LITHOSPHERIC STRENGTH PROFILES

To study the mechanical response of the lithosphere to various types of forces, one has to take into account its **rheology**, which means knowing how it flows. As a scientific discipline, rheology describes the interactions between strain, stress and time. Strain and stress depend on the thermal structure, the fluid content, the thickness of compositional layers and various boundary conditions. The amount of time during which the load is applied is an important factor.
- At the time scale of seismic waves, up to hundreds of seconds, the sub-crustal mantle behaves elastically down deep within the asthenosphere.
- Over a few to thousands of years (e.g. load of ice cap), the mantle flows like a viscous fluid.
- On long geological times (more than 1 million years), the upper crust and the upper mantle behave also as thin elastic and plastic plates that overlie an inviscid (i.e. with no viscosity) substratum.

The dimensionless **Deborah number** D, summarized as natural response time/experimental observation time, is a measure of the influence of time on flow properties.

Elasticity, plastic yielding, and viscous creep are therefore ingredients of the mechanical behavior of Earth materials. Each of these three modes will be considered in assessing flow processes in the lithosphere; these mechanical attributes are expressed in terms of **lithospheric strength**. This strength is estimated by integrating yield stress with depth. The current state of knowledge of rock rheology is sufficient to provide broad general outlines of mechanical behavior but also has important limitations. Two very thorny problems involve the scaling of rock properties with long periods and for very large length scales.

ELEMENTS OF RHEOLOGY

Definitions

Rheology describes the response of materials to an imposed stress system. This response varies considerably according to the physical conditions of deformation. Deformation can be either recoverable or permanent. A time-independent strain-stress relation describes recoverable deformation: when the stress is removed, the strain returns to zero. This includes **elastic** deformation and **thermal expansion**. Permanent deformation includes **plastic** and **viscous flow** or **creep** as well as **brittle** deformation (cracking, faulting, etc.) and requires time-dependent relations. The **flow** behavior of rocks to applied loads is empirically derived from laboratory experiments and can be compared to theoretical **constitutive equations**.

Gross geological differentiation

One can observe the nature of the deformation and describe mathematically the specific relationships between stress and strain, more precisely between the rate of application of stress and the rate of deformation, by experimentally subjecting rocks to forces and stresses under controlled conditions. Different rock types respond differently to the forces that act upon them. The response of each rock type depends on the conditions under which the force is applied. As a general observation:
- Under low confining pressures and temperatures like those at shallow depths in the crust, and on short time scales, the sample returns to its original dimensions when the load is removed (the material behavior is **elastic**) or has deformed by fracturing (the material is **brittle**).
- Under high confining pressures and temperatures like those at greater depths in the crust, the sample deformed, slowly and steadily without fracturing. It behaved as a pliable or mouldable material, that is, it deformed in a **ductile** manner. The deformed sample does not return to its original dimensions when the load is removed. At least part of the strain is permanent. The sample behaved as a **plastic** material.

- Whatever the deformation regime, two layers separated by a "weaker" layer may deform independently; they are **decoupled**. Conversely, strain and stresses are transmitted across **coupled**, physically bonded layers. The degree of coupling between the upper and lower crusts as well as between crust and mantle affects the mode of deformation and the structural style at any scale.

In summary, experiments show overall that cold rocks in the upper part of the crust are brittle and hot rocks in the lower part of the crust are ductile. This approximation will permit to outline some of the deformation mechanisms thought to exist in the lithosphere. However, deep and hot rocks can also behave elastically when they are deformed very fast. This is why seismic waves can travel through the entire Earth. Therefore, it is indispensable to understand and apply the mathematical models described in this chapter to comprehend and quantify rock behavior correctly.

Constitutive equations

Rheological equations are called constitutive equations because they describe time-dependent stress/strain behaviors resulting from the internal constitution of the material, such as thermal energy, pore fluid pressure, grain size, composition, mass, density, etc., and external parameters such as pressure, temperature, the chemistry of the environment, etc. In other words, constitutive equations involve material (**intrinsic**) properties and external (**extrinsic**) conditions. For each constitutive equation, a mechanical analog will be considered. However, a single equation describing material responses over the wide range of physical conditions would include too many parameters (visco+elasto+plastic rheology with temperature, grain size and pressure-dependent viscosity considering strain softening of friction angle) to be practicable. It is more suitable to consider few ideal classes of response (**rheologies** such as elastic, viscous, plastic, etc.) which some materials display to various degrees of approximation under various physical conditions. For these reasons, it is important to distinguish between **materials** and **responses**.

Material

Mechanical properties

Homogeneity

A strictly **homogeneous** material is one in which all pieces are identical. In other words, material composition and properties are independent of position. Materials are not **homogeneous** are **heterogeneous (inhomogeneous)**. *Isotropy*

Homogeneous materials may be mechanically **isotropic** or, on the contrary, **anisotropic**. An isotropic material is one in which the mechanical properties are equal in all directions: material properties are independent of the direction in which they are measured. Sandstones and granites can be considered as homogeneous and isotropic materials.

Layered and foliated rocks are statistically homogeneous, anisotropic materials if the scale of the layering or fabric is small relative to the scale of deformation.

Parameters

Material parameters are quantities that define some physical characteristics. Parameters such as elasticity, rigidity, compressibility, viscosity, fluidity are not constant. They are scalars in isotropic materials and tensors to account for the directional dependence of parameters in anisotropic materials. They also depend on extrinsic parameters such as temperature and pressure and are related to the rheological properties of the material. It is consequently very important to know the physical dimensions of these material parameters.

Types of material responses

A body of material will permanently deform only if stress exceeds its **strength**. In simple terms, strength is the maximum differential stress that a material can support under given conditions. Three

end-members, rheological behaviors can describe the theoretical continuum deformation: elasticity, viscous flow, and plasticity. These three behaviors refer to three ideal rheological models, which are the characteristic relationships between stress, strain and time, exhibited by analog objects under deformation. To understand deformation in rock, it is convenient to examine separately and in combination the three rheological end-members:

- Reversible elastic rheology, at small stresses and strains.
- Irreversible, rate-dependent **creep** (viscous rheology), usually thermally activated.
- Rate independent, instantaneous yielding at high stresses (plastic rheology), which is often pressure sensitive but temperature independent.

This introduction discusses only the one-dimensional macroscopic behavior.

<div style="border:1px solid">

Food for thoughts

Drop on the floor: (1) a jellybean (or a gum eraser), (2) a rusk or a cracker, (3) a ball of dough (or soft clay or silly putty) and (4) some honey or heavy syrup. They all are submitted to the same gravity forces and they all follow the same trajectory. Describe their difference when they hit the ground, and relate their behavior to those introduced up to now.

