FOLDING

Folding is a concept that embraces all geologic processes by which surfaces in rocks become curved during deformation. Since folds are permanent deformation structures with no or little loss of cohesion of the folded layer, folding refers to the essentially slow, ductile behaviour of relatively soft and/or hot rocks. Beyond the descriptive, anatomical classifications, much of the early geologic work on folding processes focused on the deformation of stratified sediments. Different folding mechanisms combine a few basic processes involving the geometrical (layer thickness and spacing) and physical (viscosity, viscosity contrast, anisotropy) properties of the rocks.

This lecture deals with some consideration on genetic, mechanical aspects concerning the development of folds. The important point to note is that stress alone is insufficient to cause folding: A planar surface must first exist to define the fold shape, and the orientation of this planar marker with respect to the stress direction controls in many ways the attitude of the resulting fold.



Folding processes

Most models of fold formation ignore body forces and the effect of the material enclosing the layers (both are treated as viscous fluids), which in practice has a very important role in determining or modifying the fold geometry. Flat layers may become curved in several ways.





Rotation

Obviously, folding rotates parts of the layers. The continuous change in orientation of the stiff layers with respect to the shortening direction first produces a marked decrease in compressive resistance of the rock mass. This is a form of bulk weakening, which accompanies the progressive modification of the internal geometry of the rock mass without any change in material properties (e.g. viscosity); it is accordingly termed **structural softening**. Structural softening is followed by an increase in compressive resistance, **structural hardening**. Softening and hardening are the conditions for onset, growth and decay of instabilities. Evidently, any other mechanism of strain softening that may be present (e.g. a change in the effective material properties caused by processes such as microfracturing, pressure solution, etc...) will also affect the stability of the system.

Lengthening

Fold hinges most often mark the site where the folds began to amplify. This is usually over a short distance so that any fold axis plunges towards the two extremities of the hinge segment. Further limb rotation and subsequent fold growth (amplification) accompanies lengthwise, not necessarily symmetrical migration of the hinge tips.

Mechanical role of layers: Active / passive folding

Any deformation involves displacement of material points, for example particles. A passive particle has no interaction with its neighbours; it only moves. An active particle interacts with its neighbours and its displacement is affected by that of neighbouring particles.

In geology, the compositional layering has mechanical properties influencing the strain pattern and the folding process. For example, boundaries between layers with contrasting strengths (viscosity) may slip or localise shear deformation, hence guiding the way curvatures develop. Folding is **active** when deformation takes place at the layer scale and the strength difference between layers directly affects the deformation pattern. Conversely, **passive folding** takes place at the grain scale while layers have no significant competence contrasts and so do not generate any stress acting across and/or parallel to the layer boundaries; layering serves merely as a geometrical strain marker. Passive folds grow during heterogeneous flow and their shape reflects the pattern of heterogeneous deformation. Passive folding is experimented by folding a stack of modelling-paste layers (think also of oil traces or scum on slowly flowing water). Active folding is experimented by bending a pile of cards that glide one upon the other.

Folding mechanisms

Folds can result from layer-parallel compression, uneven loading oblique to perpendicular to layers, or from amplification of surface irregularities during deformation flow.

Bending

Bending involves forces applied and acting at high angles to layers that may or may not have competence contrasts. A layer subjected to bending is like a notebook supported at the ends and loaded in the middle. The notebook bends downward when the load is placed in the middle.



This is the deformation commonly undergone by overloaded or too thin bookshelves. Bending is an active folding process that mostly produces very gentle folds.

Lithospheric-scale flexures

Lithospheric-scale folds produced by bending are common in cratons where vertical stresses produce broad domes, basins, swells and arches of originally horizontal planes. Flexural bending of lithospheric plates also occurs at subduction zones and adjacent to oceanic ridges, at smaller accumulations of volcanic rocks in the oceans, such as guyots and large volcanoes, and where loading by ice or sediments causes the lithosphere to undergo flexural readjustment. Flexural bending is also involved in foreland basins adjacent to mountain systems. Stresses caused by gravity (i.e. body force) are important during flexure on the lithospheric scale.

Crustal-scale bending

Crustal-scale bending produces gentle up-warping and down-warping. Examples are arching of cover rocks above an intruding pluton, drape folds and forced folds. Forced folds are formed when sediments, which cover a more rigid basement, flex and drape the deep fault scarps in response to components of vertical movement along basement faults. Above a normal fault the bending of the layers involves layer parallel extension; above a reverse fault the bending involves a buckling component.



Fault-bend folds form by the passive bending of a thrust sheet over a ramp. Such folds may be the only kind of bending folds formed in orogenic belts by tectonic forces.



Passive folding of the hanging-wall of a flat-and-ramp thrust system

Mathematically, modelling the bending of layers works well for small folds, but becomes inexact for large segments of the lithosphere because of inelastic behaviour.

Meso-scale bending

Drape folds and forced folds may occur on outcrop scale around local objects.

Buckling

Buckling is a well-known active mechanism for the development of rounded folds in a competent layer (i.e. a layer with low rate of ductile flow) enclosed in an incompetent (with high rate of ductile flow) medium of sufficient viscosity contrast. Gently pushing the two extremities of a paper sheet on a table towards each other reproduces this folding mechanism. When the force is small the sheet remains flat. As the force is slowly increased, it suddenly becomes curved. This rapid change from a flat to a curved (buckled) form at a particular force is due to the development of a **mechanical instability**.



Similarly, geological buckling involves the **flexural instability** of a stiff interface, layer or stack of layers under lateral, i.e. layer parallel compression. The presence of layers of contrasting competence produces a **mechanical anisotropy**, which is essential for buckling. The strong layer(s) fold(s) while the weaker matrix fills in gaps. Such conditions are usual geological situations and thus explain why buckle folds are very common in the Earth's crust. Natural buckling systems can be sub-divided into several groups, namely folds formed on: (i) a single interface; (ii) two interfaces, which define a single layer in a matrix; (iii) several layers; and (iv) a mineral fabric such as an earlier cleavage.

