DRIVING MECHANISMS OF THRUST SYSTEMS
Dynamics of accretionary, orogenic wedges

Tens of kilometres long displacements are reported for only a few thousands meters thick thrust sheets. With accepted values of frictional resistance, stresses necessary to push a thrust sheet on a horizontal or slightly up-inclined basal thrust exceed the failure strength of intact rocks. Accordingly, hinterlands should deform and crush before fronts of thrust sheets would move. This mechanical paradox has triggered formulation of a few working hypotheses. The question is still not fully resolved, possibly because all envisioned mechanisms are involved on a scale-dependent importance.

Concepts and definitions

Thrust systems are common in converging settings, resulting from compressional deformation of rocks once the shear strength along weak layers or planes is exceeded. In convergent plate boundaries, thrusts cutting through accretionary wedges are virtually splays of the subduction plane. This mechanism, where the accretionary hanging wall is deformed while the subducting footwall plate descends undeformed, is typical of thin-skinned tectonics depicted in mountain systems. Therefore, mechanics developed from accretionary wedges are often applied to foreland fold-and-thrust belts and even to orogenic belts (see lecture on thrust systems).

Geometry

A thrust sheet (or **nappe**; distinguish from **fold nappe** = large recumbent fold) is an allochthonous unit that has been transported and lies almost horizontally, away from its original position, on an autochthonous substratum. The bottom boundary is an originally shallow-dipping thrust fault or shear zone along which most of the displacement has taken place. The frontal part in the direction of movement is the **leading edge**. The original site from where the thrust sheet comes is the **root**. Root zones may be unidentified or have disappeared (e.g. a subducted plate); the nappe is then **rootless**. Thrust sheets are usually recognized from older rocks on top of younger rocks.

> Note
>
> Sheet and nappe are geometrical descriptions that do not imply compression or extension processes; out of place, allochthonous units can be either type of structure.

Other geometrical definitions are assumed to be known (see lecture on Thrust systems).

Symbols

Geometry

h = thickness of a thrust sheet

w = width of the thrust sheet, measured parallel to the slip direction

L = length of the thrust sheet, measured parallel to the slip direction

α = surface slope

β = basal décollement slope

$\theta = \alpha + \beta$ = angle of taper

ψ = angle between principal stress and a plane

Forces and stresses

F_N = normal force on the basal décollement

F_S = shear force along the basal décollement

F_F = frictional force along the décollement

σ = horizontal tectonic stress
σ_v = normal stress
P_f = pore fluid pressure
τ = shear stress on the basal décollement
σ^* = effective stress
$\lambda = P_f / h\rho g$ = ratio of pore fluid pressure to lithostatic pressure

Parameters and constants

ρ = bulk density
k = a measure of the yield strength of the wedge
t = time
v = décollement-parallel velocity
τ_0 = cohesive shear strength
ϕ = angle of internal friction
μ = friction coefficient (0.6 to 1 for most rocks)
$\dot{e}$ = erosion rate
g = acceleration of gravity

Thrust mechanisms

Paradox of large thrust sheets

An important question facing students of orogenic systems is the relationship between geometric characteristics of coherent thrust sheets and the mechanisms by which they were initiated and transported. Two distinct mechanisms have been discussed, corresponding to the application of (1) surface forces (**push theory**) or (2) body forces (**gravity gliding**). Both mechanisms can be reduced to the elementary problem of pushing or sliding a rigid, rectangular block over a rigid base.

Push theory

Concept

The push theory dates back to the late 1800´s, when thrust sheets recognised in mountain belts were considered to be pushed from behind by regional compressive forces. In that case, the sheet is driven forward by stress transmitted through the thrust sheet from the rear. However, to push large rock sheets seems to require forces that the rocks are unable to withstand.

Model

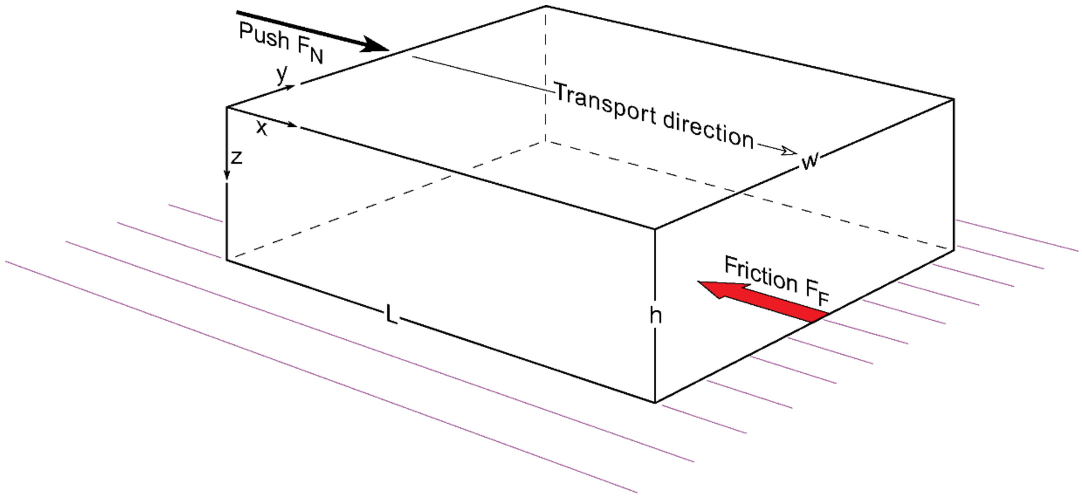
The model involves a force F_N pushing on the rear edge of a rectangular block to move it on a horizontal surface.

The block has thickness z, width w and length L parallel to the motion.
Its weight per unit volume is ρgz ; this is the normal stress on the flat base.
The coefficient of sliding friction at the base is μ.
The frictional force F_F that resists the motion of the block is proportional to the normal force F_N across the base:

$$F_F = \mu . F_N \quad (1)$$

$$F_F = \mu \left(F_{N/unit\_area}\right)\left(area_{base}\right)$$

$$F_F = \mu\left(\rho gh\right)\left(w.L\right) \quad (2)$$



Since the pushing force is applied across the back vertical face of the block, the stress on that face is the driving force per unit area:

$$\sigma^{driving}_{compression} = F_F / Lh = \mu \rho g w$$

If stress exceeds the strength of the block, the block fails before the frictional resistance is overcome. This equation allows calculating the maximum length a thrust sheet may have to move coherently.

Exercise

Calculate the maximum possible dimension L of a thrust sheet in the direction of thrusting with average values:

μ= 0.6 ρ= 2500kg.m^{-3} σ= 250 MPa (1 Pa = 1 Newton.m^{-2}, 1N = 1 kg.m.s^{-2}).

Answer: 17 km

Another formulation

Let σ_{xx} the normal stress applied on the back edge of the block.
τ_{zx} the shear stress along its base.
σ_{zz} the normal stress across the base.

The equation of equilibrium of the forces in the x direction is, in two-dimensions:

$$\int_0^z \sigma_{xx} dz - \int_0^x \tau_{zx} dx = 0$$

The Coulomb law of failure is:

$$\tau = \tau_0 + \sigma \tan\phi$$

where τ_0 is the shear strength (cohesion) and ϕ the angle of internal friction (see lecture on Faulting).

The Coulomb law of failure is formulated for normal and tangential stresses on a fault plane and can be reformulated for principal stresses (see lecture Faulting). The relationship between the maximum and minimum principal stresses at failure is linear with the form (see lecture Faulting):

$$\sigma_1 = a + b\sigma_3$$

where

$$a = 2\tau_0\sqrt{b} \text{ and } b = \frac{1+\sin\phi}{1-\sin\phi}$$

Assuming that σ_{xx} is the greatest (i.e. σ_1) and σ_{zz} the smallest stress (i.e. $\sigma_3 = \rho g z$), the vertical integral of σ_{xx} in the equation of equilibrium of horizontal forces is:

$$\int_0^z \sigma_{xx} dz = \int_0^z (a + b\sigma_{zz}) dz = az + \frac{b\rho g z^2}{2}$$

Assuming friction so that:

$$\tau_{zx} = \sigma_{zz} \cdot \tan\phi = \rho g z \cdot \tan\phi ,$$

Integrating τ_{zx} horizontally and substituting the integrals of σ_{xx} and τ_{zx} into the equation of equilibrium yields:

$$az + \frac{b\rho g z^2}{2} - \rho g z \cdot x \cdot \tan\phi = 0$$

which, solved for x, gives:

$$x = \frac{a}{\rho g.\tan\phi} + \frac{b}{2\tan\phi} z$$

This equation that defines the maximum length of a block pushed from the back is made of two terms: the first is a constant, the second is proportional to the thickness of the block, reflecting that the strength of rocks increases with depth.

