algorithm without acceleration discussed in previous section (our previous algorithm [1]). These two algorithms were tested extensively on a variety of scenarios involving different numbers of maneuvering and closely spaced targets and in presence of heavy clutter. We construct a set of scenarios with 3, 4 and 5 targets in a cluster and in the presence of moderate and heavy clutter. The scenarios are similar to those from our previous paper [1] where we searched for the limits of the IMM JPDA algorithm in terms of the number of targets in a cluster.

The first step in preparing the common frame for testing is to decide how many K-best hypotheses need to be generated. We mentioned in the end of the previous section that the value of K has to be, in some sense, optimal so that: a) it is sufficiently small to ensure acceleration of the algorithm, and, in the same time, b) it is not too small to lead to distortion in computing assignment probabilities.

As it can be seen from Figure 1, the scores of feasible hypotheses decrease very rapidly and some 5-10 percents of them (Figure 2) cover more then 95 percents of the total score sum. However, as we know neither the total number, nor the total sum, we try to derive indirect criterion for determining the value of K. One possible expression can be

$$H(n) - H(n+1) < \alpha \cdot H(n),$$

where $\alpha << 1$. Here with $H(n)$ the probability density of n^{th} hypothesis to be true is denoted. The implementation of this criterion, however, did not give stable results. The reason is that very often there are subsets of hypotheses with very close values of their scores, even in the beginning of the sorted hypotheses array. Another expression, providing for higher stability, is

$$H(n) < \alpha \cdot H(1). \qquad (10)$$

In order to tune experimentally the value of α, a range of experimental runs have been carried out. Every one run is performed with scenario with the same number of 6 targets and 12 measurements but with different reciprocal (relative) location. Averaging over 1000 runs, the following simulation results have been received (Table 4).

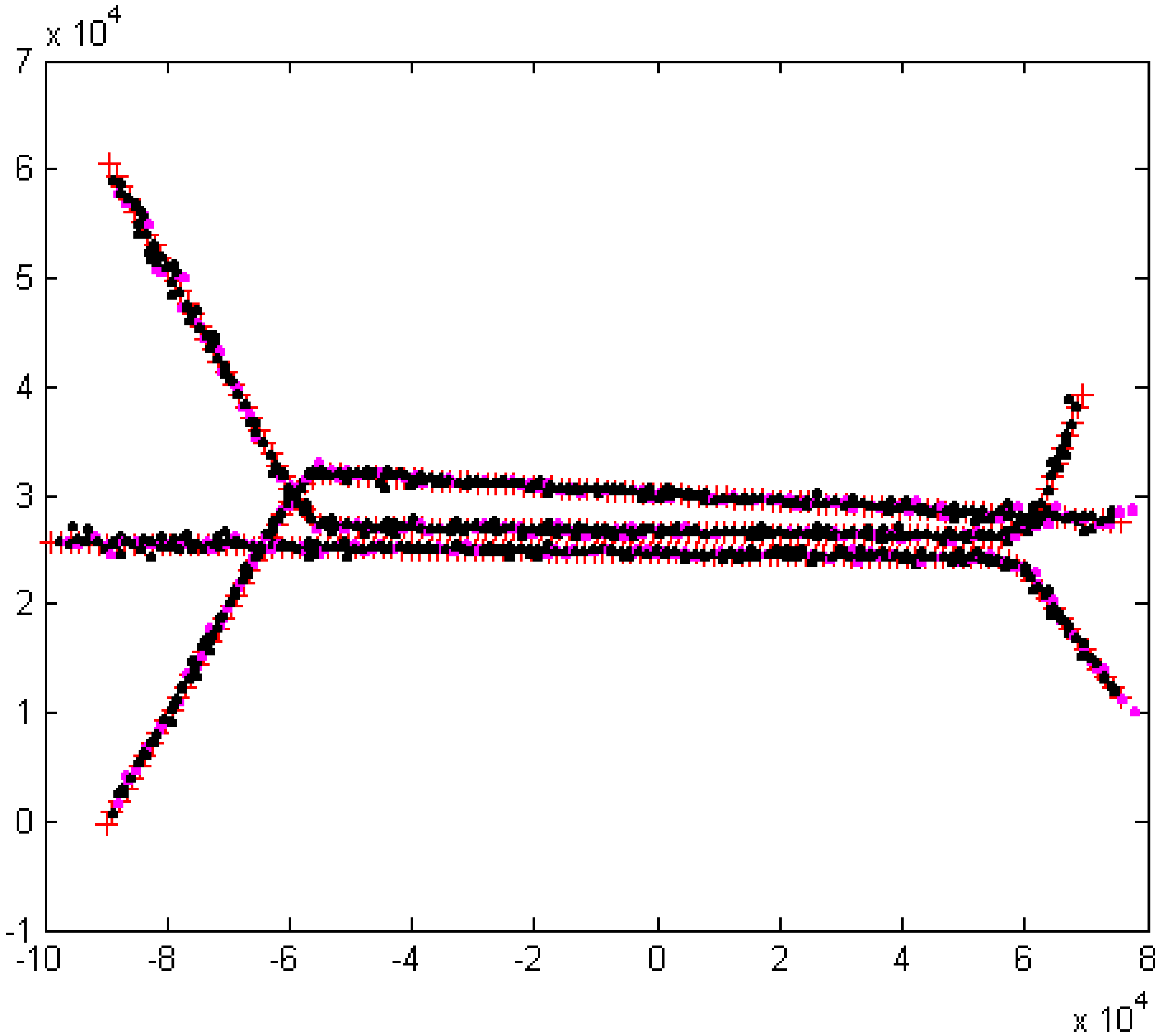
The first column of the Table 4 contains the different values of α, the second and third columns contain respectively the mean and the largest number (worst case) of the first N-best hypotheses in accordance with (10). The fourth and fifth columns contain mean and lowest values of the ratio of the total score sum of these N-best hypotheses. In opposite to the hypotheses' number, the worst case for this ratio is its lowest value.

Table 4: Number of hypotheses and ratio of the total score sum for different values of α as per equation (10)

α	N_{mean}	N_{wst}	R_{mean}	R_{wst}
0.05	32	506	0.7842	0.6185

0.01	179	1510	0.9414	0.8587
0.005	286	2074	0.9690	0.9184
0.001	632	3350	0.9942	0.9784
0.0005	779	3797	0.9973	0.9899
0.0001	1082	4459	0.9995	0.9982

Now, we can choose the most suitable value for α. For example, if we choose the value of $\alpha = 0.005$, after summation of the first 286 hypotheses we ensure, in average, the attainment of nearly 97 percent of the total score sum. If we take into account that the mean of the total hypotheses number for this experiment is 9780 we can conclude, that choosing the value of $\alpha = 0.005$ we can generate and process the first 3 percent of all feasible hypotheses ensuring 97 percent of the total score sum. Similar conclusions can be drawn for $\alpha = 0.01$. Consequent experiments confirm that the values 0.01 and 0.005 for α are equally appropriate.



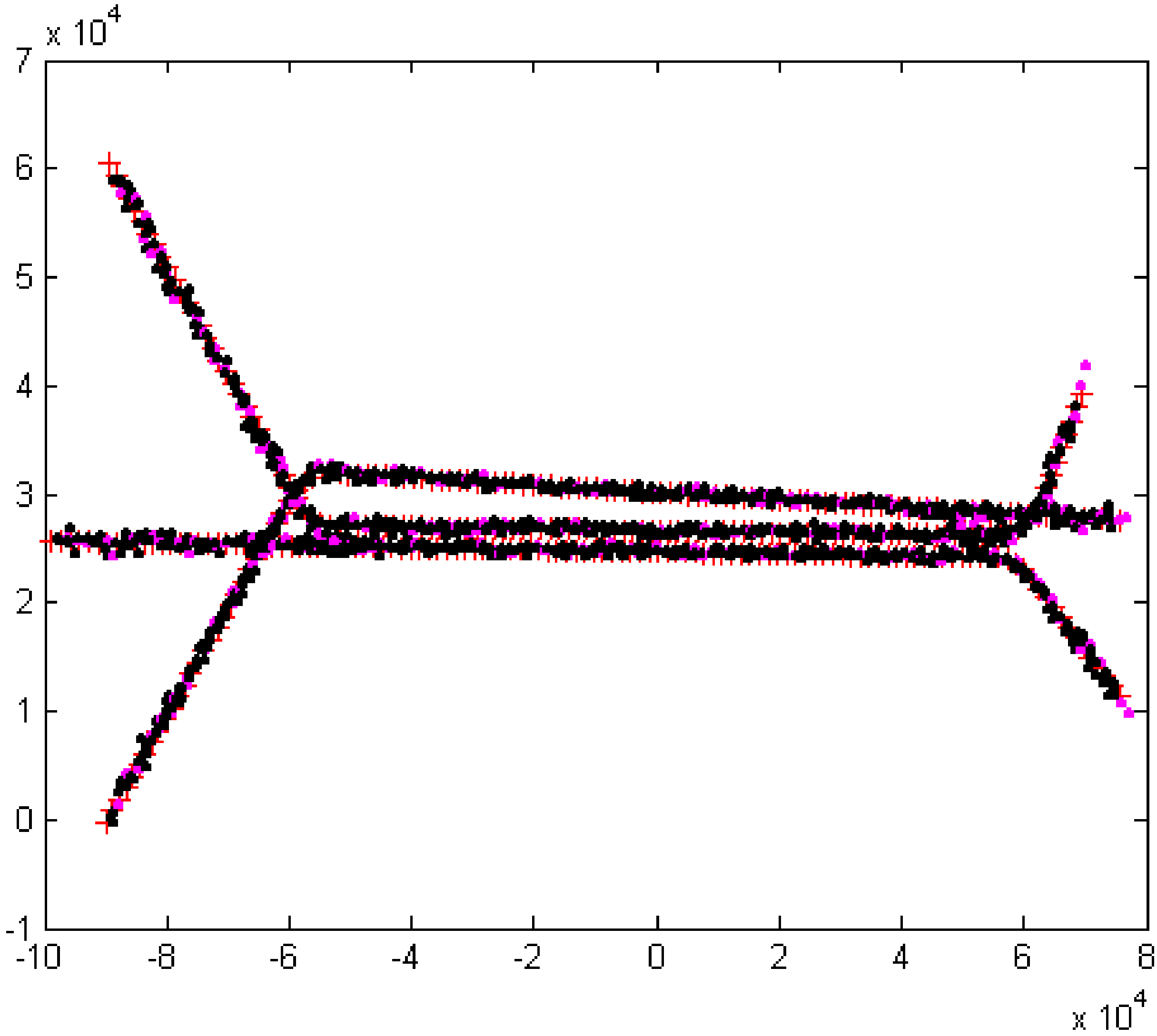


Figure 3: Three targets with crossing trajectories and Poisson parameter $\beta V = 1$ for the upper, and $\beta V = 2$ for the bottom picture

To test further the presented algorithm we constructed a range of scenarios with increasing complexity in terms of number of targets and presence of clutter. The chosen scenarios include 3, 4 and 5 targets with closely spaced and crossing trajectories (Figures 4 and 5). The included clutter has been modeled as a Poisson process with parameter βV, where B is spatial false alarm density and *V* is validation volume:

$$P(N = m_k | \beta V) = \frac{(\beta V)^{m_k} e^{-\beta V}}{m_k!}.$$

For every scenario two levels of clutter have been tested: with $\beta V = 1$ to simulate moderate clutter, and $\beta V = 2$ for heavy clutter. The results achieved can be summarized as follows:

A. Scenario with 3 closely spaced targets.

Table 5: Time per cluster in seconds for 3-targets scenario

	All hypotheses computation /targets in a cluster/		First K-best hypotheses only /targets in a cluster/	
	2 targets	3 targets	2 targets	3 targets
βV =1	0.016	0.062	0.02	0.26
βV =2	0.011	0.136	0.09	0.68

The comparison of presented algorithm with the algorithm where all feasible hypotheses are computed gives unexpected results – the latter algorithm spends less processing time (Table 5). Obviously the program frame for finding out the first K-best hypotheses is heavy and unsuitable for simple cases. Even so, both approaches give results far below the real time implementation threshold.