Interface buckling

Interface buckling occurs on many scales, from regional scale as the unconformity between the Mesozoic cover and older basement in the Alps, to small-scale examples observable in the field. In experiments of shortening two-layer analogue models, buckling of the interface starts as symmetric sinusoidal deflections that change their geometry as they amplify into alternating **cusps** (points formed by two intersecting arcs) and **lobes** (rounded bends). **Cuspate-lobate** folds are characteristic of interfaces between media of strongly differing viscosity. The cusps always point into the stronger of the two materials.



Thus, in outcrops exhibiting cuspate-lobate forms, it is possible to know at a glance the layer that was the stiffest at the time of folding. Analytical solutions indicate that cuspate-lobate folds may evolve from the kinematic amplification of sinusoidal folds subjected to shortening greater than about 10%. Analytical work has also shown that folding of the interface between two linear viscous materials is

not a mechanically instability; mechanical instabilities can develop only in materials with power-law viscous flow laws. However, the growth of cuspate-lobate folds is enhanced in materials with non-linear viscosity.

Single layer buckling

Buckled single layers are common in nature, for example, an isolated sandstone or limestone bed in a thick shale or marl sequence or a vein of igneous rock intruded into an unlayered matrix. The buckled layer maintains its thickness throughout, thus producing a parallel, concentric fold. Experimental buckle folds are usually symmetric.



Currie et al. 1962 Geol.Soc.Am.Bull 73(6), 655-674

Multilayer buckling

A multilayer is a package of different layers, which is the most common situation in geology: a sedimentary succession is often a more or less regular alternation between two or three rock types (e.g. sandstones and shales in turbidites). The alternating layers have variable thickness and competence. Theoretical and experimental studies have shown that the behaviour of a multilayer depends upon a number of factors, e.g. the number and thickness of competent layers, the spacing between the competent layers, the competence contrast among the layers and the competence of the medium confining the multilayer.



Influence of spacing

In sequences consisting of layers of dissimilar thicknesses and competence (viscosity), two different buckling modes are possible, depending on the spacing between the competent layers:

- 1) If the spacing is large (larger than the dominant wavelength W_d , defined further down this lecture for single layer folds), the multilayer behaves mechanically as a series of single layers. Each competent layer tends to buckle more or less as an independent layer folding according to the single layer buckling theory. Accordingly, thicker layers show a larger wavelength than the thinner layers. This results in "disharmonic folding". Folds of smaller wavelength and amplitude (**parasitic folds**) can develop and grow at different rates within folds of complex multilayers by such processes.
- 2) For close spacing ($\leq 1/W_d$), the competent layers may interfere with each other. Often, the larger wavelength of the stronger (or thicker) layers is superimposed on smaller wavelengths of less competent and/or thinner layers. The multilayer behaves mechanically as one multilayer, i.e., all the layers conform to the same wavelength and amplitude usually imposed by the strongest and thickest layer. There is mechanical interaction between the competent and incompetent layers. Folding is "harmonic". Even if competent layers are so close to each other that the multilayer behaves as a single anisotropic layer, the multilayer will buckle with a smaller wavelength/thickness ratio than an isotropic single layer with the same thickness.

With complex multilayers, folds of different wavelengths occur and some grow at different rates. They may interfere with each other, and often a larger wavelength is superimposed on a smaller one because the incompetent members conform to the shape changes that are prescribed by buckling of the stronger layers. Folds of different orders can develop by such interference.



Buckle folds in a complex multilayer (terminology from Currie *et al.* 1962 *Geol. Soc. Am. Bull.* **73(6)**, 655-674)

The different types of buckling behaviour are explained by considering the strain in the matrix around a single, competent layer as it buckles. The matrix is displaced by the developing fold, but the displacement progressively vanishes away from the folded layer and becomes negligible at a distance about equal to the dominant wavelength. There the matrix records only homogeneous flattening due to the bulk shortening. The zone of heterogeneous disturbance (folding strain) on each side of the buckling layer is known as the **zone of contact strain**. If the competent layers are sufficiently far apart, there is no overlap of their zones of contact strain and each layer can buckle as a single layer. If, however, the zones of contact strain of adjacent competent layers overlap, the layers can no longer buckle independently of each other. Zones of contact strain and associated zones of contact stress of adjacent layers must be compatible and, as a result, all the layers are subjected to the same stress field and develop the same wavelength. In order to determine how close the competent layers must be for multilayer as opposed to single layer buckling to occur, it is necessary to know how far the zone of

contact strain extends away from the layer into the matrix. For a viscous matrix, the disturbance in the zone of contact strain has died down to approximately 1% of its maximum value at a distance of about one wavelength from, and on either side of the folded layer.



Numerical shortening of a viscous multi-layer.

Thin lines are passive markers to visualize fold decay away from relatively stiff layers.

Influence of discontinuities: Flexural-slip and flexural flow:

A multilayer can be a pile of competent layers separated by surfaces of discontinuity or alternating layers of highly contrasting competence. The mechanical consequence is that the competent layers on either side of the surface of discontinuity or of a weak layer may easily slide relative to each other. This shear "decoupling" of layers allows a fold to accommodate a greater flexure than if the stack deforms as a single layer.

Flexural-slip describes discrete faulting, usually coincident with bedding planes and accompanying folding. A classical simulation is to bend a book or pile of paper sheets; increasing bending about the fold axis is accommodated by increasing slip between the pages of the book or sheets of the pile. The the thickness of individual sheets does not change, meaning that each sheet makes a **parallel** fold (i.e. layer surfaces remain parallel). Slip is an important part of folding because layer-parallel stresses increase with increasing rotation of the limbs.



When the shear stress exceeds shear resistance of weak layers or layer boundaries, the strong layers in the limbs slip over each other towards and usually perpendicular to the hinges, which are fixed from layer to layer. Therefore, slickensides and fibrous mineral growth or other movement indicators showing reverse dip-slip on bedding planes within fold limbs are common criteria for flexural slip. Slip values are greatest at inflexion points, on the limbs, and decrease to zero at the fold hinge. The amount of displacement increases as folds tighten and also depends on the spacing of slip planes. Note that hingeward slip implies opposite movement directions from one limb to the next, yet consistency is maintained from anticline to syncline.

Structural variations include ramp faults connecting separate layer-parallel faults and duplex contained within layer-parallel floor and roof faults. The temperature and pressure at which flexural-slip folding occurs are generally low.