Supplying experimental constants in these equations shows that pushing a block along a horizontal surface is a mechanical impossibility if the block is longer than few kilometres. The length is ca 3 times the thickness of the block if it has similar material parameters as the base.

Some authors argue that such a model is flawed because it is too simplified. In terms of geological observation, the three most critical points are that (1) the model thrust sheets move rigidly, (2) the basal slip zone is a single, straight frictional plane and (3) basal slip occurs simultaneously over the entire fault surface. In natural thrust sheets, folds and fractures are ubiquitous and total displacement results from the addition of many small slip events on limited parts of the basal thrust fault. Besides, friction may be overestimated: The resistance to motion of thrust sheets can be determined by yield stress rather than frictional stress. Fault planes often exploit mechanically weak layers such as evaporites and mudstones. The yield stress of such rocks is one to 2 orders of magnitude less than yield stresses of other rocks. Thus, the resistance to motion is significantly less in thrust faults occupying weak beds than in other rocks.

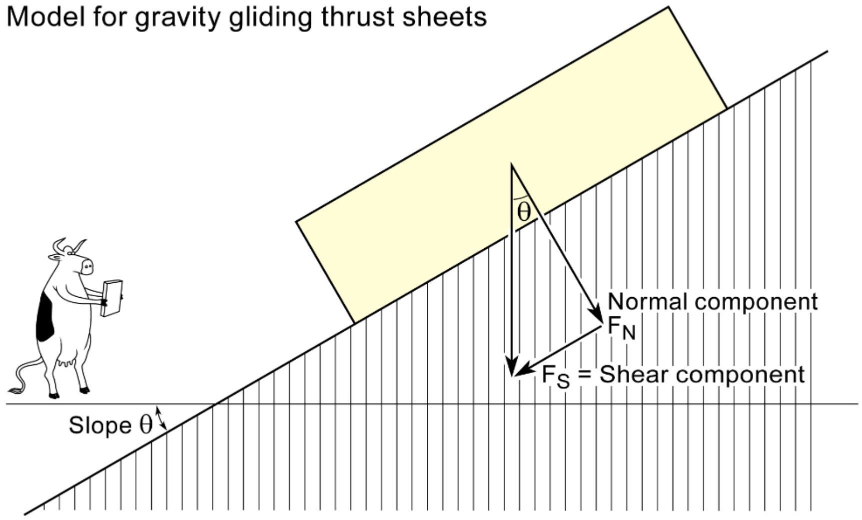
Summary

The force that needs to be applied to the back of a thrust sheet to overcome the frictional resistance along its base is larger than the strength of the rock being pushed. Hence, the rock of a thrust sheet longer than few kilometres should fail and produce internal deformation rather than slide.

Gravity Gliding

Concept

The alternative explanation is that thrust sheets are not pushed but slide down a plane inclined towards the foreland under the action of gravity (like downslope landslides or olistostrome emplacement). In this case, a body force is substituted to the pushing surface force. Gravity models solve the question of stress transmitted through the thrust sheet from the rear because gravitational forces act independently on every point in the thrust sheet.



Model

Gravity gliding occurs if the shear force F_S provided by the force of gravity is at least equal to the frictional resistance F_F on the basal thrust or décollement. If the resistance is known, the slope θ necessary to cause a thrust sheet to slide can be determined through a simple trigonometric construction:

$$\tan\theta = F_S / F_N$$

Exercise
Calculate the slope θ using the given equation (1) and assuming a coefficient of friction μ = 0.6. Experimental data for nearly all rocks yield values 0.8<μ<0.9, make the same calculation. Then calculate the topographic height of the inner zone of the Alps required to push the Pre-Alpine klippen ca. 100 km from their source.

31°, 51.5 and c 30 km; not realistic.

Another formulation

Consider a column of unit cross-sectional area perpendicular to the base of a block of thickness z, resting on a plane of inclination θ.

The weight of the column is	$\rho g z$
The normal stress across the base is	$\sigma_N = \rho g z \cos\theta$
The shear stress along the base is	$\tau = \rho g z \sin\theta$

From which:	$\frac{\tau}{\sigma} = \tan\theta$
For the block to slide it is necessary that	$\frac{\tau}{\sigma} = \tan\phi$

Which implies that sliding will occur for a tilt angle $\theta = \phi$ known experimentally to be not far from 30°.

Results are unrealistic, meeting the same mechanical difficulty as with the push force.

Problems with the application of gravity gliding

(1) Master décollements and most thrusts recognised by seismic reflection dip towards, not down-slope orogen hinterlands.
(2) The angular relationship of bedding-thrust cut-offs is incompatible with gravity gliding.
(3) Fold-and-thrust belts are laterally extensive, which is difficult to reconcile with gravity gliding.
(5) The regions of tectonic denudation, where thrust sheets originated, are rarely observed.
(6) The penetrative deformation of sedimentary cover in the internal parts of the orogen cannot be explained by gravity gliding.

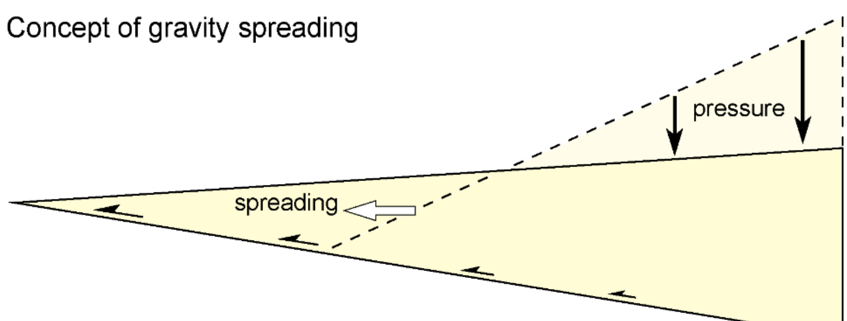
Summary

Gravity gliding is not suitable for movement of thrust sheets on a regional scale although it can produce smaller, localised thrust sequences.

Gravity Spreading

Concept

Gravity spreading is a variation of the gravity sliding theory. Gravity spreading considers the orogenic hinterland to be a rising hot, hence ductile mobile mass. The difference in elevation produces, at the same level, a higher pressure beneath the hinterland than beneath the neighbouring low lands. This difference in **gravitational potential energy** may cause the hinterland to collapse and spread outwards to attain equilibrium. Where resistance to sliding is overcome, spreading pushes the adjacent sediments into thrust sheets that develop serially towards the foreland, allowing thrust sheets to slip up-dip as long as the overall mass movement is from higher toward lower elevations.



Shortening at the front of the spreading sheet balances collapse-related extension in the hinterland. Again gravity is the driving force. The topographic slope rather than the dip of the basal décollement is the geometric control for thrust movement. In addition, spreading forces continuously diminish while the hinterland thins at the same time as thrusting thickens the neighbouring regions, which reduces the pressure difference.

Shortcomings with the application of gravity spreading

(1) Most thrust sheets have been thrust uphill.
(2) Not all regions of high elevation exhibit gravity sliding or spreading.
(3) The cumulative amount of shortening and displacement is greatest in the interior of most hanging blocks rather than at the toe as these models predict.

Because of such problems, it is generally accepted today that gravity alone is insufficient to explain the motion of hanging block wedges.

Fluid pressure

Concept

Under similar mechanical conditions, it is difficult to explain the large thrust sheets recognised by geologists. Authors working on the problem proposed that fluid pressure in rocks reduces frictional resistance.

Model

Frictional resistance is proportional to the normal stress on a surface; therefore, reducing normal stress reduces frictional resistance. This principle is expressed with the modified Coulomb failure criterion:

$$\tau = \tau_0 + \mu\sigma_N (1-\lambda) \tag{4}$$

where λ is the ratio of pore pressure to normal stress. Since pore pressure diminishes the frictional resistance, then it reduces the stress levels required for slip. If pore pressure approaches lithostatic load, then $\lambda \approx 1$ and the frictional resistance approaches zero. In that case, the possible width of the thrust sheet becomes unlimited and thrust sheets can move under push lower than the failure strength of rocks.