B. Scenario with 4 closely spaced targets.

Table 6: Time per cluster in seconds for 4-targets scenario

	All hypotheses computation Targets in a cluster		First K-best hypotheses only Targets in a cluster	
Targets in a cluster	3 targets	4 targets	3 targets	4 targets
βV =1	0.03	3.94	0.79	3.39
βV =2	0.22	124.7	3.42	9.86

It can be seen in this case (Figure 4) that when the scenario becomes denser the results become comparable (especially for clusters with 4 targets) and for the heaviest case (βV =2) the processing time for the first algorithm increases almost exponentially (Table 6). In the same time, the processing time for the new algorithm increases polinomially.

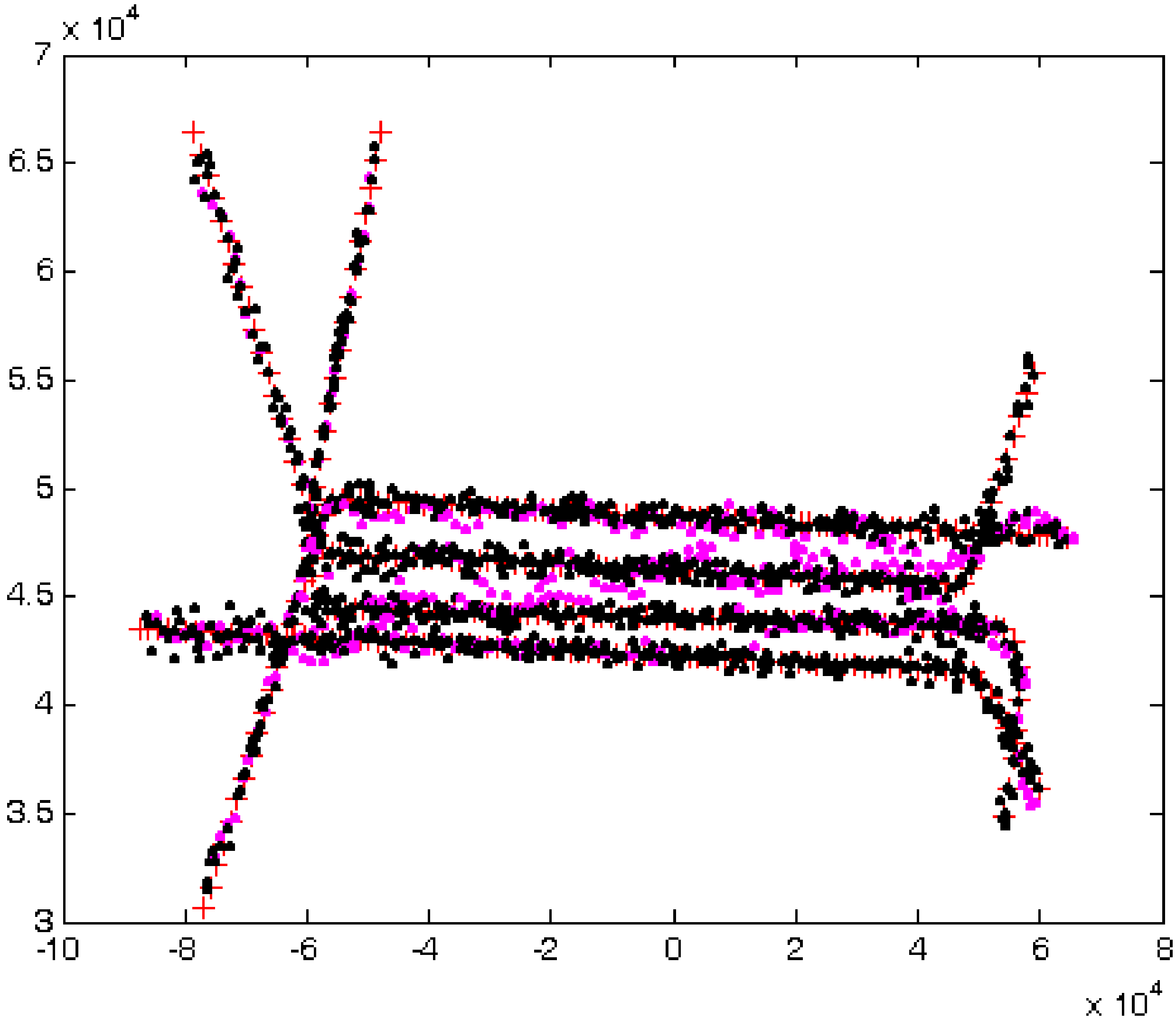


Figure 4: Four-target scenario with $\beta V = 2$

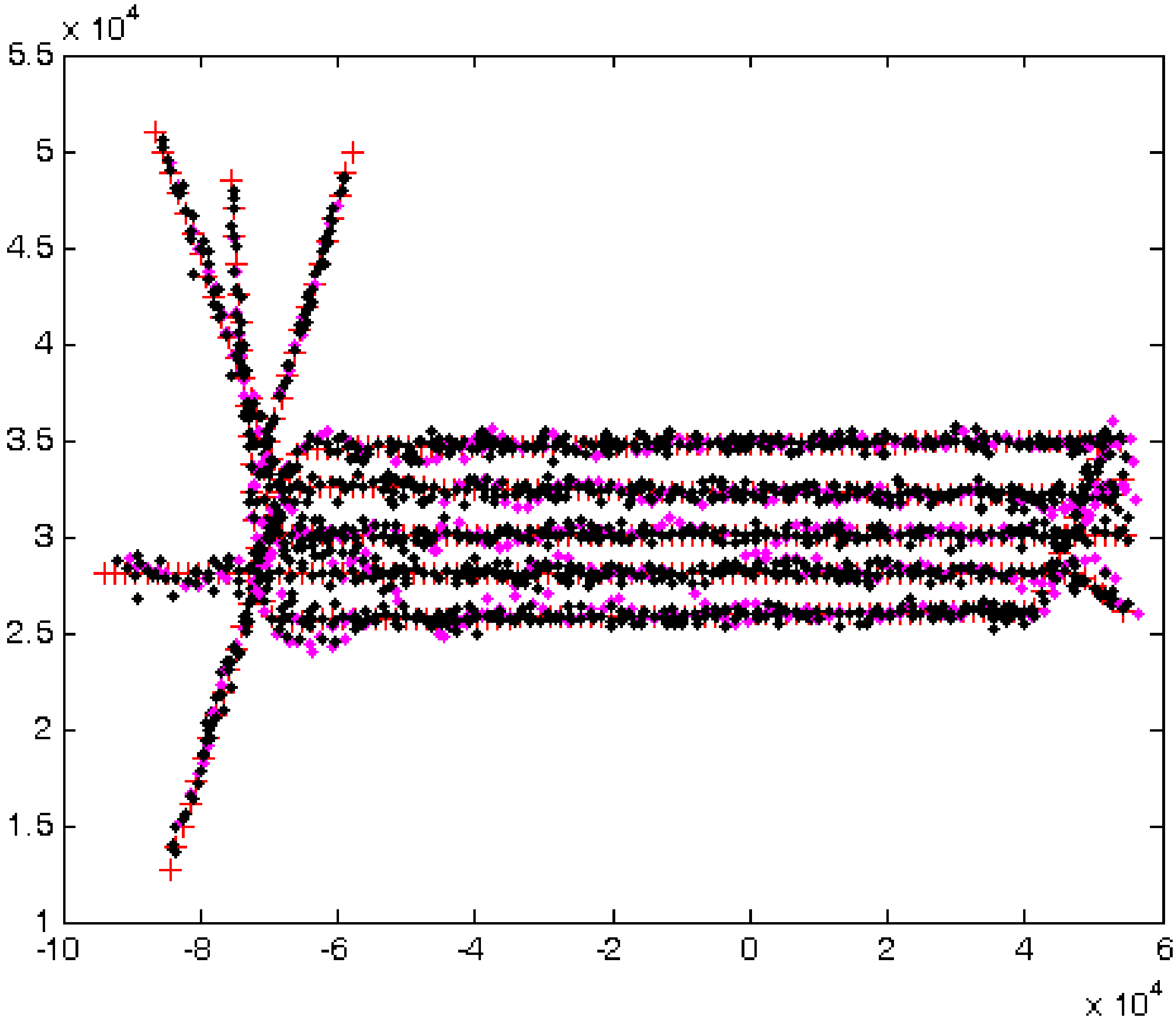


Figure 5: Five-target scenario with $\beta V = 2$

C. Scenario with 5 closely spaced targets.

For this scenario (Figure 5) only the proposed algorithm has been tested. For the most dense case, when five closely spaced targets have to be tracked in heavy clutter we compute average time per scan *t=8.7 sec*. But as it can be seen from Table 7, when in a given scan all five targets fall into the cluster the processing time becomes twice the average time. It can be stated that this case is the limit for algorithm implementation.

Table 7: Time per cluster in seconds for 5-targets scenario

	First K-best hypotheses computation Targets in a cluster		
Targets in a cluster	3 targets	4 targets	5 targets

βV =1	0.35	1.16	8.2
βV =2	1.58	6.36	15.4

5. Conclusions

In this paper a new algorithm is presented for tracking closely spaced targets in moderate and heavy clutter. This algorithm is an improved version of an algorithm previously presented earlier by the authors. Instead of all feasible hypotheses in the new algorithm only part of the hypotheses are generated. By means of an algorithm for finding the first K-best solutions of the assignment problem we generate the first K-best feasible hypotheses in terms of their probability of being true. This trade-off does not lead to observable assignment probability degradation and in the same time definitely speeds up the algorithm processing.