Flexural flow describes bedding-parallel shear homogeneously distributed within the ductile layer being folded between stiffer layers. Like for flexural slip, bedding-parallel shear in limbs is opposite across the axial plane. The strain pattern due to hingeward shear tends to develop thickened hinges between thinned limbs, i.e. flexural-flow folds are mostly **similar**. Flexural-flow is sometimes applied to the weak layers that take up bedding-parallel motion within larger parallel folds, generally under low metamorphic grade. In this case, the stiff, active layers tend to keep their thickness throughout the deformation to produce and control the overall shape of concentric and/or parallel folds while the incompetent layers undergo flexural flow. In order to maintain similarity from bed to bed, ductile material moves out of the limbs into the hinges. Natural examples of such similar folds show intense foliation in the fold limbs, which dies away from limbs towards hinge zones. The intensity of shear strain depends on fold shape and position within the fold, with shear strain equal to limb dip in radians.



Influence of anisotropy

A bedding-parallel anisotropy is an intrinsic property of multilayers. Theoretical and experimental work on homogeneous, anisotropic multilayers shows that there is a range of fold shapes that can form under anisotropy-parallel compression. The type of fold is determined by the mechanical anisotropy. Symmetric, sinusoidal folds in multilayers with weak anisotropy give way to folds with gently diverging axial planes and ultimately to box folds in multilayers with high anisotropy. Fold shapes propagate away from the folded layer much farther into an anisotropic than in an isotropic matrix.

40% shortening and viscosity contrast = 50 in both experiments



Numerical models of the effect of matrix anisotropy during folding of a Newtonian layer after Kocher et al. (2006) Tectonophysics **421(1-2)**, 71-87

Flow folding

Flow folding refers to the formation of passive folds by heterogeneous shear or bulk flow of mechanically isotropic rocks in a direction oblique or normal to planar markers. Examples are folds formed by variations in flow rate during emplacement of igneous bodies with a magmatic fabric. Flow folding also occurs in outpouring lava and in slower flowing salt, ice and water-saturated, unconsolidated sediments. In metamorphic rocks, the low viscosity contrast between layers is favoured under high-grade conditions. Theoretically, flow folds can form by heterogeneous simple shear and by pure shear without initial perturbation on layers. The highly incompetent layering acts only as passive marker in an equally viscous matrix. Only these contorted passive marker planes trace out the flow patterns within the uniformly ductile rock. Where flow lines or lanes diverge or converge, the thickness of a bed measured in the direction of flow decreases or increases in inverse proportion to the normal distance between flow lines. Synchronous refolding is a common ingredient of the viscous and unsteady flow. A classical comparison is the movement of oil scum on water.



<u>Attention:</u> Flow folds do not form by slip along foliation, which is a grain-scale modification recording folding-related strain. Foliation is no movement plane.

Shear folding

Differential slip along closely spaced planes or simple shear on closely spaced shear zones parallel to the axial surface and oblique to the folded layer produces ideally similar folds. This passive mechanism is called **shear** or **slip folding**.



Kinking – angular folding: Effect of mechanical anisotropy

Kinks have straight limbs between sharp to angular hinges whose axial planes define the **kink band boundaries** more simply termed **kink planes**. Short limbs define the **kink bands**. Kink bands occur in strongly anisotropic rock where the anisotropy is either beds with a finite thickness or foliation with very thin layers. Their particular geometry is controlled by the rotation through an angle α of a set of thin layers within the kink bands. Ideally, kinking involves no internal strain in the layers, only rotation around the kink hinges. Therefore, flexural slip in the limbs is inherently linked to kinking to insure the continuity of layers across the kink band boundaries.



The formation of kink bands is predicted by theoretical analysis of the viscous deformation of materials with a strong planar anisotropy. The models differ in the way the kink grows and in the geometry of the deformation, which is specified with two angles: β_i between the kink plane and the within-kink layers and β_e between the kink plane and layers out of the kink band. There are two main mechanisms:

- Model 1: kink band boundary migration (also termed mobile hinge).
- Model 2: kink bands as shear zones (fixed hinge)

Kink band boundary migration

The two kink band boundaries migrate away from a central nucleation line into the undeformed material. In this case angles β_i and β_e remain constant in the widening kink bands.



Kinking by migration of kink band boundaries

Kink band shear zones

The two kink band boundaries mark the fixed boundaries of a shear zone at the onset of kink band development. In this case, the kinked segment maintains a constant length during shear-induced rotation. If $\beta_i > \beta_e$, dilation must take place between the kink-band layers. Rotation larger than $\beta_i = \beta_e$ causes layer thinning, which might be a blocking factor.



Experiments revealed that kink bands commonly occur in conjugate sets with opposing asymmetry when the maximum compressive stress is (sub-) parallel to a pre-existing planar anisotropy. However, they do not develop along planes of high shear strain, which indicates that they are not true shear zones. Still, by analogy with faults, the apparent and relative displacement of long-limb layers across the kink band defines three sorts of kink bands:

- Normal kink bands in which there is a volume decrease in the kink band.
- Reverse kink bands in which there is a volume increase in the kink band.
- Neutral kinks in which the volume remains constant.



Three types of relationship between conjugate kinks and bulk deformation

Chevron folds

Chevron folds resemble kink bands for their planar limbs and for occurring in regularly bedded multilayers but the hinge zones are not angular. The required distortion (rotation) is localized in the hinge while flexural slip typically occurs, which means that individual layers of the limbs suffer no internal distortion. As the small hinge tightens between the straight limbs, there are space problems where holes open between competent layers. Flow of the weak interlayers, if any, fills up these spaces.



Dynamics of folding: development of buckle folds

A considerable body of work has shown, both theoretically and experimentally, that if a thin layer undergoing layer-parallel shortening is more competent (i.e. stiffer) than the surrounding material, this condition is unstable because some initial geometrical perturbations on the stiff layer are amplified and buckling occurs while the entire system (i.e. the layer along with the surrounding matrix) is deforming in pure shear.