</div>

Elastic deformation

Elastic rheology has wide applications in geodynamics and constitutes a fundament to the plate tectonic theory according to which the lithospheric plates do not internally deform significantly over geological time. The lengthwise compression or extension of a helical spring (Hookean body) demonstrates the linear elastic deformation and response.

Definition

Deformation is perfectly **elastic** when straining or unstraining takes place instantaneously, spontaneously once the load is applied or removed, and strain is strictly proportional to stress. An elastic medium deforming instantaneously and reversibly under local stresses has no **memory** of past deformations and stresses. Strain exists only if stress exists, whether deformation occurs in seconds or over millions of years: elastic deformation is **time-independent** and **recoverable**. Importantly also, the principal axes of strain must coincide with the principal axes of stress in isotropic materials. An elastic material **stores the energy** used to deform it. This is important: the elastic energy that was stored during the elastic rock deformation is suddenly released during an earthquake.

linear, elastic deformation
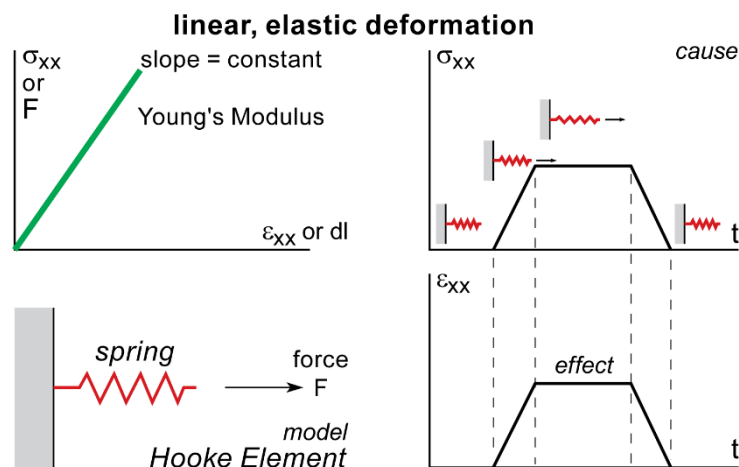
Occurrence in rocks

When a seismic (acoustic) wave from an earthquake or an explosion travels through a body of rock, the rock particles are infinitesimally displaced from their equilibrium positions. They return to these positions once the disturbance has passed; there is no **permanent distortion** of the rock.

The same kind of temporary and **recoverable deformation** occurs if a rock or mineral specimen is loaded axially in the laboratory at relatively low stress (the latter meaning differential stress) and low hydrostatic pressures and temperatures. The instant the load is applied, the specimen begins to deform. The ideal relationship between the axial stress and the **longitudinal strain** is linear. Provided stresses and strains remain small, the specimen does not break and instantly returns to its original unstrained size and shape once the deforming load is removed.

Modulus of elasticity

The linear relationship between stress and longitudinal strain in tension as well as in compression is written:

$$\sigma = E\,\varepsilon = E\left(\ell - \ell_0\right)/\ell_0 \qquad\qquad (1)$$

This equation is the Hooke's law where:

σ is the applied stress,

ε is the dimensionless extensional strain, proportional to σ,

ℓ the deformed-state length, ℓ_0 the original length, and

E is a constant of proportionality (for example the strength of the spring) known as **elasticity modulus** or **Young's modulus**, which has the same dimension ($[M]\,[T]\,[L]^{-1}$) and unit (Pa) as stress because the strain is dimensionless.

Note that $d\varepsilon/dt = 0$. There are no time-dependent effects.

Poisson number

The Poisson ratio, ν, is used to express the relationship between volume change and stress and refers to the phenomenon that elastic materials extended or shortened in one direction are simultaneously shortened or extended along the perpendicular direction. The **Poisson effect** is this lateral strain effect. The dimensionless Poisson's number is the ratio of elastic lateral, transverse shortening of an extended rod to its longitudinal extension.
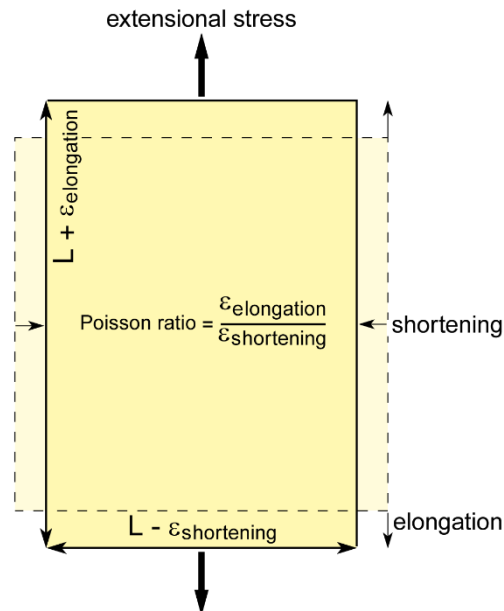
$$\nu = \varepsilon_{parallel-to-extensional-stress}\Big/\varepsilon_{perpendicular-to-extensional-stress} \qquad (2)$$

It follows from equations (1) and (2) that shortening produced by one principal stress in its direction gives rise to tensile strains, equally in the other coordinate directions in isotropic materials:

$$\varepsilon_{xx} = \sigma_{xx}/E \qquad\qquad \varepsilon_{yy} = -\nu\left(\sigma_{xx}/E\right) \qquad\qquad \varepsilon_{zz} = -\nu\left(\sigma_{xx}/E\right)$$

Since these equations are linear, elastic strain in each principal stress direction is the sum of the strains due to each principal stress:

$$\varepsilon_{xx} = \left(1/E\right)\left[\sigma_{xx} - \nu\left(\sigma_{yy} + \sigma_{zz}\right)\right]$$

$$\varepsilon_{yy} = \left(1/E\right)\left[\sigma_{yy} - \nu\left(\sigma_{xx} + \sigma_{zz}\right)\right]$$

$$\varepsilon_{zz} = \left(1/E\right)\left[\sigma_{zz} - \nu\left(\sigma_{xx} + \sigma_{yy}\right)\right]$$

Definition of the transverse shortening (Poisson effect)
for the infinitesimally small extension of an isotropic, elastic body

These equations further show that normal strains are independent of the shear stress components in isotropic materials.