Acknowledgment

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Notes:

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An Accelerated IMM-JPDA Algorithm for Tracking Multiple Maneuvering Targets in Clutter

Ljudmil Bojilov, Kiril Alexiev and Pavlina Konstantinova

Keywords: Tracking, multiple maneuvering targets, cluttered environment, assignment

Abstract: Theoretically, the Multiple Hypothesis Tracking (MHT) method is the most powerful approach for tracking multiple targets. The MHT method, however, leads to combinatorial explosion and computational overload. By using an algorithm for finding the K-best assignments, the MHT approach can be considerably optimized in terms of computational load. A much simpler alternative of the MHT approach is provided by the Joint Probabilistic Data Association (JPDA) algorithm in combination with the Interacting Multiple Models (IMM) approach. Even though it is much more simple, this approach can also be computationally overwhelming. To overcome this drawback, an algorithm due to Murty and optimized by Miller, Stone and Cox is embedded in the IMM-JPDA algorithm in order to determine a ranked set of K-best hypotheses (instead of all feasible hypotheses). The presented algorithm assures continuous maneuver detection and adequate estimation of maneuvering targets in heavy clutter. This results in a good overall target tracking performance with moderate computational and memory requirements. The article further presents corresponding simulation results.

full text

SPECIFIC FEATURES OF IMM TRACKING FILTER DESIGN

Iliyana SIMEONOVA and Tzvetan SEMERDJIEV

1. Introduction

The Interacting Multiple Model (IMM) estimator is a suboptimal hybrid filter that has been shown to be one of the most cost-effective hybrid state estimation schemes.[10] The model of hybrid system and the IMM algorithm, initially proposed by Blom,[12] may serve as a basis for synthesis of more efficient filters for tracking maneuvering aircraft. In this paper we present a number of specific features of the IMM design procedure. In section 2 we briefly describe the hybrid system (aircraft dynamics), hybrid estimation and the principle of the IMM algorithm. The models used in the IMM configuration to describe different flight phases and the measurement model used for our application are presented in sections 3 and 4 respectively. Section 5 provides a detailed discussion on the specific features of the IMM algorithm design. Finally, some examples of IMM tracking filter design and performance evaluation are presented in section 6.

2. Hybrid Systems and Hybrid Estimation

2.1. Hybrid Systems

An aircraft trajectory can be subdivided into distinct segments, corresponding to modes of flight,[10] for instance, uniform motion and maneuvers. The multiple model or hybrid system approach assumes the system to be in one of a finite number of modes.[2] Thus, the aircraft motion can be modeled by a hybrid system that is characterized by two state variables: continuous base - state variable $x(k) \in R^{n_x}$ (aircraft position, speed, acceleration, etc.) and a discrete regime variable $j = m_j(k) \in M_r = 1, 2....r$, which describes the distinct segment of the aircraft trajectory. The transitions (jumps) of the mode variable are modeled with a Markov chain. The nonlinear hybrid system is usually described by the equations:

$$x(k) = f_j[(k-1), x(k-1)] + g_j[k-1, x(k-1), v_j(k-1)] \quad \forall\, j \in M_r, \qquad (1)$$

$$z(k-1) = h_j[(k-1), x(k-1)] + w_j(k-1) \quad \forall\, j \in\, M_r, \tag{2}$$

where $x(k) \in R^{n_x}$ is the system state vector, $z(k-1) \in R^{n_z}$ is measurement vector, $v_j(.)$ and $w_j(.)$ are mode dependent process noise and measurement noise sequences, assumed to be white, zero-mean and mutually independent with covariances $Q(m_j(k))$ and $R(m_j(k-1))$ respectively. The available measurements for estimation process can be: [range and azimuth]; [range, azimuth and range rate], etc. The $f_j[.], g_j[.], h_j[.]$ are known functions. The system at time (k) is assumed to be among r possible modes,[7] i.e. $j = m_j(k) \in M_r = 1, 2....r$, where $j = m_j(k)$ denotes that the j-th sub-model is in effect during the sampling period T ending at time (k). $m_j(k)_{k=1,2....}$ is a Markov chain with completely known initial $P_j = P[m_j(0)] = j$ and transitional probabilities $p_{ij} = P[m_j(k)|m_i(k-1)]$.

2.2. Hybrid Estimation

The problem of hybrid state estimation is to estimate the base state and the modal state based on noisy measurement sequences. The application of the Multiple - model (MM) method is a major approach to hybrid estimation. It assumes the system obeys one of a finite number of models. The IMM algorithm belongs to the multiple-mode techniques and therefore provides a method to combine the estimates and covariance matrices of each mode with an interacting logic to maintain all of them ‘in track.’[8] To estimate the aircraft state, there is a bank of Kalman filters, where each filter matches a model in the set, and a procedure to estimate the probabilities that the target is in each one of the possible modes. Yaakov Bar-Shalom and coauthors provide detailed explanation of the IMM algorithm.[1,2,5,10] Following Bar-Shalom and Chang,[9] here we will give a short description of one cycle of the algorithm $(k-1) \to (k)$:

Interaction(mixing):

The mixed initial state $\hat{x}^{0j}(k-1|k-1)$ and covariance $P^{0j}(k-1|k-1)$ for the filter matched to mode $m_j(k), j = 1, ...r$ are calculated by mixing the state estimates $\hat{x}^j(k-1|k-1)$ and covariances $P^j(k-1|k-1)$ of all filters obtained at the previous time step

$$\hat{x}^{0j}(k-1|k-1) = \sum_{i=1}^{r} \hat{x}^i(k-1|k-1)\mu_{i|j}(k-1|k-1), \tag{3}$$

$$\begin{aligned} P^{0j}(k-1|k-1) = \sum_{i=1}^{r} \mu_{i|j}(k-1|k-1)P^i(k-1|k-1) + \\ [\hat{x}^i(k-1|k-1) - \hat{x}^{0j}(k-1|k-1)][\hat{x}^i(k-1|k-1) - \hat{x}^{0j}(k-1|k-1)]^T, \end{aligned} \tag{4}$$

where

$$\mu_{i|j}(k-1|k-1) = (1/\bar{c_j})p_{ij}\mu_i(k-1) \tag{5}$$

are mixing probabilities, and $\bar{c_j} = \sum_{i=1}^{r} p_{ij}\mu_{i|j}(k-1|k-1)$ is the normalization factor.

Mode-conditioned filtering:

The estimate (3) and covariance (4) are used as input to the Kalman filter (linear or extended) matched to $m_j(k)$ to obtain state estimate $\hat{x}^j(k|k)$ and covariance $P^j(k|k)$ at time k, as well as the mode likelihood $\Lambda_j(k)$:

$$\Lambda_j(k) = |S_j(k)|^{-1/2} exp\{-\frac{1}{2}\tilde{z}_j^T(k)S_j^{-1}(k)\tilde{z}_j(k)\}, \tag{6}$$

where $\tilde{z}_j(k)$ and $S_j(k)$ are innovation and its covariance of the j conditional filter.

Mode probability update:

The model probabilities $\mu_j(k)$ are updated as follows:

$$\mu_j(k) = \frac{1}{c}\Lambda_j(k)\sum_{i=1}^{r} p_{ij}\mu_i(k-1) = \frac{1}{c}\Lambda_j(k)\bar{c_j}, \quad c = \sum_{j=1}^{r}\Lambda_j(k)\bar{c_j}, \tag{7}$$

where c is the normalization factor.

Estimate and covariance combination

The combination of the updated mode conditioned estimates and covariances produces the output estimates:

$$\bar{x}(k|k) = \sum_{j=1}^{r}\hat{x}^j(k|k)\mu_j(k), \tag{8}$$

$$P(k|k) = \sum_{j=1}^{r} P_j(k|k) + [x_j(\bar{k}|k) - \bar{x}(k|k)] + x_j(\bar{k}|k) - x(\bar{k}|k)^T]\mu_j(k). \tag{9}$$

The IMM algorithm has three desirable properties: it is recursive, modular and has fixed computational requirements per cycle. The IMM algorithm can use as its building blocks Kalman filters (KF) or Extended Kalman filters (EKF) to account for nonlinearities in the measurement equation (for range - azimuth - elevation observations), and in the state equation (for coordinated turns) or probabilistic data association filters based on KF or EKF when data association is a major problem.

3. Aircraft Motion Models

Civilian aircraft in air traffic control (ATC) systems have two basic modes of flight:

- uniform motion (UM) - the straight and level flight with a constant speed and heading and
- Maneuver - turning or climbing/descending.[1]

Let us consider the linear version of equation(1) to aircraft trajectory modeling in two Cartesian coordinates (x, y):

$$X(k) = F_j.X(k-1) + G_j.v_j(k-1), \tag{10}$$

where $v_j(k-1) = \begin{bmatrix} v_{jx}(k-1) & v_{jy}(k-1) \end{bmatrix}^T$ are white noise sequences used to model uncertain accelerations. The model extension in (x, y, z) coordinates is straightforward. In this section we will present some of the most commonly used aircraft motion models.

Notations: $X \equiv X(k)$; F_j/G_j - transition/noise gain matrix for both coordinates (x, y); f_j/g_j - transition/noise gain matrix for each coordinate x/y.

3.1. Piecewise Constant White Acceleration Model

This is a second order (nearly constant velocity) model with the following parameters:

- State space vector: $X = \begin{bmatrix} x & \dot{x} & y & \dot{y} \end{bmatrix}^T$;
- Transition matrix: $F_1 = diag[f_1, f_1]$, where $f_1 = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$;
- Noise gain: $G_1 = diag[g_1, g_1]$, where $g_1 = \begin{bmatrix} T * T/2 & T \end{bmatrix}^T$.

This model assumes the variations in velocity components for each coordinate are piesewise constant zero-mean white noise accelerations.[1] The process noise variances in each coordinate are assumed to be equal: $\sigma^2_{\nu x} = \sigma^2_{\nu y} = q$ and $\sigma_{\nu x \nu y} = 0$.

- A 'nearly constant velocity motion model' (M_1) for the UM modeling is obtained by the choice of 'small' noise values: $q = q_1$.[1,6]
- The same model, but with higher levels of process noise $q = q_2$ can model 'rough' maneuvers. This model is denoted as M_2.

3.2. Piecewise Constant Wiener Process Acceleration Model

This is a third order (nearly constant acceleration) model used to describe the maneuvering phase of flight. It has the following parameters:

- State space vector: $X = \begin{bmatrix} x & \dot{x} & \ddot{x} & y & \dot{y} & \ddot{y} \end{bmatrix}^T$;

- Transition matrix: $F_3 = diag[f_3, f_3]$, where $f_3 = \begin{bmatrix} 1 & T & T*T/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}$;
- Noise gain: $G_3 = diag[g_3, g_3]$, where $g_3 = \begin{bmatrix} g_1 & 1 \end{bmatrix}^T$.