Assuming that the basal thrust fault has no or a negligible shear strength, the value of τ_{zx} at which sliding will occur is given by the reduced law of frictional sliding:

$$\tau_{zx} = \sigma_{zz} \tan\phi$$

The normal stress at the base of the block is:

$$\sigma_{zz} = \rho g z$$

With these substitutions:

$$\int_0^x \tau_{zx} dx = \int_0^x \rho g z \cdot \tan\phi \, dx$$

$$= \rho g z . x . \tan\phi$$

Introducing fluid pressure:

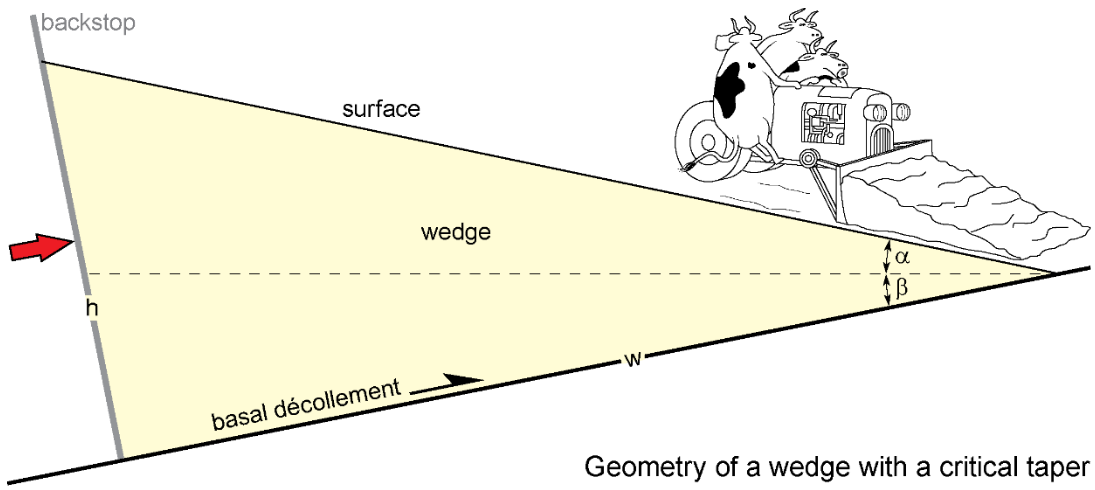
$$= \rho g z . (1-\lambda) x . \tan\phi$$

Summary

Field observations suggest that fluid overpressure cannot be a general mechanism. For example, some thrust sheets were emplaced over erosion surfaces. Conversely, very high pore fluid pressures were measured in some drills. The mechanisms may play some role, but does not provide answers for the disturbing question of large thrust sheets.

Wedged thrust systems

A crustal wedge (accretionary, orogenic and fold-and-thrust wedges) has in profile a horizontally elongated triangular shape. The upper side is topography; the lower side is a fault; the lateral side is the **backstop** (in compression) or **breakaway** (in extension) surface. The acute frontal angle is the **taper**. In three-dimensions, crustal wedges are belts in which both compressional and extensional structures are found. This structural association led to disputes on the respective roles of tectonic and gravity forces.



Geometry of a wedge with a critical taper

Concept

Wedge models consider that thrust-belt mechanics is analogous to pushing sand or snow uphill in front of a moving bulldozer. The displaced material progressively thickens at its back where it is pushed and thus forms a wedge shape. The wedge grows through internal deformation until a **critical taper** is reached; at that point, the material is able to slide stably along its base, i.e. its geometry remains self-similar, even as more material is incorporated into the growing wedge as it moves forward. The size of the taper is defined by the angle θ, which is the sum of the upper surface slope α, towards the foreland, and the dip of the décollement β (or basal slope), towards the hinterland.

$$\theta = \alpha + \beta$$

Mechanics of a bulldozer wedge

At t = 0 a bulldozer begins scraping up at a speed v a layer of dry sand of thickness h_0

The mass flux per unit length along strike into the **toe** of the wedge is: $\rho h_0 v$

The area A of the wedge is given by:

$$A = \frac{1}{2} wh$$

The height h of the wedge is related to its width w by:

$$h = w \tan\theta$$

so that the area is

$$A = \frac{1}{2} w^2 \tan\theta$$

The mass conservation law describes the growth of the wedge with time:

$$\frac{d}{dt}(\rho A) = \frac{d}{dt}\left(\frac{1}{2}\rho.w^2 \tan\theta\right) = \rho h_0 v$$

since θ does not change with time and remembering that

$$\frac{d\,y(w(t))}{dt} = \frac{dy}{dw}\frac{dw}{dt} \Rightarrow \frac{d(w(t))^2}{dt} = 2w\frac{dw}{dt}$$

The mass conservation equation can be reduced to:

$$w\frac{dw}{dt} = \frac{h_0 v}{\tan\theta}$$

This ordinary differential equation has a solution:

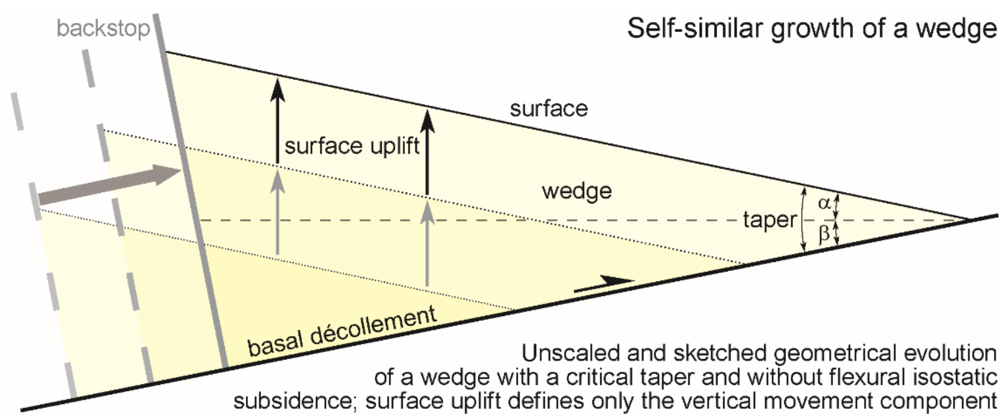
$$w = \left(\frac{2h_0.v.t}{\tan\theta}\right)^{1/2} \approx \left(\frac{2h_0.v.t}{\theta}\right)^{1/2}$$

Reminder to understand solution:

$$\frac{d(y(t))^{1/2}}{dt} = \frac{1}{(y(t))^{1/2}}\frac{1}{2}\frac{dy}{dt} \Leftrightarrow (y(t))^{1/2}\frac{d(y(t))^{1/2}}{dt} = \frac{1}{2}\frac{dy}{dt} \Rightarrow \frac{1}{2}\frac{dy}{dt} = \frac{h_0 v}{\tan\theta} \Rightarrow y = 2\frac{h_0 v}{\tan\theta}t$$

The final approximation is valid for a narrow taper. Angles are then measured in radians.

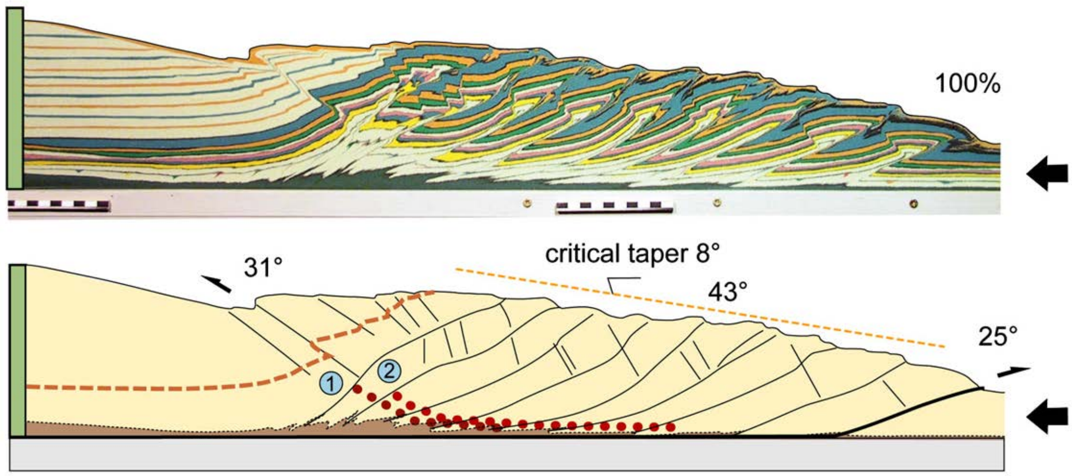
Because the critical taper is governed only by the unvarying strength of the sand and the basal friction, both the width and the height of a bulldozer wedge grow like $t^{1/2}$. The growth is self-similar in the sense that the wedge at time 2t is indistinguishable from the wedge at time t, magnified $2^{1/2}$ times.