The **Poisson number** shows how much a core of rock bulges as it is shortened. For rocks, it is typically between 0.25 and 0.33, which indicates that lateral strain is about one-quarter of the imposed strain.

Shear modulus

Equations (1) and (2) consider one-dimensional, tensile or compressional experiments. If the deformation is simple shear, the elastic resistance to sliding on a plane is also a constant proportionality: the **shear modulus** G (also called **rigidity modulus,** expressed in Pa) defined as the ratio of shear stress τ to shear strain γ :

$$\tau = G\gamma$$

Bulk modulus

In pressure tests, the rock sample is subjected to controlled hydrostatic pressure. As there is volume strain, length strain and shear strain associated with appropriate stresses, several other constants replacing E are defined for isotropic elastic materials.

For a uniform hydrostatic pressure P producing a uniform dilatation, the **bulk modulus** or **incompressibility** K is the ratio of the hydrostatic pressure to that dilatation.

$$P = K\frac{V\text{-}V_0}{V_0} = K\frac{d\,v}{v}$$

where V and V_0 are the final and initial volumes, respectively.

The inverse of the bulk modulus $k = 1/K$ is the compressibility. The units are Pa.

For isotropic elastic materials, only two parameters are independent. Therefore, four quantities E, ν, G and K are related by the expressions:

$$G = E/\big[2(1+\nu)\big] = \big[3K(1-2\nu)\big]/\big[2(1+\nu)\big]$$

Relationship between elastic strain and stress

For purely elastic deformation, strain directions coincide with stress directions. Normal stresses (e.g. hydrostatic pressure) tend to change the volume of the material and are resisted by the body's bulk

modulus, which depends on Young's modulus and Poisson ratio. Shear stresses tend to deform the material without changing its volume and are resisted by the body's shear modulus.

Strain energy

In an ideal elastic body, all the energy introduced during deformation remains available for returning the body to its original state. This stored, **internal strain energy** does not dissipate into heat, which makes elasticity the only thermodynamically reversible rheological behavior.

Ultimate strength

A body deforming elastically may brutally break: the stress value at the point of rupture is the ultimate strength.

Non-linear elasticity

Some materials have a modulus of elasticity that varies with stress and deformation but recover their initial shape along a different stress/strain path than the deformation path. Such materials are elastic with **hysteresis**.

Viscous deformation

The intricately folded rocks that exist in some tectonic environments indicate that under certain conditions rocks can undergo large permanent strains without obvious faulting or loss of material continuity. Most of the structures of interest to geologists involve strains that are **permanent** and irreversible: There is no **recovery** (i.e. elimination of strain: the rock remains strained) after removal of the deforming stress. The energy input to deform the rock is principally dissipated into heat. Hence, any rock deforming viscously emits heat.

Definition

The **ideally viscous** (**Newtonian**) behavior is best exhibited by the flow of fluids. The along-axis compression or extension of a dashpot (Newtonian body), a porous piston that slides in a cylinder containing a fluid, demonstrates this type of deformation and response. When a force is applied to the piston, it moves. The rate at which it moves depends on the stress intensity. The resistance of the fluid to the piston moving through it represents viscous resistance to flow. When the force is removed the piston does not go back: Deformation (displacement) ceases but is irreversible (non-recoverable) and permanent.
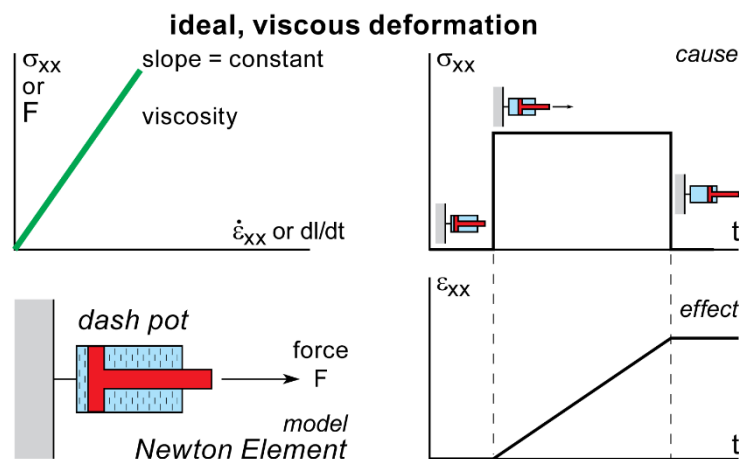
Viscosity

The ideally viscous (Newtonian) material is incompressible. In this material, the strain-rate is proportional to the applied stress:

$$\sigma = \eta \, \dot{\varepsilon} \qquad\qquad (3)$$

Where: $\dot{\varepsilon}$ is the strain-rate (i.e. $d\varepsilon/dt$, the total strain derivative with respect to time) and,

$\quad\quad\eta$, the constant of proportionality, is the **viscosity**.

The unit of viscosity has the dimension of stress $[ML^{-1}T^{-2}]$ multiplied by time, therefore $[ML^{-1}T^{-1}]$. It is 1 Pa.s. The bulk viscosity of the mantle is of the order of 10^{21} Pa.s. Typical geological strain-rates are 10^{-12} s^{-1} to 10^{-15} s^{-1}.

Equation (3) says that the higher the applied stress, the faster the material will deform. Conversely, a higher flow rate is associated with an increase in the magnitude of shear stress. The total strain is dependent both on the magnitude of the stress and the length of time for which it is applied. Large permanent strains whose amount is a function of time can be achieved. Like for elasticity, strain and stress appear or disappear simultaneously, with the corollary that any deformation along time produces local shear stress.

Note that an ideal viscous fluid has no shear strength and its viscosity is independent of stress. All of the deforming energy is dissipated into heat: the viscous behavior is **dissipative**.