This model assumes the acceleration increments for each component during k-th sampling period are zero-mean white sequence.[2]

- A 'nearly constant acceleration motion model' M_3 is obtained by the choice of 'small' $q = q_3$.[1,6]
- The model M_4 is the same but with higher levels of process noise $q = q_4$.[6]

3.3. Coordinated Turn Models (CTM)

Coordinated turn models are another way to describe the maneuvering mode of flight. The turning of civilian aircraft usually follows a pattern known as a "coordinated turn": that means the target is moving with constant speed and turning with constant turn rate. There are two basic coordinated turn models:

Constant turn rate models: here turn rate ω is a completely known design parameter. The models M_5 and M_6 are used for a left-hand turn ($\omega > 0$) and for a right-hand turn ($\omega < 0$), respectively. This assumption is suitable for a civilian aircraft because its maneuvers are constrained by flight rules, especially when approaching an airport. [5] The state space vector and noise gain coincide with those of models M_1 and M_2, but the transition matrix is different:

- Transition matrix :

$$F_{5,6} = \begin{bmatrix} 1 & \omega^{-1}\sin(\omega.T) & 0 & \omega^{-1}(\omega - \cos(\omega.T)) \\ 0 & \cos(\omega.T) & 0 & -\sin(\omega.T) \\ 0 & \omega^{-1}(\omega - \cos(\omega.T)) & 1 & \omega^{-1}\sin(\omega.T) \\ 0 & \sin(\omega.T) & 0 & \cos(\omega.T) \end{bmatrix}.$$

- The process noise variances in each coordinate are q_5 and q_6 respectively.

For military aircraft the above assumption is less natural, so the model M_7 presents the case where ω is not known. What we need here is to augment the state space vector with unknown turn rate ω.

- The discrete time state space equation is nonlinear, because the transition matrix is a function of the state component (ω):

$$X(k) = F_7(\omega).X(k-1) + G_7.v_7(k-1). \tag{11}$$

- State space vector: $X = \begin{bmatrix} x & \dot{x} & y & \dot{y} & \omega \end{bmatrix}^T$;
- Transition matrix: $F_7 = \left[\begin{array}{c|c} F_{5,6} & 0 \\ \hline 0 & 1 \end{array}\right]$;
- Noise gain: $G_7 = diag[g_7, g_7]$ where $g_7 = \begin{bmatrix} g_1 & 0 \end{bmatrix}^T$.

Here $v_7(k-1) = \begin{bmatrix} v_{7x}(k-1) & v_{7y}(k-1) \end{bmatrix}^T$ are white noise sequences used to model uncertain accelerations in x and y coordinates due to uncertainty in ω.

- The process noise variance is q_7.

In all the above models $\sigma_{\nu_x \nu_y} = 0$. Furthermore, there are two basic principles to maneuver modeling [5]: exact maneuver modeling and approximate maneuver modeling. Thus, models M_2, M_3, M_4 are based on approximate modeling,while models M_5, M_6, M_7 assume exact modeling. The reader is referred to the works of Bar-Shalom, Li and coauthors [1,2,5,6] for a comprehensive presentation of aircraft motion modeling.

4. Measurement Model

Let us consider the case when measurement (sensor modeling) equation (2) is not mode dependent:

$$z(k-1) = h[(k-1), X(k-1)] + w(k-1). \tag{12}$$

For our application the nonlinear measurement function $h[.]$ of (12) converts Cartesian target state space vector coordinates (x,y) to z=[range and azimuth]^T, $h[.] = [h_1 h_2]^T$, where $h_1 = \sqrt{x^2 + y^2}$ and $h^2 = arctan\frac{x}{y}$.

Due to the nonlinearity of the measurement(sensor) equation (12), the Extended Kalman filter is used in the IMM configuration. It requires linearization of $h[.]$ about the position prediction, that yields Yacobian.

5. Specifics of the IMM Tracking Filter Design

In this section we shall provide the practitioners with some practical rules how to chose the best IMM filter design parameters.

To obtain the best possible results, the IMM algorithm has to be properly designed. The designer need to take into account [1]:

- Accuracy in estimating both position and velocity. A trade-off between the peak errors and the errors during uniform motion is desirable;
- Timeliness of the maneuver detection and termination;
- Complexity of the implementation.

The design of an IMM estimator consist of the following steps [1]:

- Selection of the set of models describing aircraft dynamics and their structure;
- Selection of the process noise intensities for the various models;
- Selection of the Markov chain transition probabilities.

5.1. Model Set Selection

Designing IMM tracking filters, both complexity and quality of the models need to be considered. Typically, the models used in the IMM configuration include one nearly constant velocity motion model for non-maneuvering regime of flight and a set of exact or approximate maneuver models for the maneuvering phases. The models for uniform and maneuvering motion are presented in the previous section. Additionally, we need to account that increasing the number of conditional sub-models to cover the uncertain behavior of highly maneuvering objects increases considerably the computation load but does not guarantee better performance due to the competition among models. [3] The precise modeling of every aircraft trajectory segment will lead to more accurate results especially in speed estimation.[5]

5.2. Process Noise Selection

The selection of process noise standard deviations for each model is an important part of the estimator design. The process noise levels are selected based on the expected disturbances and target maneuvering magnitudes. [1] Let us consider some of the models presented in the previous section.

- The *small* process noise of model M_1 accounts for air turbulence, winds aloft changes,[2] slow turns, as well as small linear accelerations.
- The *high level* process noise of model M_2 allows for target acceleration and is applicable (with limited degree of success) to tracking maneuvering target. [2] The process noise range is usually selected as follows: $0.5.a_{max} \leq q_2 \leq a_{max}$,[2] where a_{max} is the maximum acceleration magnitude. According to Bar-Shalom and Li,[2] the *IMM configuration* with M_1 and M_2 models does obtain acceptable results for maneuvers with turn rates up to 3 deg/s; it does not, however, yield good estimates for faster turns.
- The M_3 model with *low process* noise provides more accurate estimation during a maneuver.
- A maneuver onset is a rapid jump to a non-zero target acceleration from zero and then a jump back to zero acceleration at termination. [1] So, the M_4 model with *high level* process noise can model more precisely maneuver onset and termination. The noise range should be of the order of the magnitude of the maximum

acceleration increment over a sampling period: $0.5\triangle a_{max} \leq q_4 \leq \triangle a_{max}$.[2] The *IMM configuration* with one second order model and two third order models with different noise levels [3,5,7] is best suitable for estimating more intensive ($a_n = 7$g) maneuvers with short duration, as well as longitudinal acceleration. But it leads to considerable errors for moderate turns of 1-5g. Additionally, the peak errors are not significantly reduced compared to those obtainable by using the single-model filter.[5] The explanation is that the three models are not discriminating enough. Also, the interaction step mixes the regime-conditioned estimates in a way that helps the filters based on the "wrong" models to come back on track. In this case, the a posteriori information conveyed by the innovations conditioned on the mode hypothesis do not have enough contrast. According to Kirubarajan and coauthors,[11] an *IMM configuration* which uses as it building modules models M_1, M_2 and M_4 is most suitable for highly maneuvering targets. Finally, the use of the maneuver detection model M_4 is not necessary when tracking civilian aircraft.

- The right choice of the noise levels $q_{5,6}$ of models M_5 and M_6 depends on the expected turn rate and on the number of models to be used in estimating maneuvers. The standard deviation of the process noise can be selected as $q_{5,6} = 0.5.(\omega_{i+1} - \omega_i).V = 0.5.\triangle U$,[4] where ω_{i+1}, ω_i are turn rates of two adjacent models and V is the expected linear speed . The *IMM configuration* could include one uniform motion model and a set of different constant turn rate models. According to Munir and Atheron,[4] the use of coordinated turn models with known ω is better than IMM with estimated ω when the models in the former case fully "cover" the turn rate of target motion and vice-versa.
- Because of the delay in estimating ω at the onset of the maneuver, the use of model M_7 produces rather large peaks. However, once the ω estimate converges, a very good tracking performance is obtained during turns. [5] According to an earlier work of the authors,[3] using model M_7 is best suitable for tracking aircraft performing maneuvers with moderate, a priori unknown normal acceleration ($a_n = 1 \div 5$g), as well as for more complex maneuvers with longitudinal and transversal accelerations.

The practitioner should also be aware that a high degree of smoothing and, thus, a low value of the convergence noise at uniform motion leads to high peaks during maneuvers, and vice versa.[8]

5.3. Transition Probabilities

The Markov chain transition probabilities are related to the expected sojourn time in the various modes.[1] These probabilities are chosen according to the designer's beliefs about the frequency of change from one mode to the rest. They can be subsequently adjusted by means of Monte Carlo simulations. The guideline for a proper choice is to match roughly the transition probabilities with the actual mean sojourn time (τ_i) of each mode,[2] the diagonal coefficients being determined as $p_{ii} = 1 - \frac{1}{E[\tau_i]}$. The transition probabilities p_{ij} for $i \neq j$ are selected using the identity: $\sum_{j \neq i} p_{ij} = 1 - p_{ii}$.

The choice of the transition probabilities provides a certain degree of trade-off between the peak estimation errors at the onset of the maneuver and the maximum reduction of the estimation errors during the uniform motion.[2] Following Bar-Shallm and Li, Herrero and coauthors,[2,8] the practitioner should be aware that high transition probabilities lead to low peak errors during maneuver, but at the cost of low smoothing and higher errors when tracking uniform motion. Also, the transitions between models are quickly detected and the filter is very adaptive. Let us consider an *IMM configuration* with a M_1 model and a model with unknown ω - M_7. According to Bar-Shalom,[5] if the transition matrix of this configuration has large off-diagonal entries, then it favors regime transition and results in much more volatile sample paths of the probabilities. This phenomenon is called “regime mixing.” Here, the peak errors are so low that the occurrence of the maneuver is hardly noticeable.

5.4. Initial Probabilities

If initial probabilities are set to be equal in orded to account for the worst case of uncertainty, then the initial estimation errors will be large.[2]

6. Performance Analysis

6.1. Target Motion Scenario

In order to test the capabilities of different IMM configurations we consider a class of maneuvering aircraft performing sweep maneuvers with normal acceleration up to 7g (g $\approx$ 9.8 m/s^2). The scenario of motion is depicted on Figure 1.

6.2. Sensor Parameters

The simulation involves a single track while scan (TWS) radar with scanning period of 1s. The sensor parameters considered here are : range and azimuth accuracy: $\sigma_{\mathrm{D}} = 120\,\mathrm{m}$ and $\sigma_\beta = 0.2\,\mathrm{deg}$, respectively.