The analogy links the topography of orogenic belts to the rheology of the crust and includes the effects of body forces and externally applied tectonic forces.

Sandbox simulations

Experiments with layers of sand (analogue to Mohr-Coulomb material) laid down on an inclined sheet pulled underneath the rear side simulate the transport of material from the "foreland" towards the backstop. A stable wedge of sand is then formed. Most of the deformation takes place at the toe of the wedge so that folds and faults generally form "in sequence". Repeated tests show that the wedge shape depends on the strength of the deforming material and the frictional resistance of the décollement: the weaker the décollement, the smaller the wedge angle.



Analogue model of thrust wedge by Konstantinovskaya & Malavieille 2011 *Tectonophysics* **502(3-4)**, 336-350

Analogue experiments hit conceptual limitations: Assume a constant wedge slope of 5% (5 gradians) while the wedge tip remains at sea level; self-similar growth to a 200 km long wedge raises the top of the wedge at elevations of ca 10 km. This is geologically inadmissible. Several reasons have been put forward to explain the excessive elevation. First, isostatic subsidence is not implemented. Second, sand has a constant strength throughout small-scale models, while the frictional strength of rocks varies with depth. Finally, erosion limits the height of mountain topography. These three parameters and different rheologies are more efficiently implemented into finite element simulations.

History of ideas

Generalisation from accretionary prisms to fold-and-thrust belts and orogenic wedges is based on several consistent characteristics:
(1) A basal décollement, below which there is no deformation, dips towards the hinterland
(2) Large horizontal shortening has occurred above the basal décollement, producing a wedge shaped hanging wall that tapers towards the foreland, i.e. the topographic surface slopes downward towards the foreland.
(3) The wedge is confined at its rear by a relatively rigid buttress or backstop (orogen or volcanic arc).

The first applications of wedge dynamics to geology were comparisons of the movement of orogenic wedges with dynamics of continental glaciers. Shear stress at the base is given by:

$$\tau = (\rho g h) \sin \alpha \qquad (3)$$

In this analogy, the energy for forward movement is mostly provided by the gravitational potential of the "ice sheet", hence depends on both the topographic slope and the slope of the upper surface. This comparison favours gravitational forces and involves a viscous material flowing under its own weight. The horizontal stress provided by gravity drives internal flow and downslope basal sliding but cannot produce internal shortening. Such wedges should be extremely weak and do not transmit longitudinal forces (i.e. push from behind). It is worth noting that the wedge-shape exists before wedge-flow with no consideration of where it comes from.

For an orogenic wedge, compression should be the main driving force. Models that take external, horizontal forces into consideration assumed that the wedge behaves plastically and is translated over an incompetent, very weak décollement layer. The "Coulomb" rheology of the wedge can transmit longitudinal stresses and compression can be sufficient to overcome the yield strength of the basal, weak décollement; the related equation is:

$$\tau = (\rho g h)\sin\alpha + 2K\theta \qquad (4)$$

It is equation (3) to which is added a push term depending on both the brittle strength (K) of the rock in the wedge and the angle of taper. Horizontal compression causes thickening of the rear parts of the wedge, which increases body forces. In this expression, tectonic transport adds to gravity spreading.

Introducing the role of pore fluid pressure, equation (4) becomes:

$$\tau = (\rho g h)\sin\alpha + (1-\lambda)K\rho g h\theta \qquad (5)$$

in which K is a function of the internal strength of the wedge and an inverse function of the coefficient of friction at the base. It is a dimensionless parameter, usually of the order of 2.
Setting up equality between the driving forces and forces of resistance leads to an equation that accounts for the basic wedge shape of a thrust sheet. The approximate equation for the surface slope of wedges is:

$$\alpha = \left[(1-\lambda_b)\mu_b - (1-\lambda_i)K\beta\right] \Big/ \left[(1-\lambda_i)K + \left(1-\{\rho_{water}/\rho_{rock}\}\right)\right] \qquad (6)$$

where λ_b and λ_i are the ratios of pore fluid pressure P_F to overburden along the basal décollement and internal to the wedge, respectively.

Model - Critical taper theory

The critical-taper theory describes the dynamic relations between the wedge geometry and the acting stresses (from both a push from the rear and gravitational potential energy) within the wedge.

Assumptions

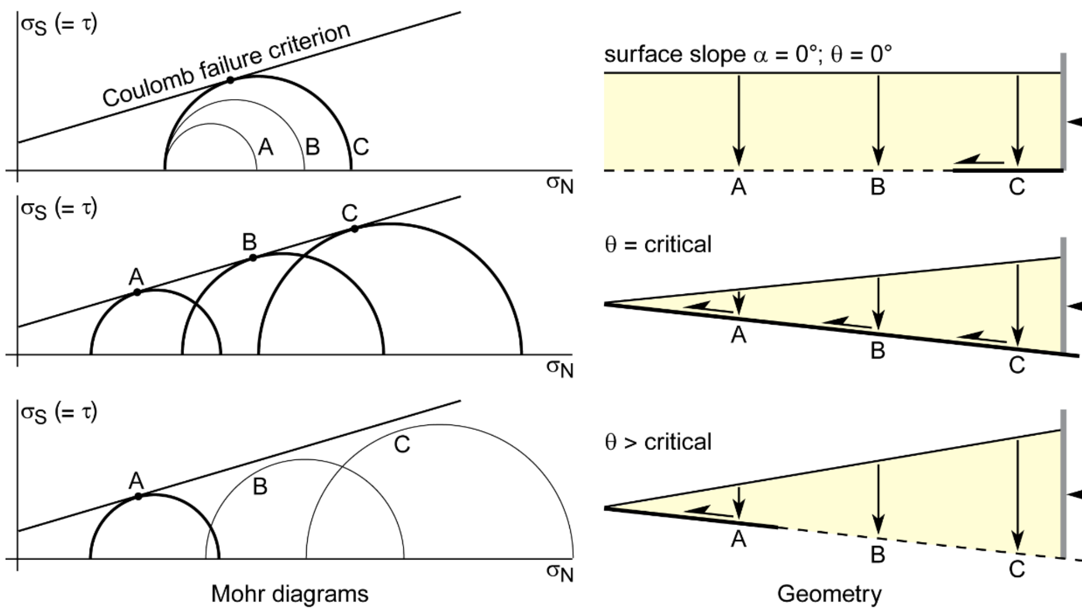
Assuming that acceleration is negligible, the equation of equilibrium requires that forces resisting motion balance the forces that push the wedge.
In modelling the evolution of a wedge, the other basic assumptions are:

- Rocks in the thrust sheet have uniform properties. Upper-crustal thrust systems are formed under brittle conditions and exhibit a plastic, Coulomb-type rheology.
- Rocks are everywhere just on the verge of shear failure, which means that the stress in the thrust sheet is everywhere as large as possible. Therefore, the strength of the rocks is a very important parameter, which also controls how much stress the material can transmit. Cohesion can be neglected because it is small compared to other tectonic and gravitational forces.
- The driving force (the tectonic horizontal push) on the vertical back-side balances the frictional resistance to sliding on décollement surface ahead of this back-side. A major part of the resistance to sliding derives from the frictional shear stress on the décollement plane multiplied by its area. A smaller part of the resistance to motion derives from the component of vertical overburden stress parallel to the slip direction on the décollement $(\rho g h \sin\beta)$. The magnitude of this component increases with increasing dip of the décollement.
- The surface slope also creates a driving force because at any given horizontal level in the wedge the overburden stress $(\rho g h)$ at a given point at this level depth is greater than the overburden stress at a point at the same level, ahead in the thinner wedge. However, the surface slope is small and this component is negligible.