The early works focused on the analysis of buckling and treated the problem by assuming that all the layers behave elastically. The assumption of linear relationships between stress and strain or between stress and strain rate simply made the whole problem tractable. The use of linear relationships leads to differential equations that are difficult enough to handle; the use of non-linear relationships between stress and strain rate in general leads to insurmountable problems mathematically. In what follows the key assumptions made are

- 1) The folds are so small that gravity is not an important factor in their development.
- 2) Compression has been parallel to the layer to start with.

3) The deformation has only involved a plane strain.

Controls of fold wavelength

One considers that a single viscous layer embedded in a matrix of lower viscosity is very thin compared to the fold wavelength (the so-called thin-plate theory). Buckling produces a fold system that has a symmetric, periodic, sinusoidal shape. The analysis deals with the nucleation, i.e. the investigation is limited to very small amplitude, first buckle folds resulting from infinitesimal deformation.

Theory

In mathematical treatment, if a laterally compressed layer is perfect, then it simply thickens during shortening without folding. An imperfection is required to induce buckle folding. This initial imperfection might be present in the layer prior to the imposition of compressive stress or may be a local **instability** that develops while compression is applied. Technically, it is simulated with one or several superposed low-amplitude sinusoidal functions that describe the layer boundaries. The theory first assumes that the medium that confines the layer resists layer-perpendicular deflection. Then, the most-stable shape is the one that needs least amount of layer-parallel stress, i.e. the least elastic strain energy in both the layer and the surrounding material to emerge spontaneously. Results indicate that, although all of the primary irregularities might start to grow, only one sinusoidal, regular fold train with one particular wavelength grows preferentially as deformation proceeds. This most stable, selected and amplified sinusoidal response is the **dominant wavelength**.

Dominant wavelength

Two key factors control the dominant wavelength:

- the layer thickness;

- the viscosity ratio (the strength contrast) between layer and matrix, which both are treated as Newtonian viscous materials.



Two-dimensional, periodic model of a single layer buckle fold

For a single competent layer of thickness h and viscosity μ_L embedded in a weaker matrix of infinite thickness and viscosity μ_M . Internal forces F_{int} (resistance of the competent layer) and external forces F_{ext} (matrix resistance) act together against the development of a fold with first wavelength W_i :

$$F_{int} = \frac{2\pi^2 \mu_L h^3 e_x}{3W_d^2 e_x}$$
$$F_{ext} = \frac{\mu_M W_d e_x}{\pi e_x}$$

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The model is shortened at a rate $\dot{\epsilon}_x = de_x/dt$ by the amount e_x in the x-direction. The dominant wavelength (**Biot-Ramberg analysis**) is the wavelength with the smallest total force $(F_{tot} = F_{int} + F_{ext})$. The following equation expresses the initial dominant wavelength W_d :

$$W_{d} = 2\pi h \left(\mu_{L} / 6\mu_{M} \right)^{1/3}$$
(1)

This relatively simple relationship has been experimentally and numerically verified and is applicable only to small-amplitude folds. Equation (1) clearly states that:

- The wavelength is independent of both the amounts of compressive load and the strain rate.

- The wavelength is directly proportional to the thickness h of the competent layer; thus, different wavelengths will arise in different layers of variable thickness, in all of which the shortening strain is constant. Thicker layers produce longer wavelengths. Variable intensities of fold development do not indicate variable intensities of deformation.

- The wavelength depends only on the cube root of the layer-to-matrix viscosity ratio.

The impact of the strength ratio on the wavelength/thickness ratio (hence fold style) can be visualised by rearranging equation (1):

$$W_{\rm d}/h = 2\pi (\mu_{\rm L}/6\mu_{\rm M})^{1/3}$$

Note that in this equation the ratio μ_L/μ_M is merely the ratio of viscosity between the layer and its embedding material. A related feature is that as μ_L approaches μ_M , the dominant wavelength approaches a value of 3.46h.

Equation (1) can be reorganized as:

$$\mu_{\rm L}/\mu_{\rm M} = 0.024 (W_{\rm d}/h)^3$$

indicating that the viscosity contrast can be approximated from wavelength and thickness measurements.



<u>Exercise</u> Calculate and draw the wavelength of folds in 1, 5 and 10 cm thick layers with viscosity contrasts of 1, 5 and 10.

Growth rate

The growth rate of a fold actually designates its amplification rate. Hence, the question is how fast the hinge points move upward, orthogonal to the direction of bulk shortening during buckling. For reasons of symmetry, the authors have considered segments of half-wavelength whose extremities are frictionless vertical planes along which two successive synclinal and anticlinal hinges glide freely by the same amount, in opposite directions (downward versus upward, respectively).

Per definition, the dominant wavelength is that one that amplifies at the fastest rate. One may immediately infer that, as in equation (1), the thickness h of the buckling layer and its viscosity contrast with the embedding matrix are parameters of the equation specifying the amplification rate. In fact, the growth rate increases with increased viscosity contrast between layer and matrix.



Kinematic amplification of a passive line during shortening / thickening

Amplification is the sum of two components: the kinematic and the dynamic growth.

- A_k is the **kinematic** (passive) **growth rate** due to the bulk thickening of the matrix. A sinusoidal passive line will acquire shorter wavelength and higher amplitude during shortening, but this deformation owes nothing to any mechanic instability. Passive amplification dominates when the rheological contrast is small. Therefore passive amplification is important in high grade rocks and water-saturated sediments.

- A_d is the **dynamic growth rate** due to amplification of the initial mechanical instability.

The amplitude A of a viscous buckle is exponentially related to time (t) such that:

$$A = A_0 e^{P_A t}$$

 A_0 is the amplitude of the initial, sinusoidal perturbation. Its presence in this equation implies that the amplitude of the initial perturbation influences the final geometry of the waveform. Finite amplitudes of folds may reflect the original variations in amplitude of existing irregularities such as ripple marks as much as the competence contrast and the imposed bulk shortening.



P_A is the **amplification factor**, which gives the amplification rate. It integrates the sum $(A_k + A_d)$

but the total expression, that also includes viscosity contrast and layer thickness, is quite complex. If amplification is exponential, it should enter an "explosive" mode when the incremental amplification should increase enormously with respect to shortening for high amplification factors. Estimates suggest that this should occur when the amplification factor of the dominant wavelength is approximately 1000 and for high values of viscosity contrast. Complex expressions avoid this problem.