6.3. IMM Tracking Filter Design

Here we introduce three different sets of models, respectively IMM2, IMM3, IMM–CT, to describe the target motion scenario. Their design parameters are:

- IMM2 - *mode set*: $[M_1, M_{3I}]$; -*process noises:* $\sigma_1 = 2.5\,\mathrm{m/s^2}, \sigma_{3I} = 20.5\,\mathrm{m/s^2} -$ I - intermediate noise; -*transition probabilities* $p_{11} = 0.9, p_{21} = 0.20$.
- IMM3 - *mode set*: $[M_1, M_3, M_4]$; -*process noises:* $\sigma_1 = 2.5\,\mathrm{m/s^2}, \sigma_3 = 7\,\mathrm{m/s^2}$, $\sigma_4 = 40\,\mathrm{m/s^2}$; -*transition probabilities* $p_{11} = 0.9, p_{12} = 0.05, p_{21} = 0.15$, $p_{22} = 0.75, p_{31} = 0.20, p_{32} = 0.05$.
- IMM-CT -*mode set*: $[M_1, M_5, M_6]$; -*process noises:* $\sigma_1 = 2.5\,\mathrm{m/s^2}$, $\sigma_{5,6} = 3.5\,\mathrm{m/s^2}, \omega_{5,6} = \pm 0.233\,\mathrm{rad/s}$; -*transition probabilities* $p_{11} = 0.9$, $p_{12} = 0.05, p_{21} = 0.20, p_{22} = 0.80, p_{31} = 0.20, p_{32} = 0.00$.

6.4. Performance Evaluation and Analysis

The IMM filters' efficiency was evaluated according to the root mean-square(RMS) error both in position and in velocity. The results presented here are based on Nr = 100 Monte Carlo runs. RMS is defined as
$\sigma_{\mathrm{pos}} = \sqrt{\frac{1}{N} \cdot \sum_{i=1}^{N} [x^i(k) - \hat{x}^i(k|k)]^2 + [y^i(k) - \hat{y}^i(k|k)]^2}$, where the superscript i denotes the results from run i, and x(k), y(k) are true target positions(x,y).The equation for RMS error in estimating velocity is derived likewise.

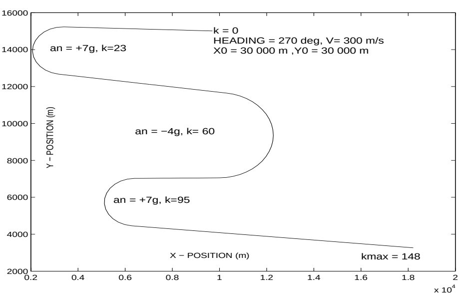

Figure 1: Target Trajectory.

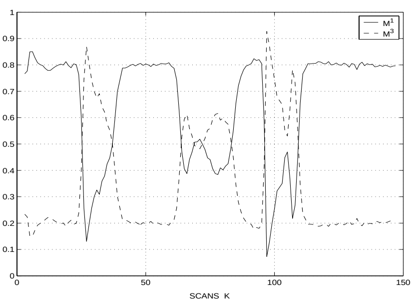

Figure 2: IMM2. Mode probabilities.

The analysis of simulation results is summarized as follows:

- The average mode probabilities for each case over scans are depicted on Figures 2, 3 and 4. The correct mode has the largest probability during each segment. [5]
- Comparison of RMS position and velocity errors are shown on Figures 5 and 6, respectively. The general behavior of these error curves is typical for IMM algorithms. Natural transients are observed at the onset and termination of maneuvers. The peaks at the start and the end of a maneuver are caused by the delay of mode probabilities switching from one mode to another. After switching, the slower decrease of the errors corresponds to the convergence of maneuver filter. [5]

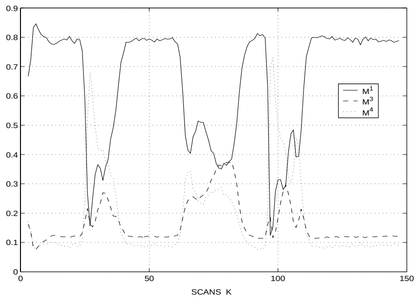

Figure 3: IMM3. Mode probabilities.

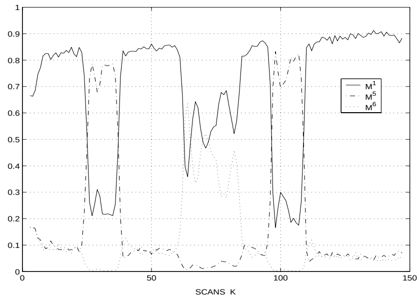

Figure 4: IMM–CT. Mode probabilities

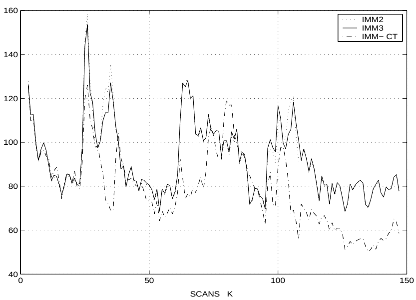

Figure 5: RMS position errors [m]

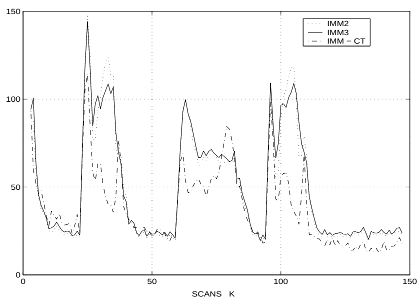

Figure 6: RMS velocity errors [m/s]

As expected, the use of model M_4 in IMM3 reduce the peak errors both in position and in velocity. Also, the use of exact maneuver models as in IMM-CT reduces significantly the peak errors and the errors during uniform motion.

7. Conclusions

Considerable number of publications is devoted to the design of IMM tracking filters. In this article we summarize results, conclusions and experience of various authors in order to provide researchers and designers with a fast and easy way to determine the advantages and the capacity of different IMM structures in variety of target motion scenarios. This is an objective not addressed in available bibliography on IMM tracking filters. The simulation results are obtained using the comprehensive MATLAB tool developed by the team at the Central Laboratory for Parallel Processing of the Bulgarian Academy of Sciences. This tool could be used as a basis for a synthesis of many other motion scenarios and IMM tracking filters.

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Specific Features of IMM Tracking Filter Design

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Keywords: hybrid systems, hybrid estimation, aircraft motion models, IMM tracking filter design

Abstract: The interacting multiple model (IMM) algorithm is one of the most cost-effective and simple schemes for tracking maneuvering targets. So the knowledge of the specifics of its design is important in order to achieve more accurate estimates of aircraft parameters. This article presents the specifics of the IMM tracking filter design. Results, conclusions and experience of different authors have been generalized. Based on results and recommendations provided herein, in a fast and easy manner the user is able to determine the advantages and the ability of different IMM structures given the target motion scenario. In addition, the authors have designed, studied and proved the behavior of three IMM configurations using a tailored but comprehensive MATLAB tool.

full text

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A MATLAB TOOL FOR DEVELOPMENT AND TESTING OF TRACK INITIATION AND MULTIPLE TARGET TRACKING ALGORITHMS

Kiril ALEXIEV

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1. Introduction

Digital computer simulation is a valuable tool, used for the design, analysis, and testing of complex systems whose behavior cannot be easily evaluated by means of analysis. Simulation includes three main steps, as follows: input data generation, modeling of examined system and performance evaluation with proper visualization. By their very nature, radar data processing algorithms are well suited to computer simulation. Simulation programs are often coded in general purpose high-level languages, such as C++, Pascal, Ada, or Java. This is mainly due to the popularity of these languages among programmers and computer users, as well as to their availability and portability. But the most complex algorithms can be easily coded by means of Matlab. The Matlab language can be learnt quickly and after that the engineers can fully exploit its power with high productivity. Matlab compiler translates *.m files to C code for real time implementation purposes. In spite of the fact, that the resulting code consists almost completely of calls to Matlab *.dll functions and the performance is similar to that of standard *.m files, the translation can be regarded as a good initial step of migration to C code.

The purpose of this paper is to describe simulation tools for analysis and design of a radar data processing system, to outline the techniques used to generate the input data, to simulate the algorithms and to analyze the results for evaluation of system's performance.

This tool can be useful to practicing radar engineers for the purposes of both analysis and design.

2. Architecture of the simulation programs

The simulation of radar data processing can be defined as a set of algorithms which allows:

- Complex scenario generation;
- Recognition of a pattern of successive detections as pertaining to the same target (track initiation);
- Estimation of the kinematics parameters (position, velocity and acceleration) of a target, thus establishing the so called “target track”;
- Extrapolation of the track parameters;
- Distinguishing of different targets and thus establishing a different track for each target;
- Adaptive scheduling of the time dwells of a phased-array radar in order to follow a maneuvering target with constant accuracy and to interleave in an optimum manner the tracking phases with search looks and other radar functions;
- Efficient managing of the detections or the tracks, provided by the different radar sets of a netted system looking at the same portion of the controlled space, in order to provide a better picture of the latter.

A generalized scheme of the proposed Matlab tool is presented on figure 1.

3. Input data simulation

There are two different approaches to the input data simulation. The first of them uses data, recorded from real radar. This approach simulates the real operating conditions of the testing system and there are no errors caused by data modeling. But there is a severe drawback – it is very difficult, dangerous, expensive and some times impossible to explore estimated algorithms in a complex scenario. Such a scenario is of low probability, although it can exist in real life critical conditions. Another, more mild drawback is that the true target path and the true target maneuver parameters are unknown and the researcher has no exact reference data for accurate evaluation of the algorithm under exploration.[2] Nevertheless, the Matlab tool has an entry for real life data, using a common data format for data exchange.

The use of simulated data has considerable flexibility in the selection of a complex target and clutter scenario and an a priori known reference input is provided. The simulation program generates hundreds of targets moving rectilinearly or maneuvering with given transversal and longitudinal accelerations.

The radar parameters (scan rate T and detection probability P_D) can vary in wide intervals. The simulation program has the ability to synchronize position of generated targets in the space and thus to create complex and critical scenarios. However, only an approximate representation of the operating conditions can be obtained. Another simulation program is used for noise and clutter generation. The noise can be generated in the whole surveillance volume or only in the current targets gates. The last feature is very useful for testing algorithms with hypotheses generation. Another useful feature of the simulation program is the possibility of generating given number of trajectories with fully random parameters or parameters, randomly chosen in given intervals. In this way, the input data are generated for Monte Carlo analysis of explored algorithms.