Stress argument

A Mohr circle whose diameter is equal to the differential stress $(\sigma_1 - \sigma_3)$ represents the state of stress in Coulomb material. To get sliding, the force applied from the rear must reach the frictional force on the basal décollement. The horizontal stress ($\sigma_{xx} = \sigma_1$) is added by the push from the rear, so we can take the vertical stress ($\sigma_{yy} = \sigma_3$) as the minimum principal stress. Both the pressure and the vertical stress at the base of a wedge are therefore highest at the rear, where the décollement is deepest. Since the tectonic horizontal stress is nearly constant for a zero décollement strength, the size of the Mohr circle decreases from rear to front (i.e. the differential stress ($\sigma_1 - \sigma_3$) decreases from the rear to the front of the wedge).

Consequences are illustrated by considering three wedge shapes: (1) with a flat "décollement" surface, (2) with the critical taper and (3) with a taper larger than the critical value. In these three geometries, a point in the back, mid-way and in the frontal part of the wedge are considered. The position of the Mohr circle along the normal stress axis is determined by the pressure, which is proportional to depth.



Conceptual representation of thrusting location according to the critical taper

Shape 1: The angle of taper is zero ($\theta = 0$). The depth to the décollement is the same everywhere, and therefore σ_{yy} is constant. The minimum stress for all Mohr circles is the same, so they are all left-aligned. $\sigma_1 - \sigma_3$ is highest in the back and smallest in front. First failure will occur at the rear of the wedge. The back of the wedge then gets thicker by thrusting while the front remains undeformed; the wedge taper θ increases.

Shape 2: With a critical taper ($\theta = \theta_{crit}$), the back Mohr circle is larger but shifted to the right of the frontal one because it is deeper (higher pressure). In fact, all Mohr circles simultaneously touch the failure envelope. The whole décollement can be activated.

Shape 3. The angle of taper is larger than the critical value ($\theta > \theta_{crit}$). Now the pressure increase from back to front is so large that the back Mohr circle does not touch the failure envelope, even though it has the largest differential stress. In this case only the front part of the décollement fails. That means that the wedge is stretched and the angle of taper decreases.

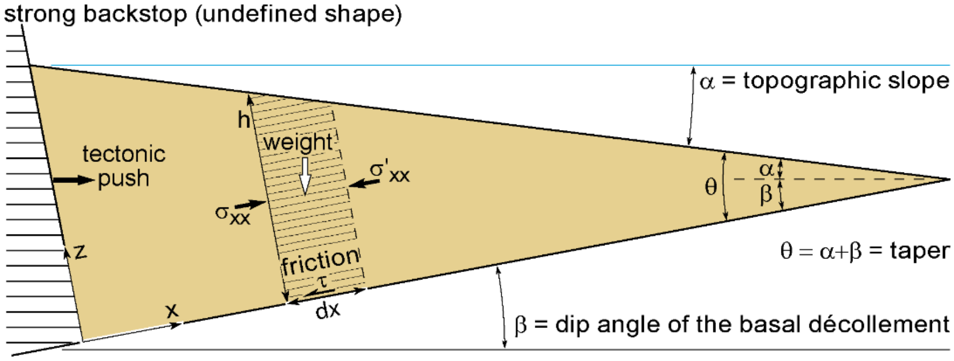
In summary, if the wedge is too shallow (shape 1), thrusting at the rear thickens the wedge and increases the taper; if the wedge is too steep (shape 3), thrusting at the front decreases the taper; the wedge is in balance at the critical angle of taper. In conclusion, wedges develop towards the critical angle of taper, which is related to the failure properties of the material and the friction along the basal fault.

Dynamics of wedges

The Coulomb wedge seems appropriate for accretionary wedges and for upper crustal fold-and-thrust belt.

Analysis

Assuming brittle behaviour, the dynamics of a wedge are controlled by four parameters: the two angles of the critical taper α and β and the coefficient of internal friction of the basal décollement μ_b and of the wedge itself μ_{wedge}. A change of one or more of these factors generates internal deformation of the wedge caused by internal stress release to regain or to maintain stability. The gravitational potential energy, due to elevation of the hinterland, creates both horizontal and vertical stresses.



Geometry and parameters of a modelled accretionary wedge

Analytical formulation

The Cartesian coordinates are x along the base of the wedge and z orthogonal to x.
We consider the balance of forces on a small segment of the wedge between x and x+dx.
The gravitational body force as an x-component per unit length is:

$$F_{grav} = -\rho g z . \sin\beta . dx$$

with z the local wedge thickness. The contribution from potentially overlying water or air is ignored. The effect of pore pressure is also ignored and can be easily implemented in the following equations.
The force on the sidewalls at x and x+dx is:

$$F_{comp} = \int_0^z \left[\sigma_{xx}(x,z) - \sigma_{xx}(x+dx,z)\right] dz$$

This force is compressive, pushing in the x direction because the x face is larger than the x+dx one.
The surface force exerted on the base results from gravity acting on the surface and from the horizontal compression acting from the rear of the wedge:

$$F_{base} = \left(\tau + \sigma_N \sin\theta\right) dx$$

We assume that the base is governed by the frictional sliding condition without cohesion:

$$\tau = \mu_{base}\sigma_N$$

The basal force then reduces to

$$F_{base} = \sigma_N \left(\mu_{base} + \sin\theta\right) dx$$

The balance condition is: $F_{grav} + F_{comp} + F_{base} = 0$

F_{grav} and F_{comp} act in the x direction, whereas F_{base} acts against them, in the –x direction. Taking the limit as $dx \rightarrow 0$ the balance conditions reduces to the exact solution:

$$\frac{d}{dx}\int_0^z \sigma_{xx}\, dz = -\left[\rho gz.\sin\beta + \sigma_N \left(\mu_{base} + \sin\theta\right)\right]$$

for small angles we employ the approximations:

$$\sin\alpha \approx \alpha \qquad \sin\theta \approx \theta \qquad \cos\theta \approx 1 \qquad \sigma_N \approx \rho gz$$

so that:

$$\frac{d}{dx}\int_0^z \sigma_{xx}\, dz \approx -\rho gz\left(\mu_{base} + \beta\right)$$

The failure criterion for dry sand with an angle of internal friction ϕ is:

$$\sigma_3/\sigma_1 = (1+\sin\phi)/(1-\sin\phi)$$

In a narrow taper, the principal stresses are approximately horizontal and vertical that is:

$$\sigma_3 \approx \sigma_{zz} \approx \rho gz \qquad \text{and} \qquad \sigma_1 \approx \sigma_{xx} \approx \rho gz\frac{1+\sin\phi}{1-\sin\phi}$$

Using $dz/dx \approx \theta$:

$$\frac{d}{dx}\int_0^z \sigma_{xx}\, dz \approx -\rho gz\left(\frac{1+\sin\phi}{1-\sin\phi}\right)\theta$$

and we obtain the approximate critical taper equation for a dry sand wedge in front of a strong buttress:

$$\theta \approx \left(\frac{1-\sin\phi}{1+\sin\phi}\right)\left(\beta + \mu_{base}\right)$$

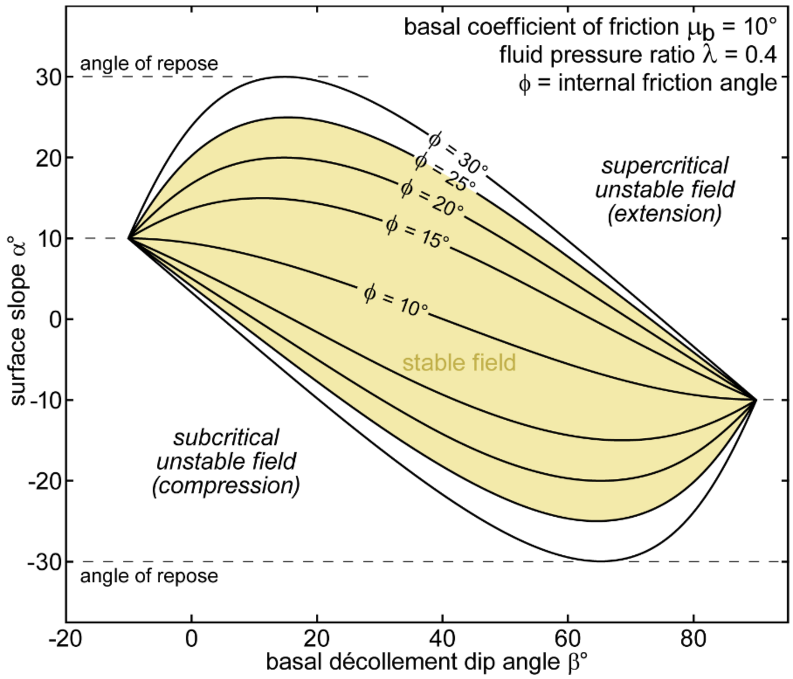
The strength of the basal décollement controls the frictional resistance acting against the forward-directed forces.