History of buckling

A contrast in competence among the associated layers is essential for buckling. The development of a buckle fold is an unstable process conveniently divided into four stages:

- Layer-parallel shortening.
- Nucleation of the buckling instability.
- Amplification of the buckle-fold.
- Locking up and shortening in pure shear.

1) Incubation: Initial homogeneous shortening

In experimental buckling, compressed layers do not produce folds for the first 20% or so of shortening. Instead, the individual layers increase their thickness essentially to compensate homogeneous, layer-parallel shortening. The amount of homogeneous, elastic and inelastic strain before buckling begins is a function of the strain rate and of the relative mechanical properties of the layers undergoing buckling. The layer thickness remains constant; therefore, there is no shear strain within and parallel to the shortened layers.

2) Nucleation

Buckle initiation is difficult and generally requires some form of perturbation on the initially shortened/thickened layer. Nucleation involves rotation of the layering at selected sites where there are inherent (e.g. initial bedding variation) or generated (e.g. local fluctuation in applied boundary stress) heterogeneity in the deformation. A certain wavelength of perturbation is selectively amplified. This amplification builds the buckle folds; the selected wavelength is related to the mechanical character of the stiff layers.



Geometrical model of the evolution of a buckle fold (construction with constant thickness and constant length along the neutral surface)

3) Amplification

Amplification is the progressive vertical growth of the fold. Theoretical studies of folding and the resulting concept of dominant wavelength are normally only valid for the first increment (i.e. nucleation) of buckling, when fold amplitude is so small that it is practically invisible. Once buckling has been initiated, shortening can continue by rotation of the limbs. Buckling becomes progressively easier, and the dip of limbs increases rapidly compared to the rate of bulk shortening. Buckling is at that stage a structural softening process, i.e. the layer resistance against shortening diminishes with progressive strain (amplification) while the material properties remain constant. The buckle-fold is amplified at a rate that depends on the ductility contrast between the stiff and soft layers (faster amplification for larger contrast). A 15° dip for the limbs is about the limit in amplitude for which the dominant wavelength analysis expressed by (1) becomes inoperative.

The weak matrix layers continue to shorten homogeneously while the fold amplifies. Progressive shortening of the system is thus composed of two parts: one part is directly associated with the bending of the layers to form folds, the other part consists of an additional strain at each point with a

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component of shortening approximately normal to the axial plane and a component of extension in the axial plane and normal to the fold axis.

Propagation involves spreading of the area of folding.



4) locking-up

Further shortening occurs as much by flattening of the fold as by flexure. Flattening can tighten the original buckle fold while thinning the limbs and thickening the hinge. Correspondingly, the progressive development of natural folds involves more than one mechanism. When the limbs become parallel to each other (i.e., the fold is isoclinal), no further shortening by limb rotation can occur. Dynamic, folding amplification stops but shortening may continue by homogeneous flattening of the buckled layer and matrix. Final decay is reached when the layering at any point has rotated into a nearly stable position and the resistance to deformation has increased. This is often referred to as "**locking up**".

5) Late stage pure shear

If the competence contrast is not too high, the limbs are then thinned while the hinges are thickened, which causes kinematic amplification. Since there is extension perpendicular to the fold axis, the competent layer may show boudinage in the plane of fold profile. Depending on the orientation of the bulk strain axes, there may or may not be extension parallel to the fold axis.



5bis) refolding

Alternatively continued compression may cause the buckled layer to buckle again. The layer has then a new effective thickness nearly equal to the height of the flattened buckles and so, in accordance with the buckling theory, will buckle with a larger wavelength.

Refolding under changing stress directions can lead to complex structures (interference patterns).

Multilayer

Complicated mathematical expressions are required to describe the behaviour of a multilayer sequence because they must include all variables, notably the spacing of stiff layers and the degree of cohesive strength between layers within the sequence. Therefore, experimental deformation of multilayered models has been crucial to identify some of the physical factors that control the shapes of folds. Models consisting of layers of different thicknesses and mechanical properties are complex systems that show specific behaviours: At stage (2), the buckling instability is related to the mechanical character and location of the thickest, stiffest layers within the sequence. At stage (3), while the stiffest, thickest layers buckle as single units, the multilayer sequence as a whole will undergo flexural-slip folding. The nature and degree of development of minor structures in the relatively soft layers will depend on the local strain environments created during the folding of stiffer layers.

Viscous rheology and folding

People who used numerical methods to examine how folds grow usually assumed the materials to be ideally viscous.

Stress distribution

Their result shows a complex relationship between folding and stress orientation.

- In the hinge zones, the maximum compressive stress is parallel to the layer on the concave sides of folds where layer-parallel shortening occurs, and it is roughly perpendicular to the layer on the convex sides where layer-parallel elongation occurs.
- In the limbs, the maximum compressive stress tends to rotate with the limbs until limb dips become steep, at which point it returns toward its original orientation and tends to be at high angle to the bedding.

The magnitudes of the stress also vary across the fold and throughout the course of the deformation. These changes reflect the fact that the competent layer bears a large proportion of the force applied to the system when the layer is parallel to the shortening direction, but its strengthening effect decreases as the limbs rotate to higher angles.

Influence of viscosity contrast

Numerical modelling demonstrated that initial, homogeneous layer shortening absorbs much of the bulk deformation and folding becomes a less important process where the viscosity contrast between layer and matrix is low.



Single-layer buckle folds

It has been shown that any sort of shape can develop. Because of the temperature dependence of the rheologies, an increase in temperature changes the mechanical behaviour of the system, thereby affecting the geometry of the folds that develop. The shape of the fold can vary from class 1B through class 1C to nearly class 2 depending on the viscosity ratio, the amount of shortening and the wavelength thickness ratio.

- Where the competent layer is much stiffer than the matrix $(\mu_L/\mu_M > 50)$, the amplification rate of buckling is very fast and the competent layer deflects vigorously into the lower-competent

surrounding material. Folds with a large wavelength compared to the thickness of the competent layer first develop, whereby the length of the competent layer is not or little changed. During further deformation limbs rotate up to more than 90°. Large wavelength, rounded forms are produced, such as **ptygmatic folds**.