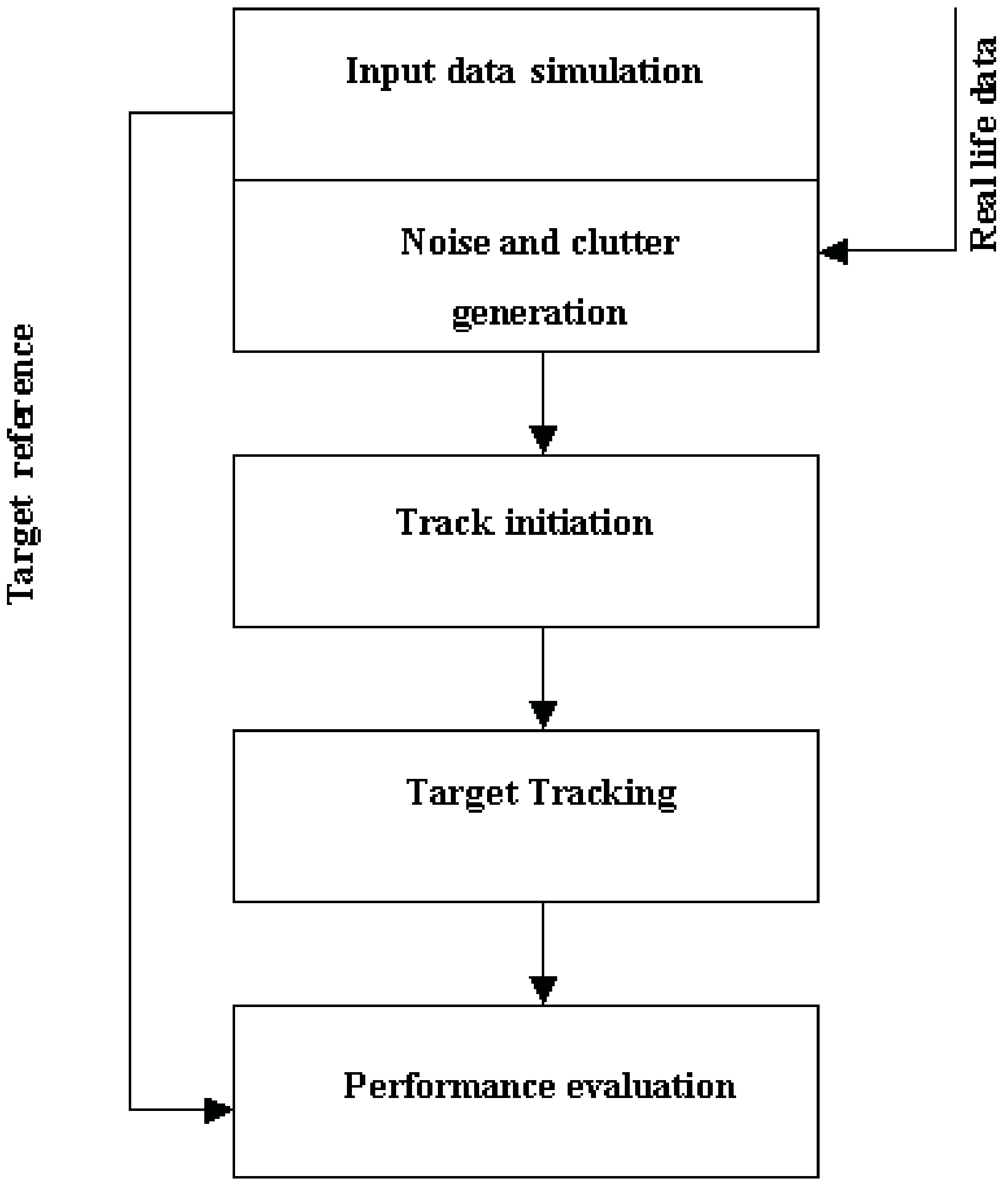

Figure 1: Flowchart of the Matlab radar data processing tool

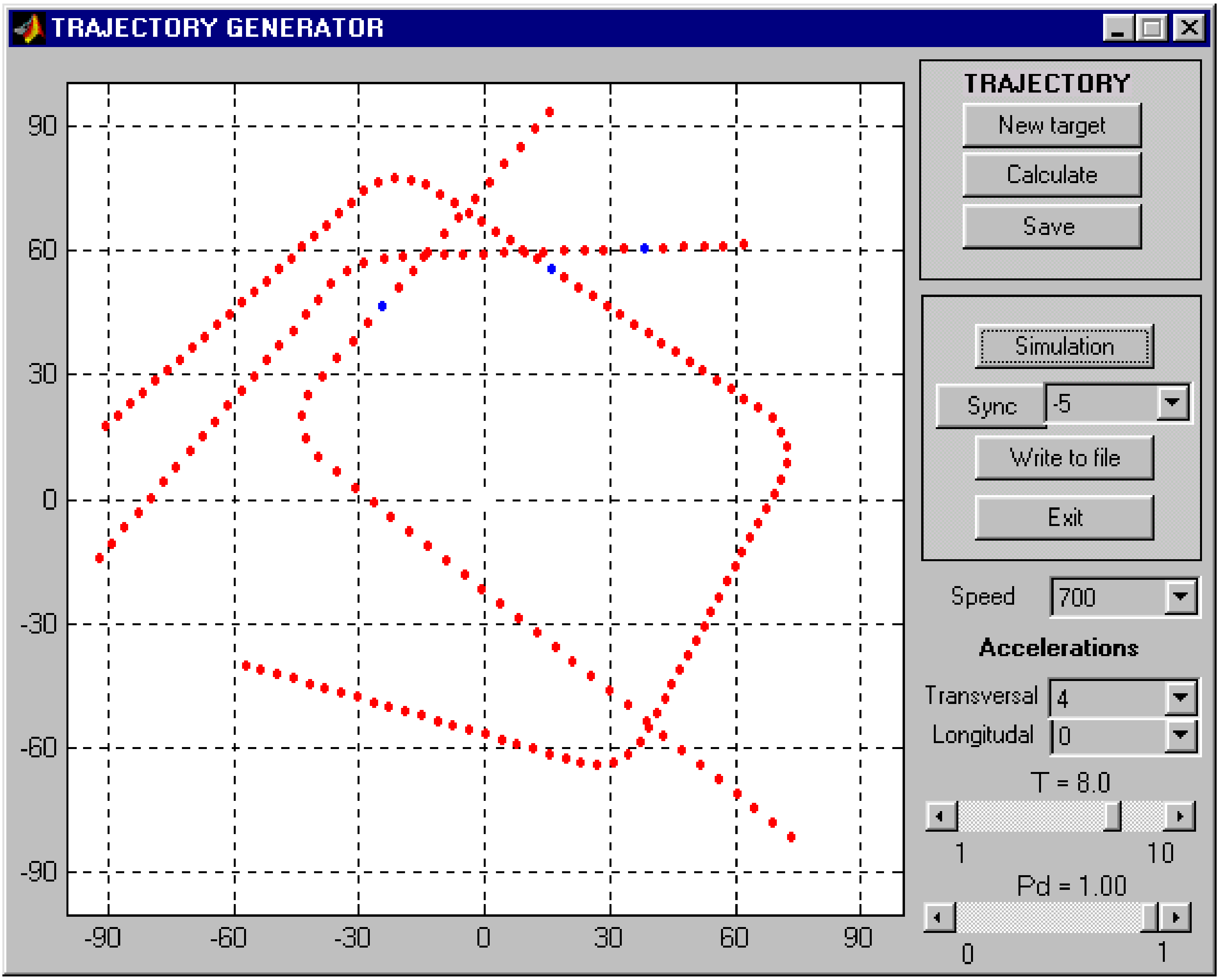

Figure 2: The graphic user interface of target simulation program

The trajectory generator is generally better suited for algorithm estimation and tuning. Recorded real sensor data can be used as a more realistic test in an advanced stage of the design or as a last approval of system characteristics.

The trajectory is assumed to be planar; it may consist of straight and circular sections. Initially, several points on the map define its sections. Every point consists of target position, target speed, target transversal acceleration and time.[9] The time is calculated using longitudinal acceleration and parameters of two sequential points. In every point (except the first and the last one) the direction of the target is changed. The maneuver is considered with constant longitudinal speed and constant transversal acceleration. The target motion is modeled by computing the position, velocity and acceleration at the equal time instants T. The time interval between two consecutive detections of a target may differ slightly from radar scan period. This simplification does not affect the accuracy of the simulation since the Kalman filtering does not require a constant sampling interval, being based on the effective detection instant. This assumption compares to a modulation of the antenna scan period, which is commonly encountered in practice, e.g. due to the wind.

The radar sensor is modeled by a program, which takes into account measurement error distribution. Analyses of radar measurements range and azimuth errors showed that the best approximation of the error distributions is a double Gaussian distribution.[3] This model is used in the radar modeling.

4. Track initiation programs

Track initiation programs associate sensor measurements with potential track trajectories using different correlation techniques. The task is to find several measurements ordered in space and time. The most common approach, considered as classical, uses N sequential measurements (usually 4-5 measurements) and implements weighted least squares to find initial approximation of target state. The classical approach can be very computationally intensive, because the number of hypotheses grows exponentially with the number of measurements under consideration. This hypothesis growth can be overcome by careful hypothesis pruning. A gating technique is introduced in order to reduce the combinatorial problem, but this algorithm does not solve the problem completely.

In dense target and clutter environments, however, the number of hypotheses remains too big enough and the classical approach fails to initialize the trajectories, thus leading to poor results. In this case, different type of track initiation procedure has to be used. The vast surveillance volume is fragmented in a set of cells and the combinatorial problem is decomposed on many such problems of smaller size, solved in small fragments. Two types of such track initiation procedures are implemented. The first of them uses uniform surveillance volume fragmentation. The measurement selection method typically uses a mosaic grid to group the measurements in subsets. The track initiator uses these subsets for potential track determination. The problems of optimization in this case are determination of cell size and how to process measurements on (or near to) the cell borders. The second algorithm uses template matching technique like Hough transform,[5] Fourier transform, etc. The cells in the surveillance volume in this case correspond to the initialized target trajectories. Both methods require additional computer resources to resolve the combinatorial problem in the case of dense target and clutter environment. The main parameters to be estimated are the probability of detection of a trajectory, the probability of false track detection as a function of the number of considered measurements N, radar probabilistic characteristics like P_D and P_{fa}, the size of gate, cell or template and so on.

5. Target tracking programs

The modern surveillance systems using radar as sensor require rapid and highly accurate data to be subsequently processed. Location, velocity, maneuver and possible identification of each target of interest can be provided by radar data processing with accuracy and reliability greater than that available from single-look radar report. Furthermore, radar data processing can enhance the signal-processing function by removing false detections caused, for example, by residual clutter.

This suit of programs reproduces the radar data processing algorithms, which allow the formation of estimated target states on the basis of incoming measurements provided by radar sensors.

Advanced multitarget/ multisensor/ multiplatform tracking algorithms have to possess the following characteristics:

- Tracking hundreds of targets;

- Work successfully with potentially long revisit rates (several seconds);
- Continuous tracks of weaker targets at lower SNRs (low value of P_D);
- Continuous tracks of closely-spaced targets;
- Creates common fused tactical picture of surveillance volume of several sensors.

Several estimation procedures are implemented in the proposed Matlab tool. They use $\alpha - \beta$, $\alpha - \beta - \gamma$, Kalman and extended Kalman filters in different realizations. All these procedures are intended to improve estimation accuracy. Note, that only in Europe more than 30 different trackers are currently in use.[4] Some of them use the same filters but work with different coordinate systems (polar, orthogonal) or the state vectors have different length. Most of these filters are available for use in the Matlab subroutine library.