For anisotropic materials, a system of nine linear equations replaces equation (3).
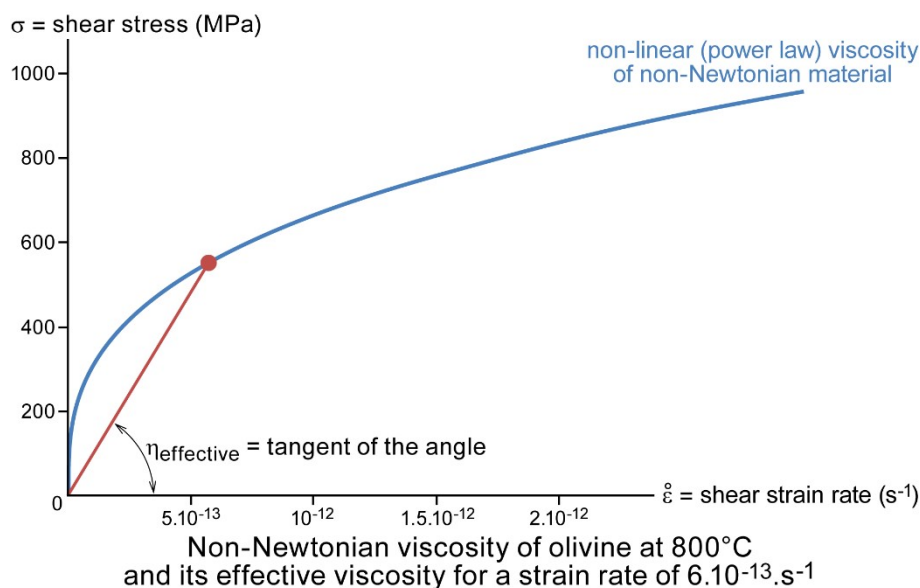
Non-linear behavior

The linear viscosity is a close approximation to that of real rocks at high temperatures (1000-1500°C) and slow strain-rates (10^{-12} to 10^{-14} sec^{-1}). Such physical conditions exist in the lower lithospheric mantle. Two important facts complicate the viscous behavior of upper mantle and crustal rocks:

(i)   Viscosity is a strong exponential function of temperature (the Arrhenius relationship). Viscosity decreases with temperature.

(ii)   The proportionality between stress and strain-rate is typically not linear but governed by a law stating that stress raised to some power is proportional to strain-rate. The stress exponent of rocks is typically between 3 and 5 so that the application of a doubled stress results in an 8 to 32 fold increase in strain-rate.

The simplest equation to describe the nonlinear behavior is:

$$\sigma^n = \mathbf{A}\dot{\varepsilon} \quad \text{or} \quad \sigma = \mathbf{B}\dot{\varepsilon}^{\frac{1}{n}} \tag{4}$$

where **A** and **B** are complex material functions. Equations (4), like equation (3), relate the stress to the strain-rate. The flow power index n is a dimensionless material characteristic.

Non-Newtonian viscosity of olivine at 800°C
and its effective viscosity for a strain rate of 6.10^{-13}.s^{-1}

28

The result is that the viscosity of rocks in the related stress/strain-rate plot is a curve. Because of the nonlinear behavior of rocks, one uses the **effective viscosity** η_{eff}, which is the ratio of the shear stress to the shear strain-rate. The effective viscosity is a material coefficient, not a material property. It is a convenient description of the viscous behavior under known conditions of pressure, temperature, stress, and strain-rate. Yet, importantly, the effective viscosity depends itself either on the stress or the strain-rate. This is apparent when reformulating equations (4):

$$\sigma = A\sigma^{1-n}\dot{\varepsilon} \quad \text{or} \quad \sigma = B\dot{\varepsilon}^{\frac{1}{n}-1}\dot{\varepsilon}$$
$$\sigma = \eta_{eff}(\sigma)\dot{\varepsilon} \quad \text{or} \quad \sigma = \eta_{eff}(\dot{\varepsilon})\dot{\varepsilon}$$

In the corresponding curve one readily sees that the rheology is strain-rate hardening because the stress increases with increasing strain-rate but it is also strain-rate softening because the effective viscosity decreases (softens) with increasing strain-rate.

Plastic deformation

Plasticity deals with the behavior of a solid.

Definition

The **ideally plastic** material is a solid that does not deform until a threshold strength = stress (the **yield stress)** σ_c is reached; the solid is incapable of maintaining stress greater than this critical value σ_c. There is no permanent deformation if the applied stress is small (i.e. smaller than the yield stress); at the yield stress, permanent and irreversible deformation proceeds continuously and indefinitely under constant stress (as for viscous deformation); it is therefore theoretically possible to get unlimited plastic deformation. The amount of strain, i.e. of plastic flow is a function of time, increasing as long as the yield stress is maintained. Plastic flow on a macroscopic scale strictly is spatially continuous (uniform). Geologists extended the concept to discontinuous, non-linear frictional sliding (e.g. faulting where the force is not proportional to displacement). In both cases, deformation is a shear strain at constant volume and can only be caused by shear stress. For rocks and soils, so-called **non-associated plastic flow laws** involve volume change during plastic flow.

Yield criterion

Ideal plastic behavior is rate-independent. It assumes there is no deformation below the yield stress and that during deformation the stress cannot rise above the yield stress, except during the deformation acceleration. Yet, the strain-rate is independent of stress. The constitutive equation where the stress for flow is a constant is the **Von Mises yield criterion**:

$$\sigma \leq K$$

which requires that the magnitude of stress cannot exceed the yield stress $K = \sigma_c$. This particular stress is the characteristic **strength** of the material. The strength is not a constant but a dependent variable; it is a function of the three principal stresses applied and of the temperature, pressure, of the nature of the material, the chemical composition of the adjacent rocks and, finally, of the history of the deformation (i.e. the intermediate steps followed to attend the yield value). Time does not appear in the constitutive equation. Neither strain nor strain-rate are related to stress.

Model

The conceptual model to simulate this type of deformation and response is a weight resting on a flat and rough surface and pulled horizontally (Saint-Venant body). The weight does not move as long as the applied force is less than the frictional resistance. At a threshold force, the weight begins moving, and a constant force that just overcomes the frictional resistance keeps it moving. When the force is removed or decreases below the threshold magnitude, the weight stays in its new position.

ideal, plastic deformation
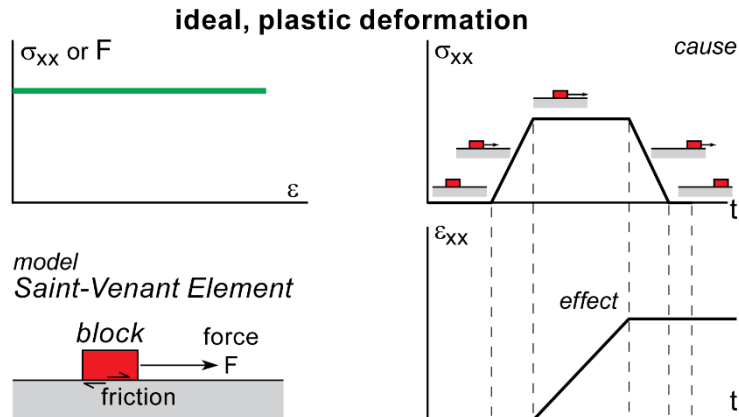
The analogy is not a case of plastic deformation. It describes the relationships between stress, strain (displacement) and time, but the weight remains undeformed. A plastic body is deformed while displaying similar relationships between stress, strain and time. The flow rule is a function of stress:

$$\dot{\varepsilon} = \begin{cases} 0 & \text{if} \quad \sigma \leq K \quad \text{and} \quad \dot{\sigma} = 0 \\ \dot{\Lambda}.f(\sigma) & \text{if} \quad \sigma = K \quad \text{and} \quad \dot{\sigma} = 0 \end{cases}$$

In which $\dot{\Lambda}$ is a positive and indeterminate proportionality factor. This indeterminacy, along with the existence of K, distinguishes perfectly plastic from viscous materials.