- Where competence contrast is low $(\mu_L/\mu_M < 10)$, the amplification rate is slow. Then folding is unlikely to develop. Instead, most of the deformation will consist of layer shortening and thickening partly expressed by low-amplitude, short wavelength folds on the boundaries of the competent layer. With further shortening these folds take alternating round and sharp shapes. These are **cuspate-lobate folds**. The deflection of softer rock into more competent rock produces the cusps that point into the stiffer rock. In three dimensions a linear **fold mullion structure** forms parallel to the fold axes.

The differences in behaviour however, are minor. Accordingly, the models provide useful insight into the geometry of natural folds, which also indicates that assuming a linear viscous rheology is probably a reasonable first-order approximation.

The question arises as to what difference initial layer shortening will make to the dominant wavelengths predicted by equation (1). Workers who have taken layer shortening into account show

that the dominant wavelength changes with the amount of strain and equation (1) has been rewritten in the following form:

$$\mathbf{W}_{d} = 2 \pi h \left\{ \left[\mu_{L} \left(s - 1 \right) \right] / \left[6 \mu_{M} 2 s^{2} \right] \right\}^{1/3}$$

$$\tag{2}$$

where s = cube root of λ_1/λ_3 ; λ_1 and λ_3 being principal quadratic elongations perpendicular and parallel to the layer, respectively. It is clear from (2) that the dominant wavelength will change with strain. The theory predicts that as the deformation proceeds folds with progressively larger thickness to wavelength ratios will become those most amplified.

Since there is thickening, the notion of dominant wavelength must be adapted to the new layer thickness h_n , for which a **preferred wavelength** W_p is calculated:

$$W_p/W_d = (h_n/h)S$$

in which S is the principal stretch parallel to the layer.

Multilayer buckle folds

A multilayer may have a large, moderate or small competence contrast among the different layers. The competence contrast between layers influences the shape of folds. Like for single embedded layers, the thicker and stiffer multilayers tend to produce larger wavelength rounded folds. Hence, buckle-folded multilayers show a wider range of fold shapes than buckle-folded single layers. For example, multilayer, both in nature and in model experiments may show round-hinged folds, chevron folds, kink bands and conjugate folds.

Influence of elasticity

Recent work integrates the short-term elasticity and long-term viscous creep in visco-elastic behaviour. For perfectly elastic materials, the dominant wavelength is expressed in terms of the elastic modules contrast. Equation (1) becomes:

$$W_d = 2\pi h (E_L / 6E_M)^{1/3}$$

where E_L is the Young modulus of the layer and E_M the Young modulus of the matrix. A purely elastic solution makes no sense in permanently folded rocks. The dominant wavelength developed on an elastic layer within a viscous matrix is given by:

$$W_{d} = 2\pi h \left(G/P \right)^{1/2} \tag{3}$$

where G is the elastic shear modulus of the competent layer and P the layer-parallel stress. The viscosity of the matrix plays no role.



Folding

The dominant wavelength for a visco-elastic layer embedded in a viscous matrix is the response of combined elastic and viscous behaviours. Usually, one uses the dimensionless ratio R_{Wd} :

$$R_{Wd} = (4\mu_L \dot{\epsilon}/G)^{1/2} (\mu_L/6\mu_M)^{1/3}$$

in which a large part of the first term is:

$$D_e = \mu_L \dot{\epsilon}/G$$

called the **Deborah number**. This number determines whether deformation is effectively elastic or viscous. It can be considered as the ratio of the time scale of stress relaxation (μ_L/G) to the time scale of deformation ($1/\dot{\epsilon}$). If De>>1, stress relaxation takes much more time than deformation, which is effectively elastic. Conversely, if De<<1 stress relaxation is much faster than the deformation time and the deformation is effectively viscous. The Deborah number is not suitable to resolve if folding of a visco-elastic layer is effectively viscous or elastic because folding involves an additional time scale, that of amplification controlled by the growth rate. The dimensionless ratio suitable to determine the effective deformation behaviour of a visco-elastic layer is the ratio R_{Wd} of the viscous (equation1) to elastic (equation 3) dominant wavelengths. This ratio can be modified by using P = 4 $\mu_L \dot{\epsilon}$, assuming that the layer-parallel stress is effectively viscous and only the flexural stresses during folding are visco-elastic. Then:

$$R_{Wd} = (P/G)^{1/2} (\mu_L/6\mu_M)^{1/3}$$

The full analysis shows that a visco-elastic layer tends to have essentially either elastic or viscous folding behaviour for $R_{Wd} < 1$ and $R_{Wd} > 1$, respectively. In other words, the folding behaviour of a visco-elastic layer is comparable to either pure elastic or to pure viscous layer depending on whether the strain rate is relatively fast or slow. In summary, equation (1) is largely valid for most geological cases.

Fold trains

Experimental shortening of layered models made from rock analogue materials provide much information on the amplification to locking stages of buckle folds. Such experiments show that folds that form during the compression of the models do not generally develop synchronously. They form in a serial manner, either one after the other where the amplification of one fold stimulates the initiation and amplification of another next to it, or one after the other at random positions within the model. In general, folds develop sequentially outwards from the location of the initial instability.

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Serial development of buckle folds ultimately arranged in a fold train. Computer simulation with power-law viscous layer and matrix with stress exponents = 3. Competence contrast = 25 for the bulk shortening rate. After Schmalholz & Schmid 2012 *Phil.Trans. R. Soc.* **A28(370)** 1798-1826

Large-scale buckling - Characteristic wavelength of large-scale folds

During the development of large-scale folds, the upward movement of a buckled layer in an antiform or the downward movement in a synform must be affected by the gravity. Intuitively we understand that under the effect of gravity the antiform will tend to subside and the synform will tend to rise. As a result the buckle-folds will tend to flatten out. The layer-parallel compressive force will oppose this tendency.