- 1) if μ_{base} increases (i.e. the décollement becomes stronger), the wedge cannot slip forward any more. Then the internal deformation thickens up the wedge so that θ increases until forward forces increase enough to overcome the new frictional resistance. This may explain out-of sequence thrusting. A stronger base will favour transport of material further under the wedge before accretion.
- 2) if μ_{base} decreases, then internal deformation ceases and the wedge simply slips forward on the basal décollement. If there is a more dramatic decrease (e.g. if the base encounters salt), then the wedge may collapse. This causes local compression at the toe and extension higher up in the wedge.
- 3) For ϕ = 30° (typical value for dry sand) then the critical surface slope is

$$\alpha \approx \frac{1}{3}\left(\mu_{base} - 2\beta\right) \qquad (7)$$

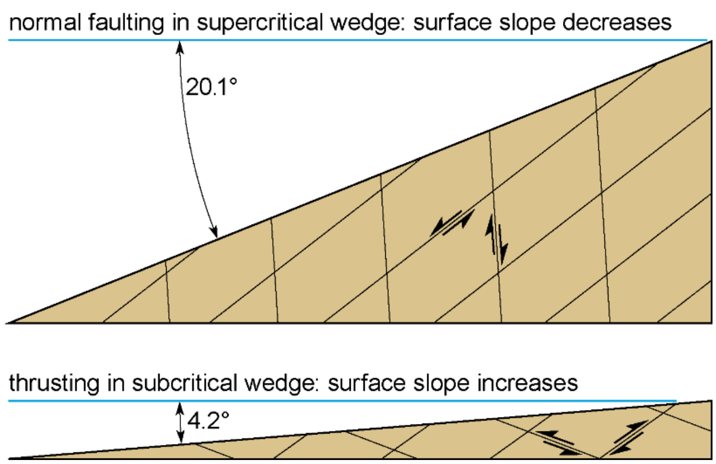
Out-of-equilibrium wedges

Conditions in nature are rarely constant so that an active wedge will always be adjusting. According to equation (7), equilibrium conditions of a tapered wedge may be represented on a plot of α versus β. The surface slope angle α can be calculated for all basal slope β for any given basal friction angle.



Two antisymmetric curves enclose a central area, which contains the surface slope angle and basal friction combinations for which the wedge remains stable, i.e., endures no internal plastic deformation. Outside the stability domain, the wedge is super- or subcritical.

- Above the upper limit of the stable domain, the wedge has excessive thickness (the wedge is **supercritical**); then the basal plane is unable to support the load and the wedge is predicted to thin, collapse forward by gravity spreading and deform by normal faulting.



- Below the lower limit of the stable domain, the wedge is too gentle (**subcritical**); insufficient gravitational force is transmitted down to the basal plane to generate shear stresses needed to activate the basal décollement. Then, the topography builds up while the wedge deforms internally by forming folds, thrusts and penetrative strain.

The stability-domain is shifted if the strength of the basal décollement (or the strength of the wedge material) changes. Then the wedge will deform to increase or decrease its surface slope until the critical taper is reached.

- An increase in the sliding resistance increases the critical taper.
- An increase in the wedge strength, on the other hand, decreases the critical taper, since a stronger wedge can be thinner and still slide over a rough base without deforming.

Shortcoming

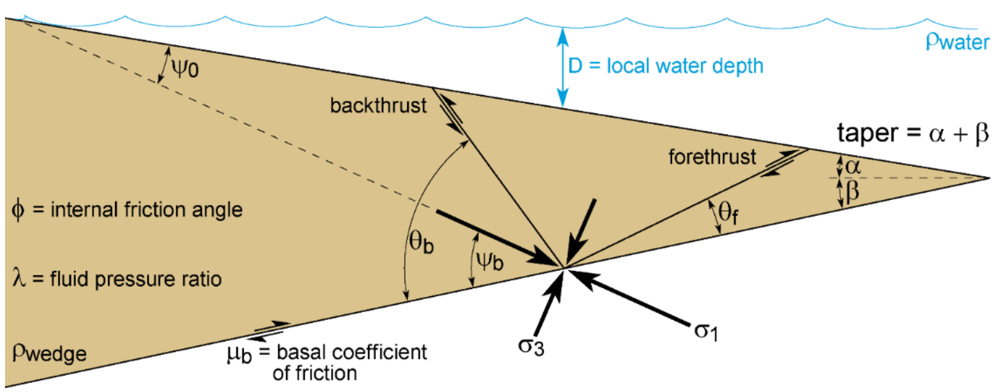
A difficulty with the critical wedge model is its assumption of uniform plasticity. The time-independent Coulomb material is a homogeneous and isotropic lithology incorporating the friction coefficient, the pore pressure and the stress orientation for both wedge and basal thrust. Additionally, reactivation of faults in weakened material is not taken into account. Since the continental lithosphere is rheologically stratified, such models unlikely provide an adequate enough description of how an entire orogenic belt deforms. In general, one assumes that the internal strength of a wedge is defined by its weakest lithology and reactivated weakened faults.

Effects of material accretion and erosion on stability

The mechanical conditions and shape of wedges with Coulomb-type rheology change with changing vertical load. Consequently, deformation style and associated mass transfer patterns may vary with addition or removal of wedge material. An active wedge will grow by addition of material to its toe and/or to its base.

Frontal accretion

As the wedge pushes forward it encounters undeformed sediment on the down-going lithosphere. These sediments are then incorporated at the tip of the wedge. This process is referred to as **off scraping**. The accreted material tends to lengthen the wedge and lower the taper angle so that the frontal region will be in compression. The load on the fault plane is still insufficient to allow slip and the wedge has to thicken up to regain equilibrium. If the longitudinal stresses are large enough, the wedge itself may deform internally by out-of-sequence thrusting and thrusts that cut through previous structures.



Backthrusting

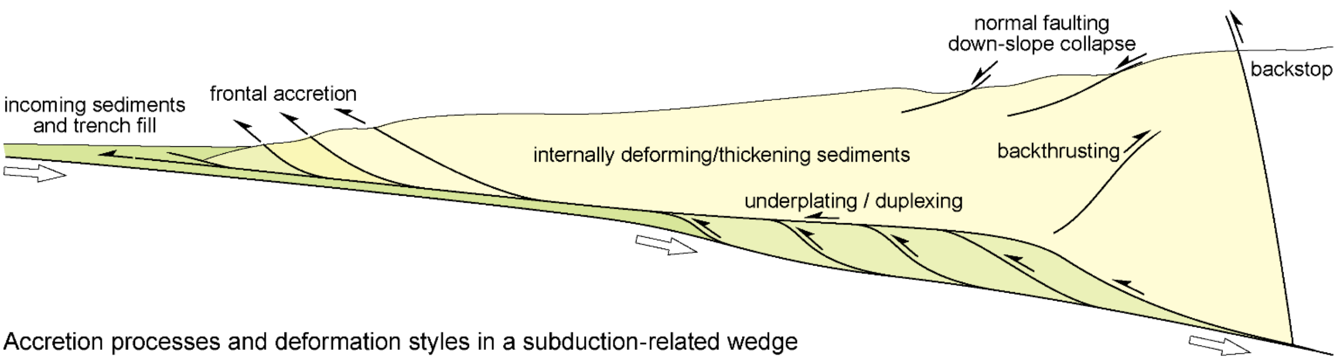
Backthrusting can be important for the internal thickening of the wedge. This is seen in sandbox experiments. Movement on each successive thrust (also preceded by significant folding) is immediately followed by the development of a backthrust. Movement on the backthrusts continues as the wedge grows forwards allowing it to preserve its surface slope. These effects would be greatly increased in an isostatically compensated wedge.