Note that the stress does not determine the strain-rate, they are not proportional, but the stress and strain-rates behave analogously. An ideally plastic material displays another characteristic kind of deformation: strain only takes place in localized regions where the critical value of stress is reached.

Viscoelastic deformation

Real rocks, whether shallow or deep in the Earth, combine the properties of ideal viscous, plastic and elastic bodies. Their strain has elastic properties on the time scale of seismic waves, and plastic or viscous components or any combination of these behaviors on the long, plate-tectonics time scale.
Rocks are **elasto-visco-plastic**. A material that combines viscous and elastic characteristics is **viscoelastic**.
Viscoelastic behaviors are useful in modeling the response of the earth's lithosphere, which exhibits elastic deformation over a short time-scale but gradually flows on long terms. Mechanical models of such materials are a spring and a dashpot arranged in series or in parallel.

Firmo-viscous (i.e. strong-viscous) behavior (Kelvin body)

A spring and a dashpot in parallel (known as the Kelvin body) simulate the strong-viscous (firmoviscous) deformation.

When the force is applied both the spring and the dashpot move simultaneously. The deformation (i.e. the displacement) is the same for both. However, the dashpot and spring stresses are in parallel and thus the total stress is the sum of the stress in the spring and the stress in the dashpot. From adding equations (1) and (3), the Kelvin model has the constitutive equation:

$$\sigma = E\varepsilon + \eta\dot{\varepsilon}$$

Note that: If $E \rightarrow 0$ (the spring has zero stiffness), the flow is viscous.
              If $\eta \rightarrow 0$ (i.e. for low viscosities) the material is elastic.

linear, firmo-viscous deformation
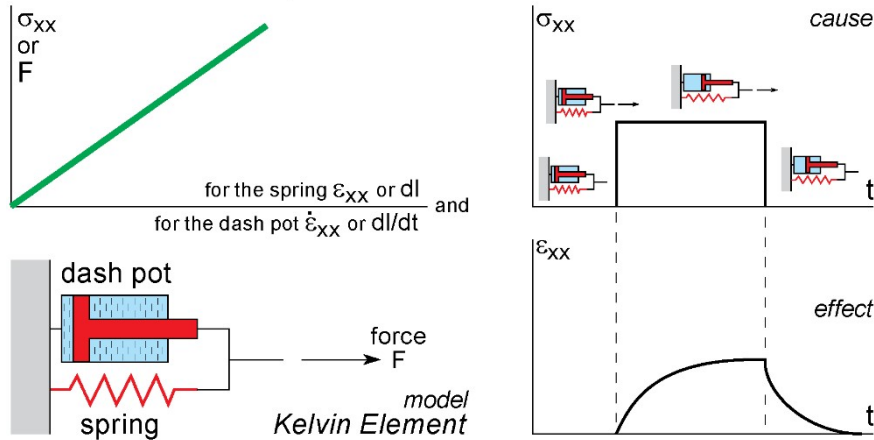
Because of the parallel arrangement of the dashpot and the spring, the Kelvin body is characterized by a complete recovery of its geometry when an applied force is removed. However, the dashpot retards the elastic shortening of the spring. Stress gradually decreases until all of the strain is recovered. When the force is removed, the strain does not immediately disappear. This is known as the **elastic after-effect.** Note again that a suddenly applied stress will induce no instantaneous strain because of the dashpot in parallel to the spring.

Some unconsolidated geological materials approximate this behavior. However, this model is inappropriate to describe large tectonic strains because stresses become excessively large if the strain of the elastic spring becomes very large.

Visco-elastic behavior (Maxwell body)

A visco-elastic material obeys the viscous law (i.e. strain is a function of time) but behaves elastically at the instant of stress application and for a stress of short duration.

<u>Rheology</u>

A spring and a dashpot arranged in series (Maxwell body) represent this type of behavior. This arrangement indicates that the spring and the dashpot take up the same stress (the forces and the stress they represent are equal in the spring and the dashpot) but the total strain (as well as the strain-rate) is the sum of the spring deformation and the dashpot deformation.

linear, visco-elastic deformation
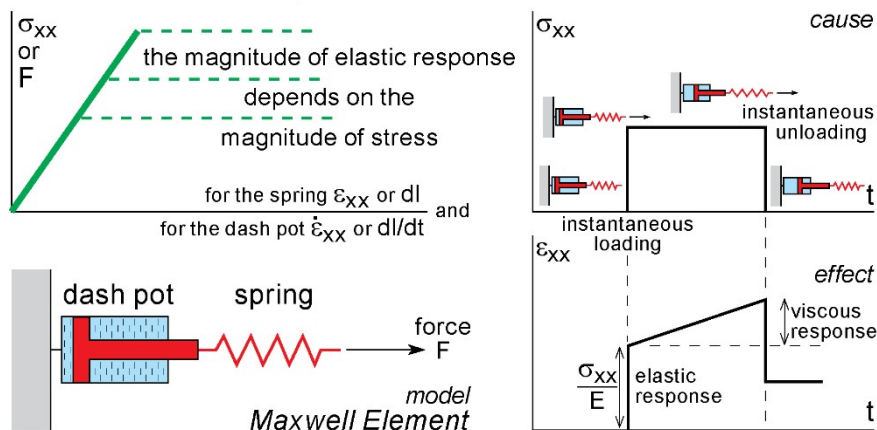
This model is befitting large tectonic strains since there is no limitation from the elastic spring.

When a Maxwell body is subjected to stress, the linear strain-rate $\dot{\varepsilon}$ is composed of two parts: (1) The spring is instantaneously and elastically extended at a strain-rate proportional to the stress rate $\dot{\sigma}$; (2)

the dashpot responds only to the instantaneous level of stress and moves at a constant rate controlled by the viscosity for as long as the force is applied. Using equations (1) and (3), the corresponding constitutive equation is:

$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E} + \frac{\sigma}{\eta} \tag{5}$$

Note that: If $E \rightarrow \infty$ (the spring is rigid), the material behaves like a viscous material for long-term loads.