Classically, plots are associated to the potential tracks using the "nearest-neighbor" algorithm. Wrong nearest-neighbor assignments, however, cause tracking filter divergence. Such is the case when there are false alarms in the target gate or in the case of closely spaced targets.

The first of these cases can be resolved using the probabilistic data association (PDA) algorithm.[1] This is a basic algorithm for plot-to-track association, which uses all measurements in the target gate. PDA allows more than one measurement to be associated to a track, each with a different probability and corresponding weight, according to its distance to the target prediction. The PDA filter is very simple and robust against false alarms.

The Joint Probabilistic Data Association (JPDA) algorithm is another advanced technique, implemented as a tracking algorithm in the Matlab tool. This algorithm resolves the case of closely spaced targets with common measurements. In this case measurement to track association for one track cannot be performed independently of other tracks in the cluster (cluster is a set of closely spaced tracks). *Joint* means that all possible measurement track combinations have to be evaluated. Furthermore, the track state vector update must, in principle, be done also jointly. Through appropriate approximations in the JPDA algorithm, however, the latter may not be necessary. Still, the complexity of JPDA grows exponentially with the number of tracks and measurements involved in the resolution situation. The advantage of JPDA is that, even in resolution situations, the track quality can be maintained at a high level. Several modification of this algorithm have been realized and estimated.[7,8]

The Interacting Multiple Models (IMM) filter is a robust filter, used for tracking of maneuvering targets. It assumes that a target is in one of a number of modes of movement, each of which may be modeled by its own equations of motion. This approach uses several filters. Every filter corresponds to a mode of movement of the target. All filters process each measurement. The particular filter innovation and the probability of holding target in (or moving target to) this mode define the weight of particular filter estimate on the common estimate. In the next interaction step, the information from all particular filters is combined and fed back into the filters. The choice of filters and suitability of their parameters remains a difficult problem to solve. It is obvious that robustness of IMM filter is achieved at the expense of estimate accuracy. For example, if a filter matches exactly with target motion mode, its estimate is deteriorated by influence of the other filters, which give poorer estimates. Another disadvantage of the IMM filters is the increased computational complexity. The IMM filter may also be used in conjunction with PDA filter and JPDA filter.[1,6] The researchers have on hand several versions of the described algorithms in the Matlab tool library.

6. Statistical estimation and visualization programs

The input data simulation program works with real life data (received by radars) or simulates the movement of targets and calculates the values of measurements. The second case is used for Monte Carlo estimation of the algorithms. Sufficient number of statistically independent trials is performed in order to achieve a significant sample of output values from which reliable statistics can be estimated. The estimates are compared with reference values of the tracks and models. The accuracy and detail of every model may vary from a coarse functional description of the system to a very accurate one, according to the purpose of the simulation and required accuracy of the results.

The visualization of results can be achieved by means of power Matlab graphic output. The next two examples demonstrate the capabilities of the presented tool.

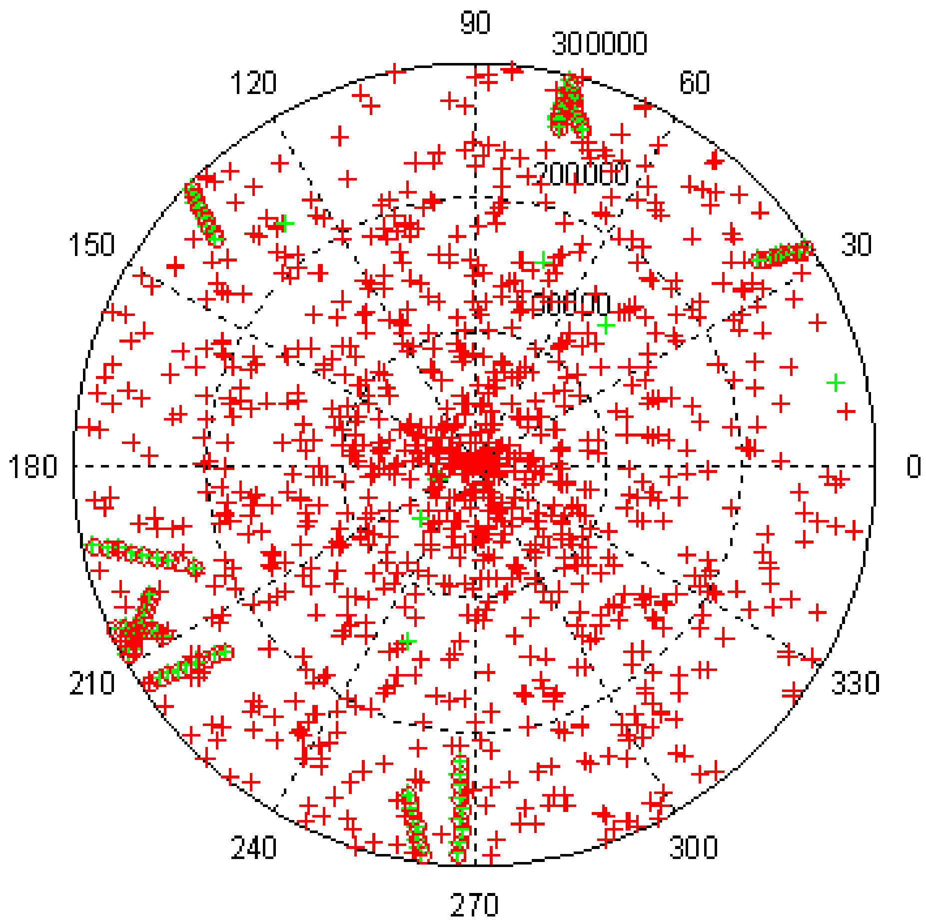

Figure 3: The test scenario includes ten approaching targets with randomly generated trajectory parameters and noise

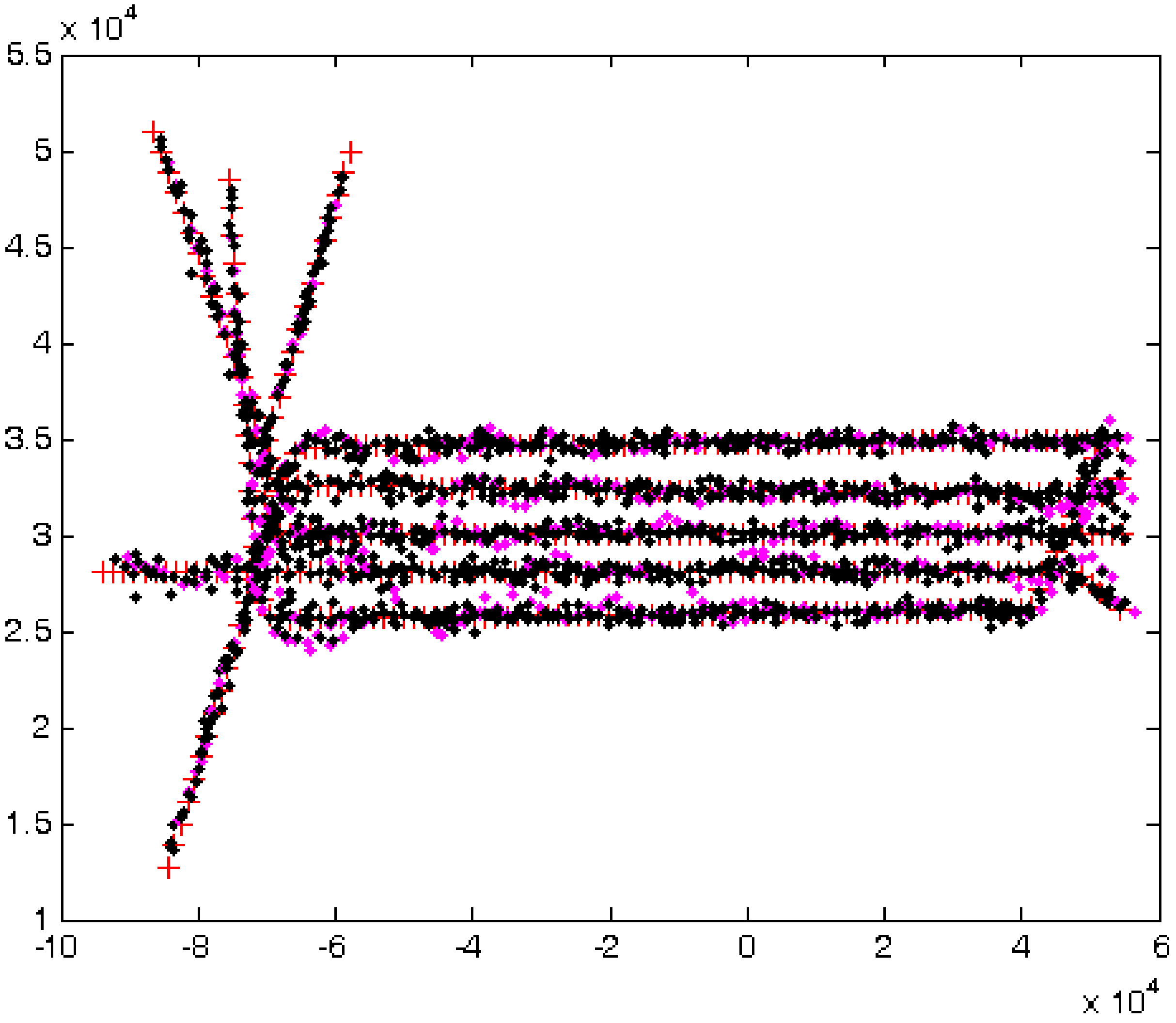

Figure 4: IMM JPDAF algorithm for the case of five-target scenario with $\beta V = 2$

Conclusion

A Matlab simulation tool is presented for multiple target tracking algorithm exploration and estimation. The built-in library of scenarios, models and algorithms provides an opportunity for easy implementation and testing of new versions of the track initiation and target tracking algorithms, comparative analysis and prompt estimation of their characteristics. This simulation tool protects us from surrogate target tracking system implementation.

Acknowledgment

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A MATLAB Tool for Development and Testing of Track Initiation and Multiple Target Tracking Algorithms

Kiril Alexiev

Keywords: radar simulation and modeling, automated design, synthesis, analysis tool

Abstract: Computer simulation is a valuable tool for design, analysis, and testing of complex systems whose behavior cannot be easily evaluated by means of analysis. The simulation includes input data generation, modeling of the system under examination and performance evaluation with proper visualization. The most complex algorithms can be easily coded by means of Matlab. The language of Matlab can be learnt quickly and, after that, the engineers can fully exploit its power with high productivity. The Matlab compiler translates *.m files into C code for real time implementation purposes. This article describes architecture and simulation tools for analysis and design of radar data processing systems, outlines the techniques used to generate the input data, and presents simulation results to analyze and evaluate the performance of various algorithms. The presented tool can be useful to practicing radar engineers for analysis and design purposes

full text

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I N M E M O R I A M

Professor Emil Atanasov Semerdjiev, D.Sc.