Underplating

Underplating occurs when sediments on the subducting plate are added to the accretionary wedge. The basal thrust of the wedge propagates into the down-going sediments, forming a duplex structure and adding the sediments to the bottom of the wedge, after the sediments have travelled down some distance with the underthrusting slab.

> ATTENTION: The name brings confusion with magmatic underplating, although both processes are utterly different.

Tectonic underplating is particularly important in accretionary wedges. It is probably the only way to get sediments down to conditions of blueschist facies metamorphism. Seismic evidence shows that sediments are being underthrust for several tens of km beneath active subduction wedges. Mass balance arguments in various places have also indicated the necessity of the process of underplating. However the actual process of accretion of tectonic underplating and what triggers it is poorly understood.



Accretion processes and deformation styles in a subduction-related wedge

Extension – Normal faulting

If the wedge surface becomes too steep because of excessive thickening, then the basal thrust plane is unable to support the load and the wedge collapses forward (more thrusting at the toe and extension in the thickest, highest, inner part of the wedge). The prediction is that extension favours exhumation of deeply buried (blueschist facies) rocks.

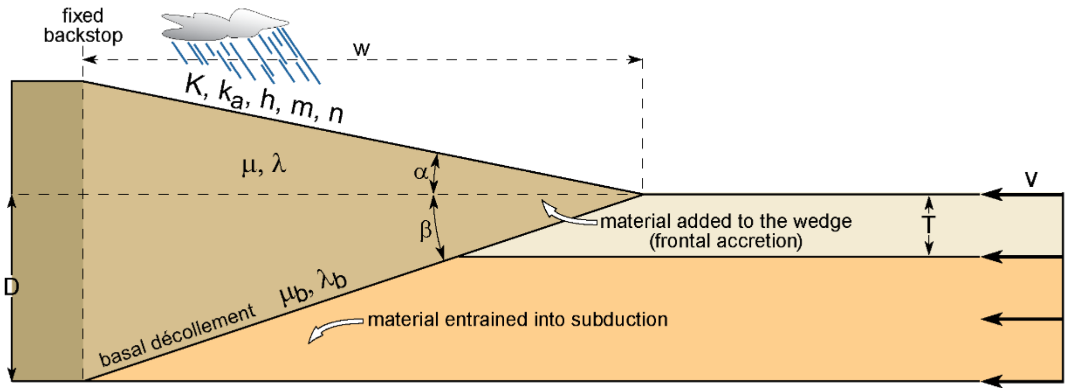
Erosion

Mechanical erosion by turbidity currents carries sediments downslope into the trench. Erosion reduces α and so makes a wedge subcritical. An eroding wedge will attain a dynamic steady state when the average erosive efflux balances the accretionary average influx rate of new material at the toe. The steady state width of a uniformly eroding wedge at an average rate $\dot{e}$ is approximated (for very small angles) with the flux balance condition:

$$\dot{e}.w = T.v$$

with T thickness of incoming material and v the convergence velocity of the subducting plate with respect to the overriding plate.

A steady state wedge must continually deform both to accommodate the influx of new material in the toe and to maintain its critical taper against erosion. This is where climate has a role to play in the growth and dynamic of exposed wedges; submarine wedges suffer nearly no erosion.

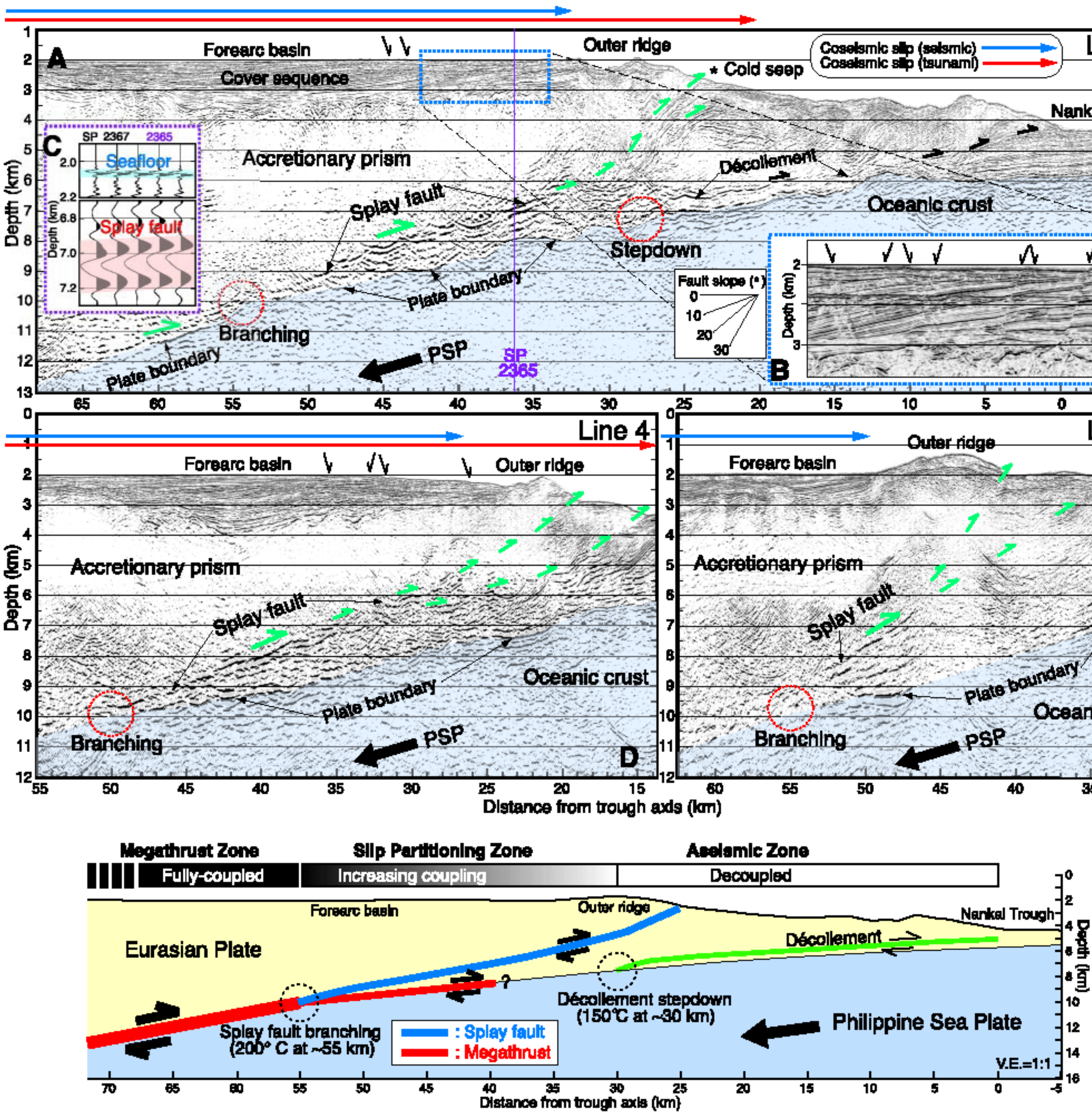


An active wedge must always be pushed from behind to remain in dynamic equilibrium. If compression ceases then the wedge will collapse to some degree.

Seismicity in active subduction zones

Different structural styles characterize the seaward (outer wedge) and landward (inner wedge) parts of present-day accretionary prisms. The outer wedge, is a few tens of kilometres wide, and has a relatively steep slope denoting imbrication on thrust faults. The inner wedge is generally wider and has a shallow slope denoting little deformation of previously accreted sediments occasionally underlying piggy back forearc basins. Modelling suggests that the outer and inner wedges have different, yet mutually influencing behaviours.