If $\eta \rightarrow \infty$ (i.e. for high viscosities) the material behaves like an elastic material for loads of short duration.

Note also that, in contrast to the Kelvin body, a suddenly applied stress will induce an instantaneous elastic strain because the spring is free to respond.

Relaxation time

For simulating a permanent deformation, the spring is extended and then held at a given extension (i.e. the material is under constant imposed strain). The spring deforms instantaneously and elastically as soon as a constant force F is applied. The spring instantaneously stores energy gradually converted into permanent viscous deformation as the dashpot moves at a constant rate through the fluid until the spring has returned to its unstressed length. Provided no force is still applied, the spring only may recover its original length, but the dashpot maintains some irreversible flow. The viscosity of the liquid retards elastic recovery, which is rapid at first but decays as the tension in the spring decays. When the application of force F stops, stress neither vanishes nor persists; instead, it dissipates exponentially. The reduction in stress that occurs with time under a fixed, constant strain or displacement is called **stress relaxation.**
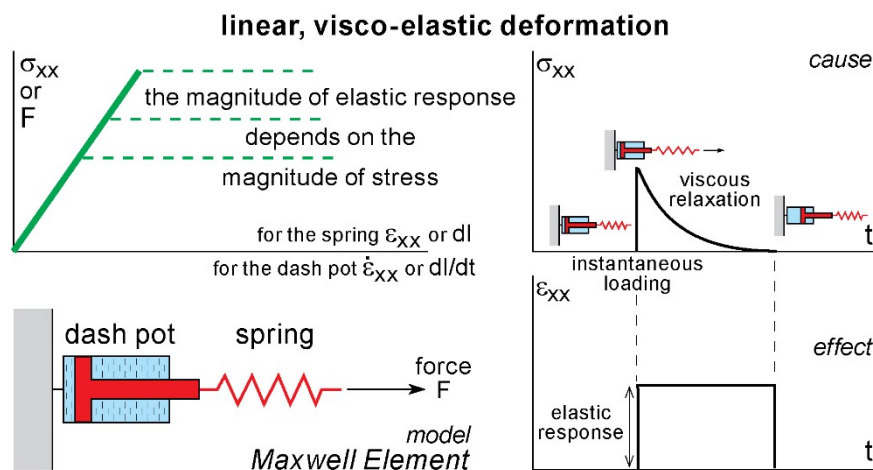
The time taken for the stress to decay to e^{-1} of its initial value is known as the **Maxwell relaxation time** t_M. e is the base of natural logarithms. The relaxation time t_M can be obtained by dividing viscosity by the shear modulus:

$$t_M = \eta/G$$

Exercise

Calculate the relaxation time for olivine-dominated rocks (Earth mantle): $G = 10^{12}$ dyn.cm^{-2} *and* $\eta = 10^{22}$ poise.

The dimensionless **Deborah number** D incorporates both the elasticity and viscosity to characterize the **fluidity** of a visco-elastic material. Its definition is the ratio of a characteristic material time (how long does it take to naturally relax/deform) to a characteristic process/observation time (time of the material response in experimental or numerical tests):

$$D = \mu\dot{\varepsilon}/G$$

with μ the viscosity, $\dot{\varepsilon}$ the strain-rate and G the elastic modulus. Small Deborah numbers define viscous materials that easily flow; high Deborah numbers define non-Newtonian to elastic materials.

The Deborah number depends on the observation time. What does it mean? Water behaves as a fluid under natural and "usual" conditions (i.e. above freezing temperatures). As an experiment, jump from high into a pool or a river or the sea: you can be injured as if you crashed against a solid. This is "observation time": fluids behave closer to solids under some circumstances.

Anelastic behavior (Kelvin-Voigt body)

Although the pre-seismic strain in the upper crust is mostly recoverable, the complete response of rocks is not instantaneous. This type of non-perfectly elastic behavior where unstraining is recoverable but not instantaneous (time-dependent) is called **anelastic**.

The mechanical model of a standard linear solid comprising a spring (a Hookean body) in series with a unit of parallel dashpot and spring (a firmo-viscous Kelvin body) represents the anelastic behavior. A plot of strain against time illustrates this for a specimen loaded axially. When the force is applied there is an instantaneous elastic response due to the first spring. Stretching of the second spring is inhibited by the viscosity of the fluid in the dashpot with which it is in parallel. Later flow comprises elastic deformation delayed by the action of viscosity. When the force is removed, there is again an instantaneous elastic response and then the strain decays asymptotically back to zero.
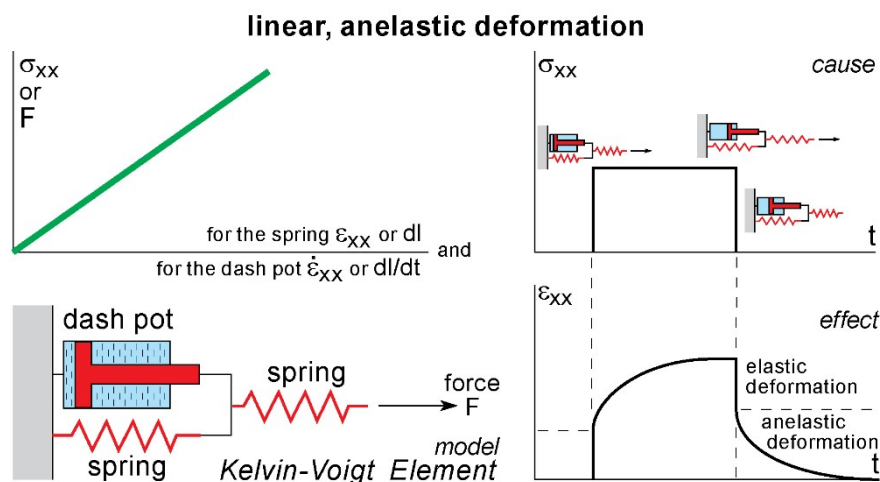

Anelasticity is of great importance in many rock mechanics problems associated with mining, tunneling, and quarrying. Anelastic behavior is associated with reversible, time-dependent slipping along grain boundaries (internal friction). This strain response absorbs energy from seismic and any sound waves moving through a rock. The magnitude of this **attenuation** depends on environmental parameters such as temperature, pressure, and the frequency of the propagating wave, for example as they pass through parts of the upper mantle of the earth. This behavior is also termed recoverable **transient creep.** The anelastic motion of a fluid phase, as either a melt or a saturated hydrous fluid, will give rise to energy losses from sound waves propagating through rock and to a parallel decrease

in the velocity of the waves through the rock. This phenomenon has been widely postulated as the source of the upper mantle low-velocity zone.