Consider an elastic layer of thickness h floating on a viscous substratum of density ρ . Let $\Delta \sigma$ be the deviatoric stress necessary to produce elastic buckles in the layer. The characteristic wavelength for buckling of an elastic layer under the joint action of the force of gravity and the lateral deviatoric stress $\Delta \sigma$ has been analytically been shown to be:

$$W_{d} = \pi h \left[\left(2\Delta\sigma \right) / \left(h \rho g \right) \right]^{1/2}$$
(4)

while the deviatoric stress is given by the expression

$$\Delta \sigma = \left\{ \left(Eh \rho g \right) / \left[3 \left(1 - \upsilon^2 \right) \right] \right\}^{1/2}$$
(5)

As before, E and υ are Young's modulus and Poisson's ratio of the elastic layer, respectively. Experimental results agree remarkably with the theory.

Equations (4) and (5) show that a larger compressive stress is required to produce folds of greater wavelength. Since the deviatoric stress cannot exceed the strength of the rocks, equation (5) sets an upper limit to the size of buckle folds.

We may consider the case of buckling the earth's crust. A granitic rock has a strength of the order of 5.10^9 dynes cm⁻². The average crustal thickness is 30 km and its bulk density is c. 3 g.cm⁻³. Hence the maximum value of wavelength for crustal buckling is according to equation (4) about 100 km.

Driving forces

The chief driving forces involved are (1) lateral compression and (2) gravity.

Most natural folds result from lateral shortening. Plate tectonics (mostly convergence) or igneous intrusions normally cause lateral compression.

Gravity produces folding instabilities of two kinds: density instabilities and relief instabilities.

The most common example of density instabilities arise where dense rocks lay over less dense rocks. To establish gravitational equilibrium, the denser material will sink and the less dense material will rise. In the course of the process, layered material will fold. Eventually the low-density rocks may pierce overlying rocks in an antiform called diapir.

Relief instability arises where rates of erosion are slow compared with strain rates of the rocks involved, so that high tectonic relief tends to be levelled by lateral flow toward lower ground. Such levelling may proceed by sliding along fault surfaces (as in landslides) or by lateral spreading, which may results in **collapse folding**. Drape and forced folds that are shaped by faulting of their core also accommodate basement relief and are mainly controlled by gravity forces. Gravity driven folds formed near the surface are usually flexural folds accompanied by fracturing.

Folding in relation to stress

Stress in folding must be clearly separated from the forces acting on the system. If compressive stress parallel to layering may be true on a regional scale, the assumption on the outcrop scale is less secure. Indeed, a component of shear stress parallel to bedding is often operating during folding, and it changes at every stage.



Geometrical parameters of a buckle fold and the exerted stresses

During the initial stages of folding, when layers are virtually flat, stress is homogeneously distributed within both the layer and the matrix. The stress distribution becomes significantly heterogeneous once amplitudes become finite for the whole fold amplification. Numerical simulations mapping the magnitudes of the differential stress (i.e. $\sigma_1 - \sigma_3$, the diameter of the Mohr circle) show that it varies by more than two orders of magnitude in the layer with a high amplitude fold.

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Strain distribution in symmetric folds

Since a folded layer was initially planar, deformation-folding, which produces variable strain states along and across the layer, is fundamentally heterogeneous. One expects that the distribution of additional, folding-related strains throughout the folding body depends on how the mechanical properties of the layers change with progressive deformation. If there is little change then the additional strains are likely to be homogeneous. If, on the other hand, strain hardening or weakening is important in high strain areas, or there are changes in mechanical properties associated with the development of anisotropy such as foliation or crystallographic preferred orientations, then the additional strain field will likely be heterogeneous. The multiplicity of interfering geometrical, such layer thickness, and rheological parameters leads to extreme complexity. Again, the problem is simplified to a few reference symmetrical cases where strain ellipses are therefore symmetric with respect to the axial plane. Two cases dominate discussions: strain concentrated in the hinges or strain concentrated in the limbs.

Buckle folds: tangential longitudinal strain

The strain produced within the hinge of a buckled competent, homogeneous and isotropic layer involves tangential, layer-parallel extension around the outer arc and tangential, layer-parallel contraction around the inner arc. An immaterial surface of no strain, the **neutral surface**, exists within the folded layer between the extended outer arc and the shortened inner arc. This surface, the **neutral surface**, moves toward the core of the fold as it tightens while there is no elongation of lines perpendicular to the layer (constant thickness condition). The limbs remain undistorted on both sides of the strained hinge. For low viscosity contrasts the tangential longitudinal strain is an important contribution to the overall deformation.



The inward migration of the neutral surface with fold tightening implies distinguishing an incremental and a finite neutral surface. The fact that limbs are not strained suggests that neutral surfaces are continuous around the folds. Numerical modelling suggests that, instead, they both are discontinuous and terminate either at the bottom or top interface of the layer in the hinge to limb transition zone. Modelling further suggests that the incremental neutral surface migrates ahead of the finite neutral surface across viscous layers.



in Frehner (2011) Journal of Structural Geology, 33(10), 1501-1508

The effects of layer-parallel strain may be recognized by the development of hinge-parallel, layerperpendicular (radial) tension fractures on the convex, outer side of the neutral surface and smallscale bulges, mullions, folds or thrusts towards the concave, inner side. Since the curvature of a fold is maximal at the hinge and decreases to zero at the point of inflection, the absolute value of tangential longitudinal strain decreases from the hinge to the inflection point. It decreases also as it comes closer to the neutral surface.

The important features of the buckling-associated strain are:

1 - Deformation involves only bending about the fold axis; there is ideally no extension parallel to the fold axis so that there is plane strain throughout the fold. Consequently, the fold axis is parallel to the intermediate principal axis of strain λ_2 at each point in the fold.

2 - The layer being folded maintains its initial thickness measured normal to the layer at each point. However, the layer has been extended on the outer arc and shortened on the inner arc of each fold. Therefore, the $\lambda_1\lambda_2$ planes of the strain ellipsoids fan across each fold. The fan diverges away from the fold hinge on the outer side of the neutral plane, and converges toward the fold hinge on the inner side. Strain increases with distance measured normal to the neutral plane.

3 - The neutral surface maintains its initial area in which there is no strain.

4 - An initially straight lineation, lying in the surfaces to be folded and inclined at θ to the fold axis before folding, becomes curved during the folding and the angle with the fold axis remains constant in the neutral surface only. On a stereographic projection this lineation plots as a small circle with the semi-apical angle θ about the fold axis. Lineations on surfaces other than the neutral surface plot as more complicated curves. For surfaces on the outer side of the neutral surface this angle is increased, depending on the amount of strain. For surfaces on the inner side of the neutral surface this angle is decreased, again according to the amount of strain.