Exact stress solutions have been derived to understand subduction-related great earthquakes (remember the 2004 Indonesia and 2011 Japan disasters), which generally occur at the base of the inner wedge. With the assumed elastic–perfectly plastic rheology, the seismogenic subduction fault alternates in time between interseismic locking and coseismic slip, a typical stick-slip seismic history. During the interseismic period, inner part of the fault is locked while the shear stress builds up toward the level of fault failure. Low internal friction and high pore fluid pressure in the fault zone imply failure under low stress levels, too low to cause significant shortening strain within the overlying inner wedge material. During an earthquake, this fault segment has a velocity-weakening behaviour: its frictional resistance against slip decreases with increasing slip rate. At the same time, the push from the inner part forces the outer, frontal part of the fault to slip. There, its strength (internal friction) must increase to resist slip: it has a velocity-strengthening behaviour. If the earthquake is small, the outer wedge experiences a brief phase of elastic compression, hence approaches but does not reach a critical state. If the earthquake is big, the outer wedge is immediately compressed at the beginning of the coseismic slip, up to a stress level triggering failure in the wedge material. After the earthquakes, the inner fault is locked and the outer segment may creep to relax the coseismically generated stress, bringing the outer wedge back from a critical to a stable regime.



Depth migrated profiles of the Nankai subduction zone with sketched section interpreting seismic distribution and behaviour in the inner and outer parts of the accretionary wedge.
From Park *et al.* 2002 *Science* **297(5584)** 1157-1160.

With such fluctuations of fault stress, the wedge is not permanently in a critical state. The outer wedge geometry is controlled by the peak stress produced by largest earthquakes on the outer subduction fault while the inner wedge remains in the stable regime and acts as an apparent backstop.

Accretionary prisms - Geological applications

Two much-studied, internally compressive wedges in steady-state conditions are Taiwan and the Barbados.

Taiwan

The Taiwan fold and thrust belt is developing along the ongoing collisional boundary between the stable continental margin of China, on the Eurasian plate, and the Luzon Island arc, on the Philippine Sea plate. Collision started at 4Ma in the north and is going on at the southern end of the island; southward migration is due to the divergence between the arc and the margin. Further south, the oceanic crust of the South China Sea is subducting beneath the Luzon arc along the Manila trench forming a submarine accretionary wedge on the East Side of the trench. The Taiwan fold and thrust belt, which forms half the island, is the onland expansion of this accretionary prism as the arc encounters the thick sedimentary succession of the Chinese continental slope and shelf. At the south end of the island the fold belt has just risen above sea level and gradually rises to the north.
Rapid tropical erosion strongly affects the growth of the wedge. The characteristics of the fold and thrust belts are the following:

z = 7 km	Convergence rate: 70km Ma^{-1}
w = 90 km	Erosion rate: 5.5 $km.Ma^{-1}$
$\alpha = 3°$	$\beta = 6°$

The critical taper model can be used to determine the range of basal and internal friction, and to show that the latter exceeds basal friction by 20 to 30 %.

Barbados

The Barbados accretionary wedge lies above the North American oceanic plate where it subducts below the Caribbean plate, east of the Lesser Antilles Arc. There has been continuous accretion of the wedge since the Eocene (40Ma). The characteristics of this accretionary complex are:

z = 7 km in the south to 1 km in the north	Convergence rate: 2 km Ma^{-1}
w = 300 km	Erosion rate: 5.5 $km.Ma^{-1}$
$\alpha = 2°$	$\beta = 1.5°$
$\rho = 2000\ kg.m^{-3}$	$\lambda = .95$

Makran

The ca 1000 km long Makran accretionary wedge, in southern Iran and Pakistan, is 200-300 km wide onshore and ca 150 km wide offshore. Accretion is supposed to be continuous since convergence between Arabia and Eurasia began in the Late Cretaceous. The characteristics of this accretionary complex are:

z = ca 10 km in the north to < 1 km in the south	Convergence rate: 2-3 km Ma^{-1}
w = 300-350 km	Erosion rate: 0.7 $m.Ma^{-1}$
$\alpha = < 2°$	$\beta = 2°$
$\rho = 2000\ kg.m^{-3}$	$\lambda = .98$

Orogenic Wedges

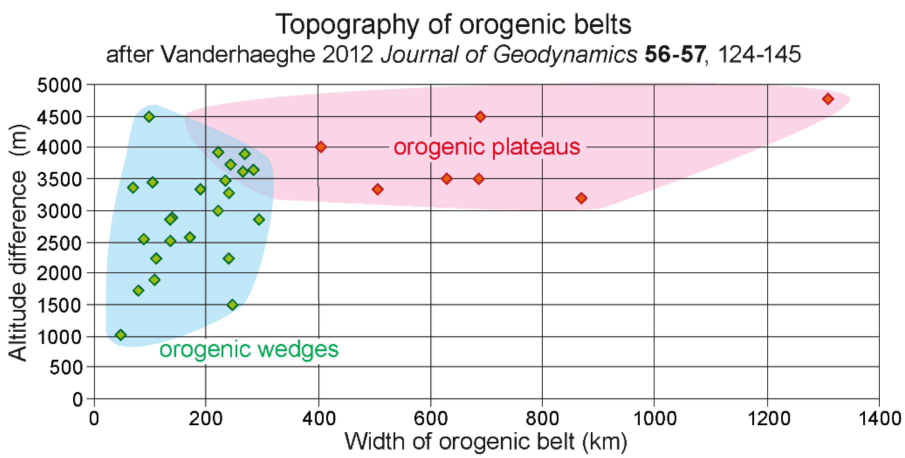
The behaviour of **orogenic wedges**, i.e. convergent complexes, has been approximated as a rigid buttress behind a wedge-shaped prism resting on a subducting lithospheric slab. This comparison was based on the observation that:
(1) Mountains belt have averaged topographic surface sloping towards the foreland.
(2) Large horizontal shortening has taken place in front of a relatively rigid backstop.

(3) Deformation proceeds from rear to front as long as material is added at the front.
(4) Metamorphic grade increases from front to rear, as deeper rocks are exposed in the hinterland.

A bulk viscous rheology is more appropriate than a Coulomb rheology, in particular for long-lived orogens. Simplifications lead to the approximated stability criterion:

$$\sin\alpha = \tau/\rho g z$$

which is identical to the glacier sliding of equation (3). In an orogenic wedge, tectonic underplating is very important and leads to thickening of the wedge that, from a limiting plateau level, becomes unstable and collapses. Orogenic collapse may be the central mechanism for the exhumation of high-pressure rocks found within orogenic belts.



Conclusions

The question concerning thrust sheets is: How could forces be transmitted through relatively weak sheets of rock over distances many times their thickness without drastically deforming them? The main answers are:

- The force of friction on the base of the thrust sheet could be lower than it is assumed.
- Coulomb friction might be overestimated and being replaced by viscous or plastic ductile flow of weak rocks.
- The effect of pore-fluid pressure greatly modifies the problem.
- There is no push from the rear but gravitational forces are the driving forces.
- Thrust sheets do not move *en masse* but caterpillar style by the propagation of localised domains of slip along the fault.

Most of these answers are considered in the concept of plastic, wedged thrust systems. The wedge has to attain a critical taper to exceed the basal shear strength and initiate basal displacement. In a critically tapered, stable wedge, equilibrium between three main elements exists:

- Frictional resistance to sliding along the base, which refers to the basal traction of the compressive wedge.
- Forces pushing at the rear of the wedge, which express the regional tectonics.
- The shape of the wedge, which is controlled by various factors such as frontal or basal accretion, internal deformation, sedimentation, surface and tectonic erosion.

Accretionary prisms generally display a tapered cross section whose shape depends on strength of both the basal thrust zone and the wedge material and on the forces acting on the wedge. Moving thrust wedges are thus in a state of dynamic equilibrium. The taper angle is measured between the basal décollement, dipping towards the orogen, and the upper surface that slopes towards the foreland. The surface slope of thrust belts is actually affected by the slope of the basal thrust while there is a permanent interaction between wedge deformation at the subcritical taper and slip periods at the critical taper. Addition of material at the toe, slumping, hinterland collapse and erosion tend to lower the topographic slope, hence to reduce the taper angle. Internal deformation through folding and faulting steepens the topographic slope, hence increases the taper angle. Internal deformation also thickens the hinterland that may collapse under its own weight and produce downslope slumps and normal faults so that the wedge thins down during thrusting.

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