Elasto-plastic behavior (Prandtl body)

The **elasto-plastic** deformation is simulated by the series arrangement of a spring and a weight (the Prandtl body).
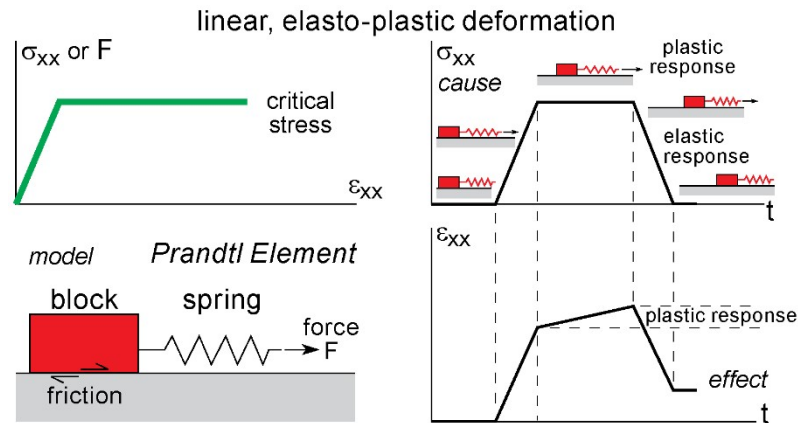
Stress below the **yield strength** first stretches the spring. Then the weight is pulled at **yield stress** close to the frictional resistance of the weight and comes to rest in a new position where the stress state is the same as in the initial state. Yield defines the point where permanent deformation begins.

Viscoplastic deformation

To eliminate the indeterminacy on $\dot{\Lambda}$ in the flow rule of perfectly plastic materials, one must introduce hardening in the model. The conceptual model (Bingham body) to simulate this type of deformation and response is a weight resting on a flat and rough surface (Saint-Venant body) in parallel with a linear viscous dashpot (Newton body).
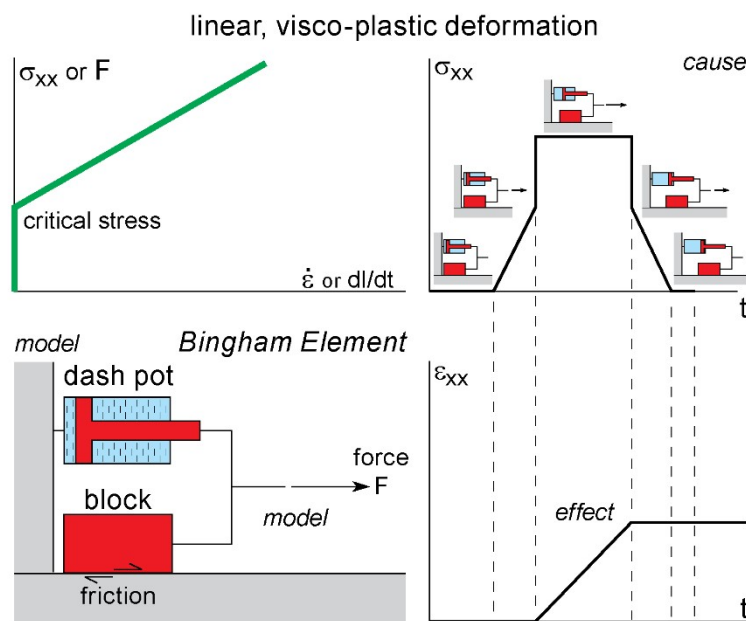
If the stress is smaller than the yield stress of the weight, the model is rigid. At the yield stress, an overstress $(\sigma - K)$ is exerted on the dashpot. Accordingly, the flow rule of a viscoplastic material is:

$$\dot{\varepsilon} = \begin{cases} 0 & \text{if} \quad \sigma < K \\ (\sigma - K)/\eta & \text{if} \quad \sigma \geq K \end{cases}$$

Visco-elasto-plastic deformation

If a viscoelastic material has also yield stress, the behavior is **visco-elasto-plastic**. Materials that behave in this manner are Bingham materials. They are represented by a spring, a dashpot and a friction block on a raw surface (Hookean + Newtonian + Saint-Venant bodies) arranged in series or in parallel. The deformation is distributed between the mechanisms according to their material properties.

On applying a load there immediately is elastic deformation (from the spring). If the applied force is greater than friction resistance of the weight, this element will come into play together with the dashpot and visco-plastic flow will take place at a constant rate. The strain/time curve is similar in shape as for elasto-viscous behavior in both loading and unloading phases. The main difference is that flow begins only after the force applied on the Bingham material has exceeded the frictional resistance of the Saint-Venant body, which simulates the limit of plasticity. Below this limit, only elastic deformation can take place, whereas flow takes place no matter how low the load is on the elasto-viscous body. In this respect, visco-elasto-plastic models portray the properties of rocks more accurately than other models.

For the two-dimensional case

The following mathematical relations are written out for two-dimensional spaces, in the x- and y-direction.

The starting point for the analysis of deformation rates is the spatial velocity tensor **L**:

$$\mathbf{L} = \begin{vmatrix} \dfrac{\partial v_x}{\partial x} & \dfrac{\partial v_x}{\partial y} \\ \dfrac{\partial v_y}{\partial x} & \dfrac{\partial v_y}{\partial y} \end{vmatrix}$$

The symbol v stands for velocity. With **L**, the relative velocity of a material point Q at x+dx can be indicated opposite to a material point P at x:

$$d\mathbf{v} = \mathbf{L}d\mathbf{x}, \quad d\mathbf{v} = \begin{vmatrix} dv_x \\ dv_y \end{vmatrix}, \quad d\mathbf{x} = \begin{vmatrix} dx \\ dy \end{vmatrix}$$

L is a second-order tensor and can be divided into a symmetrical and an antisymmetrical tensor:

$$\mathbf{L} = \mathbf{D} + \mathbf{W}$$

$$\mathbf{D} = \frac{1}{2}\begin{vmatrix} \dfrac{\partial v_x}{\partial x} + \dfrac{\partial v_x}{\partial x} & \dfrac{\partial v_x}{\partial y} + \dfrac{\partial v_y}{\partial x} \\ \dfrac{\partial v_y}{\partial x} + \dfrac{\partial v_x}{\partial y} & \dfrac{\partial v_y}{\partial y} + \dfrac{\partial v_y}{\partial y} \end{vmatrix}, \quad \mathbf{W} = \frac{1}{2}\begin{vmatrix} 0 & \dfrac{\partial v_x}{\partial y} - \dfrac{\partial v_y}{\partial x} \\ \dfrac{\partial v_y}{\partial x} - \dfrac{\partial v_x}{\partial y} & 0 \end{vmatrix}$$

The coordinates of the symmetrical part **D** indicate the changes for lengths and angles of material line elements (i.e. deformation) and the tensor **W** the rotation.