4 June 1956 – 30 August 2001

The talented scientist with many achievements is no longer among us. Bulgarian science lost one of its best experts in the field of multiple sensor data fusion.

Emil Semerdjiev received M.Sc. and Ph.D. degrees in avionics from Zhukovsky Air Force Engineering Academy, Moscow, Russia, in 1978 and 1981. In 1990 he received Doctor of Sciences degree in Radar Data Processing and Navigation from the “G.S. Rakovski” Defense and Staff College, Sofia, Bulgaria. From 1982 to 1990 he led the Mathematical Modeling and Computer Programming Department of the Institute of Special Electronics of Bulgarian Defense Industry. Since 1995 he was Head of Mathematical Methods for Sensor Information Processing Department at the Central Laboratory of Parallel Processing, Bulgarian Academy of Sciences. In parallel, he was teaching Information Processing in Air Traffic Control System at the Department of Air Transportation at the Technical University of Sofia. He led numerous research projects in the areas of multisensor data fusion, multiple target tracking, advanced parallel data processing, information modeling and learning, classification, information security and information warfare.

Prof. Semerdjiev was member of the International Academy of Information Processes and Technologies (IAIPT), Moscow, and the Armed Forces Communications and Electronics Association (AFCEA). He was among the creators of the International Association of Information Fusion (ISIF), which unites scientists from all over the world. He was the only representative from South East Europe at the first international forum FUSION'98 in Las Vegas, USA, conducted under the patronage of research centers of the US Army, Navy and Air Force and the biggest aircraft and shipyard companies. He was plenary speaker at the international forums in Las Vegas (1998), Paris (1999), Pitlochry, UK (2000), etc.

Professor Semerdjiev was author and co-author of over 100 papers published in international and Bulgarian journals. His work is of great value to all students, specialists and scientists, working in the field of sensor data processing.

LIST OF SELECTED PUBLICATIONS

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Emil Semerdjiev, *et.al.*, “Stochastic Sampling Algorithm for Tracking two Closely Spaced Maneuvering Objects in Clutter,” in *4th International Conference on Information Fusion* (Montreal, Canada: August 2001).

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CENTER OF EXCELLENCE
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In 2000, the Central Laboratory for Parallel Processing (CLPP) of the Bulgarian Academy of Sciences has been recognized as a Center of Excellence for learning, scholarship and research in the Eastern-European region. It has a leading position among the scientific institutions in Bulgaria in the fields of Computer Science, Scientific Computations, Supercomputing, Linguistic Modelling and Communications and Control. It performs research, consultations, projects and high quality education in its fields of interest. The activities of CLPP are oriented mainly to the creation and usage of advanced mathematical and computer technologies. During the last years fundamental results in different areas of the Theoretical Informatics and Scientific Computations were obtained. They were motivated by the developed by the Laboratory staff methods, algorithms and software as well as by introduced advanced information technology and computer facilities. Nowadays, the scientific strategy of CLPP consists in the development of multiprocessors systems and new high-performance algorithms for supercomputer (vector and/or parallel computers with shared and distributed memory, including here clusters of workstations) The results obtained by Laboratory staff are applied in various fields - ecology, engineering, computer technology, information systems, etc. The CLPP Computer Center maintains the information network of the Bulgarian Academy of Sciences. Annually, scientists from the Laboratory publish about 140 papers, and about 100 are in refereed international journals and proceedings of highly recognized international conferences.

History

The Central Laboratory for Parallel Processing was established in 1985. It was founded as Center for Informatics and Computer Technology (CICT) by group of scientists headed by acad. Blagovest Sendov who was the first Director of this institution. The main idea was to coordinate research in the field of Computer Science and Compute Technology done by scientists from Bulgarian Academy of Sciences, Bulgarian Universities and some Institutes belong to the Industry. In a short time many senior scientists as well as young mathematicians, engineers and computer science specialists were attached to the group and in only one year CICT took a leading position in Bulgaria in the field of Computer Science and Computer Technology. In 1996 CICT was renamed as Central Laboratory for Parallel Processing. CLPP has been, and still is, an active participant in a number of research and educational projects of the EU programs INCO-COPERNICUS, TEMPUS, PECO, GO EAST GO WEST, etc. as well as in NATO Scientific Programs. A lot of the scientists of the Laboratory has been on long-term specializations in USA, UK, Denmark, France, Germany, the Netherlands, etc. CLPP has organized many international conferences and workshops. Some of them are periodical as Parallel and Distributed Processing, Network Information Processing Systems,

Structure

CLPP is headed by a Director, Deputy Director and Scientific Secretary. General and scientific police of the Laboratory is formulated by the Directors Board including all heads of Departments and 24-members Scientific Council. Members of the Scientific Council are most of the full and associate professors of CLPP as well as some of most famous scientists in the fields of interest from other Institutes of the Bulgarian Academy of Sciences. CLPP consists of six Departments: Distributed Computing Systems and Networks, Parallel Algorithms, Scientific Computing, High-Performance Computer Architectures, Linguistic Modelling, Mathematical Methods for Sensor Information Processing and Computer Center. CLPP disposes of the following computer facilities: CRAY ORIGIN 2000, IBM 4381 with 65 local terminals, VAX 3100 with 24 terminals, at about 45 PC's AT 486 and 586, 18 SUN Microsystems workstations including one 4-processors stations, SGI-O2 station, HP 9000-C class. All of them are connected through a local LAN Ethernet to the International network.

Staff

The total staff of CLPP is 112 persons including one academician, one corresponding member of the Academy, six full professors, 24 associate professors, 39 research assistants and 20 specialists with university education. Every year students from Sofia University (Faculty of Mathematics and Informatics) and the Technical University of Sofia prepare their MSc theses in the Laboratory having as supervisors Senior Scientists from the staff. Permanently there are five-six PhD students in the field of Numerical Mathematics, Computer Science and Computer Technology and their applications. Most of the Senior Scientists of CLPP are affiliated with various national and international administrative organizations such as the National Research Fund, the Higher Evaluation Commission, the Executive Council of the Bulgarian Academy of Sciences, AMS, British Computer Society, ISCU, IMU, Int. Commission for Math. Education, Green Gross, etc. Many full and associate professors, as well as some of the research associates, lecture on their fields of interest in several Universities in Sofia, Plovdiv, Varna, Bourgas and Blagoevgrad for undergraduate, graduate and postgraduate students.

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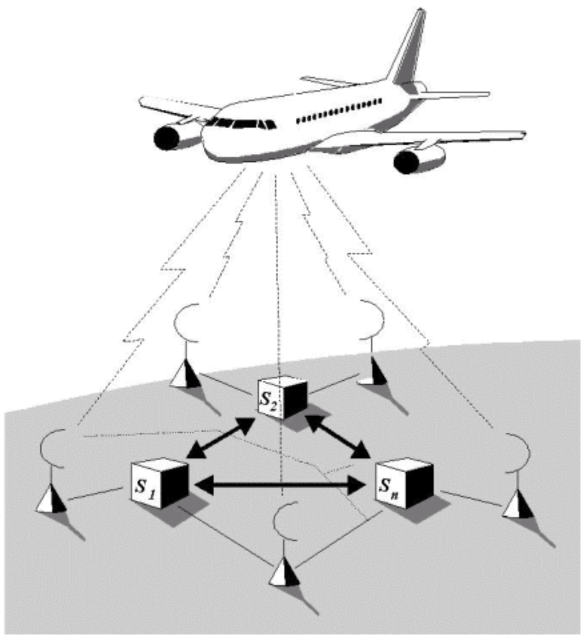

Objectives:

- Exchange of knowledge and expertise
- Solution of challenging problems
- Enhancement of research capabilities
- Education and training

Methodology:

- Research co-operation
- Visits exchange
- Conference organization

Actions:

- Study of advanced sensor's interfaces
- Design of new methods and algorithms
- Development of software libraries
- Results dissemination

Partners:

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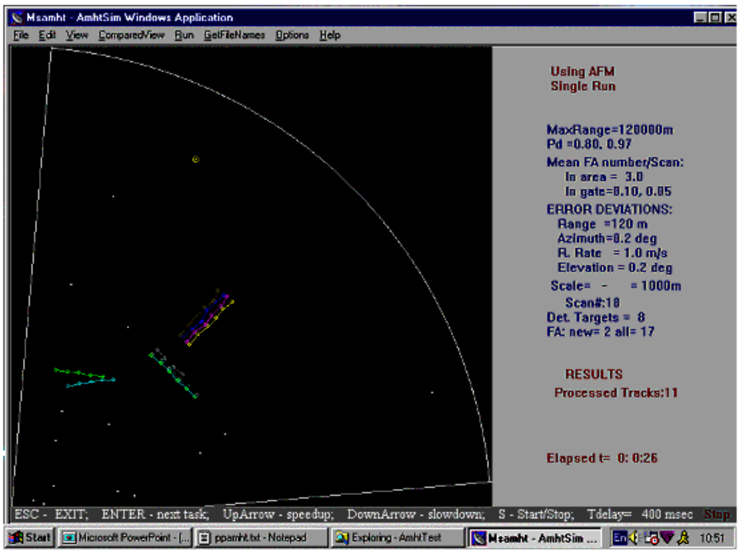
- University of Bristol, UK;
- University of New Orleans, US;
- ONERA, France

Deliverables:

- Modern education
- Information center and Web site
- International conference
- Advanced algorithms and software



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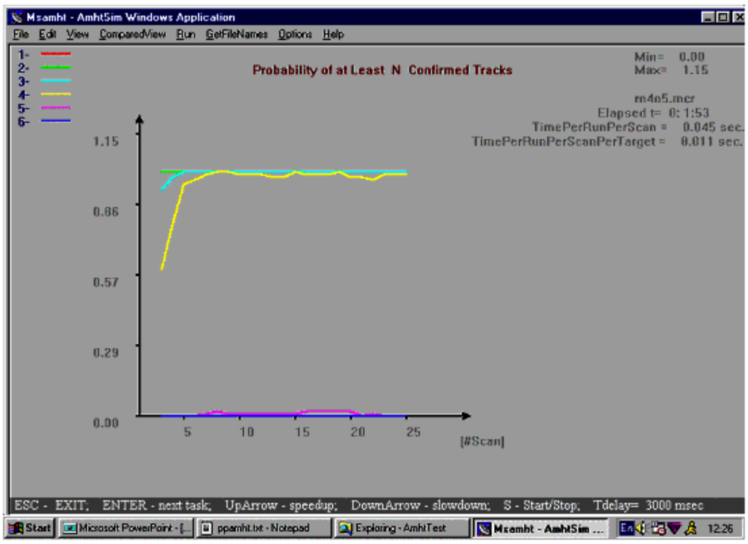
- Tracking in Clutter



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