Flexural-slip and/or -flow

Folding of homogeneous but anisotropic layers involves bedding-parallel slip, which affects the strain distribution: shear strain is concentrated along distinct bedding planes. The magnitude of layer-parallel shear strain is maximum at the middle of a limb and decreases towards the inner and the outer arcs. On any surface parallel to the layer the shear strain is a maximum at the inflection point and decreases to zero at the hinge. The important strain features of flexural slip folds are outlined below: 1 - There is no or very little internal strain in the competent layers that maintain their initial thickness, without any distortion at every point around the fold (parallel, buckle folds in stiff layers).

2 – Bending about the fold axis and shearing on surfaces in directions normal to the fold axis involve plane strain at all points in the fold. The fold axis is parallel to λ_2 of the strain ellipsoid.

3 - The folded surface is parallel to a circular section of the strain ellipsoid at each point. The $(\lambda_1 \lambda_2)$ planes of the strain ellipsoids define a divergent fan.

4- The amount of bedding parallel slip or simple shear strain γ increases from 0 at the fold axis to a maximum at the inflexion points. The sense of shear reverses at the hinge. The amount of slipdisplacement along a top layer-boundary is therefore easy to calculate. In the plane of fold profile, the location where slip is to be calculated is specified by the inclination α of the layer surface at the chosen site. Slip is then determined using the following formula:

slip = (thickness of the folded layer). α in radians

The radian measure of an angle is the ratio of the length of the intercepted arc of a circle centred at the angle to the circle's radius. In a circle of radius 1, $1^\circ = 2\pi/360 \approx 0.01745$ radian). α is the dip in case of upright folds.



Strain in a buckled viscous layer at 50% shortening (viscosity contrast with matrix = 100) Finite Elemente Simulation by M. Frehner, ETH-Diplomarbeit, 2005

5 - Since there is no distortion within the planes of the folded layers (these are circular sections of the strain ellipsoid at each point) an initially straight lineation lying within the surfaces to be folded, and inclined at θ to the fold axis before folding, remains at this angle to the fold axis throughout the fold. The lineation therefore lies on a cone of semi-apical angle θ and with the fold axis as the axis of this cone. On a stereographic projection the lineation plots as a small circle.

Folding

Heterogeneous simple shear, in which the intensity of shear varies through the rock, can form and amplify folds. Shear strain is uniformly distributed across homogeneous, anisotropic layers in flow folding.

The main strain features of shear or slip folding are:

- 1 Since deformation is simple shear, there is a plane strain everywhere and the shearing planes are circular sections of the strain ellipsoid at each point in the fold there is a plane strain everywhere.
- 2 There is no reason for the shearing direction to be orthogonal to the fold axis. The only constraint is that the shearing direction is not parallel to the layer. The λ_2 axis is everywhere in the shearing planes, normal to the direction of shearing and may or may not be parallel to the fold axis.
- 3 Since the shearing plane is a circular section of the strain ellipsoids, the layer maintains constant thickness when measured parallel to the axial plane in profile. This implies thickened hinges and thin limbs.



4 - An initially straight lineation is distorted such that each point on the lineation is displaced in a systematic way parallel to the slip direction. The lineation is therefore distorted so that it lies in a plane defined by the original orientation of the lineation and the slip direction. It forms a great circle distribution on the stereogram.

Kinks

Conjugate kink bands are common structures in finely foliated rocks. One may define the directions of λ_1 , λ_2 und λ_3 from the symmetry on both sides of the axial plane and the relative movements in conjugate kink bands: λ_1 bisects the usually acute angle that contains the extension direction, λ_2 is the intersection of the axial planes and λ_3 bisects the usually obtuse angle that contains the shortening direction.

Summary

Folding reflects permanent strain in a heterogeneously deforming layered body, commonly but not always under lateral compression. In geology, folding normally involves a multilayered sequence in which the individual layers possess different mechanical properties and thicknesses. This means that,

Folding

in general, only one or two layers begin to fold initially and control the deformation from then on. Other weaker layers are more or less constrained to behave in the manner that the stronger layers determine.

The following principles are important:

- 1) Depending on the thickness and the mechanical properties, a single layer or a multilayer may develop larger or smaller buckle folds. Other conditions remaining the same, a thicker or a stiffer layer will form larger folds. Similarly, a coarsely bedded thick multilayer will form larger folds than a thinly bedded thin multilayer. The wavelengths of folds are large (compared with the total thickness of the multilayer) when the confining medium is weak or very incompetent. If the confining material is very competent the folds are much smaller than the total thickness of the multilayer.
- 2) The growth rate of the fold-amplitude depends largely on the competence contrasts with the consequence that a stiff layer or multilayer will buckle faster than a weaker layer or multilayer.
- 3) Buckled multilayers show a wider range of fold shapes than buckled single layers. Thus in natural situations, where layers of dissimilar thicknesses and viscosity are associated, folds of different wavelengths and growing at different rates may interfere with each other. Folds of different order can develop by such interference.
- 4) The shape of folds in buckled multilayer is essentially controlled by two factors: the competence contrast or ease of gliding within multilayer, and the nature of the medium within which the multilayer is confined.
- 5) There is a complex relationship between fold geometry and orientation and finite strain.

Three general physical mechanisms are responsible for folding. They are buckling, probably on a primary singularity, bending and amplification. The history of deformation at any point in a fold is complicated due to continuous changes in the states of stress and strain. It is important to note that the shape of the folded layer by itself does not enable the strain to be established at each point. Additional types of information are required. Our knowledge of strain variation throughout folded rocks rests on the results of strain analyses and on the strain distribution observed in various modelling experiments.

The following factors should be taken into account when analysing folds in any rock:

The rheological properties of the deforming material

The mechanical anisotropy of the rock mass

The stress field acting on the rock including the body-weight forces

The influence of any heterogeneity

The boundary conditions

Never forget that the scale of observation allows access to small parts of folds whose characteristics may change in other places.

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