Stress-changes in a material are generally caused by deformations and not by rotations. The linear viscosity µ of a liquid is the proportionality factor between stress and deformation-rate defined for the one-dimensional case as:

$$\sigma_{xy} = 2\mu \mathbf{D}_{xy} = \mu \frac{\partial v_x}{\partial y}$$

Extension to two dimensions for non-compressible materials provides:

$$\sigma_{ij} = -p\delta_{ij} + 2\mu \mathbf{D}_{ij}, \quad i,j = 1..2$$

in the so-called index notation and written out:

$$\sigma_{xx} = -p + 2\mu \frac{\partial v_x}{\partial x}$$

$$\sigma_{yy} = -p + 2\mu \frac{\partial v_y}{\partial y}$$

$$\sigma_{xy} = \sigma_{yx} = \mu \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right)$$

where p is pressure, δ_{ij} the Kronecker delta, and $\sigma_{xx}, \sigma_{yy}, \sigma_{xy}$ and σ_{yx} the components of the two-dimensional stress tensor.

Brittle/ductile behavior

Rocks like many usual materials are reacting elastically only for small strains. When the yield stress is attained, there are two possible behaviors:
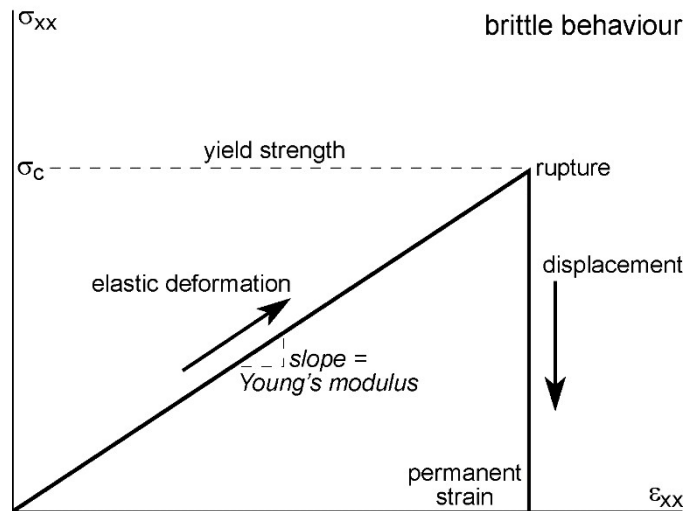
− rupture, when the continuity of the deformation is lost. This behavior is **brittle**, where elastic deformation leads to **failure** before plastic deformation and the material loses cohesion through the development of **fractures** or **faults**.

− a plastic, irreversible flow (**creep**) while the continuity holds. The deformation is similar to the usual viscous flow, but it can be observed only when the yield stress is reached. This behavior is **ductile**.

Brittle behavior

The maximum stress a rock can withstand before beginning to deform permanently (inelastic behavior) is its **yield point** or **elastic limit**.

At this stress level, and low confining pressure and low temperature, most rocks and minerals break into fragments. Localized deformation at the yield stress is permanent; therefore, the brittle behavior yields a plastic deformation. With this definition, brittle behavior refers to a state of stress at which rupture, i.e. loss of cohesion occurs.

The brittle mode of deformation includes both fracturing and sliding, hence governs the development of joints and faults. Open fractures are generally parallel to the axis of loading and there is no offset parallel to the fracture surface. Faults are inclined to the axis of loading and show a localized offset parallel to their surface.

σxx   brittle behaviour

yield strength
σc - rupture

displacement

elastic deformation

slope =
Young's modulus

permanent
strain   εxx

Cataclastic flow is achieved by distributed fracturing and the relative movement of rock fragments. The mechanical properties of rocks deforming in the brittle regime are nearly insensitive to temperature, but very sensitive to strain-rate and confining pressure. Indeed, friction critically depends on the pressure acting across planes. Therefore, the fracture strength of rocks at the Earth's surface is the lowest and is controlled by the failure criteria only, but it increases with depth due to increasing **lithostatic pressure**. In reality, friction is controlled at any depth by the **effective pressure**, the difference between the lithostatic pressure and the **pore fluid pressure** acting against it.

Ductile behavior

The **ductile** behavior is a non-mechanistic, phenomenological term to describe the non-brittle modes of deformation; rocks exceeding their yield point deform by distributing the permanent strain in a smoothly varying manner throughout the deformed mass without any marked discontinuity. The term **ductility** is used in geology to indicate the capacity (% of strain) of rock to undergo permanent deformation without the development of macroscopic fractures. The term does not refer to the microscopic deformation mechanisms:
- **Ductile flow** commonly involves deformation of individual grains by several solid-state deformation mechanisms such as crystallographic slip, twinning, or other processes in which atomic diffusion plays a part.
- **Diffusion flow** refers to the transport of material from one site to another. The three diffusion processes are volume diffusion, grain-boundary diffusion, and pressure solution. A set of conditions (relative heat, pressure, time, fluids, etc.) must be met before the rock deforms.
- **Granular flow** applies to the pervasive microcracking permitting movement of microfragments and grains, which is often compared to the flow of dry sand. Grain boundary sliding plays a major role. The mechanism under high confining pressure is sometimes called **superplastic flow** to account for friction on the moving particle (grain) boundaries.

The ductile behavior is dominantly temperature-dependent and prevails in the deeper crustal and lithospheric levels or regions with a high thermal gradient. The stress magnitude, strain-rate and the mineral composition of the rock medium are other important controlling parameters. The effect of increased temperature and decreased strain-rate is to promote **thermally activated** processes such as crystal slip and atomic diffusion. Creep is the time-dependent flow of solid material under constant stress. Creep can be by viscous or by plastic flow. **Plasticity** refers to the property of crystals to deform permanently by slip along lattice planes. **Crystal plasticity** is due to the movement of dislocations and is not systematically proportional to time.

Flow of ductile material
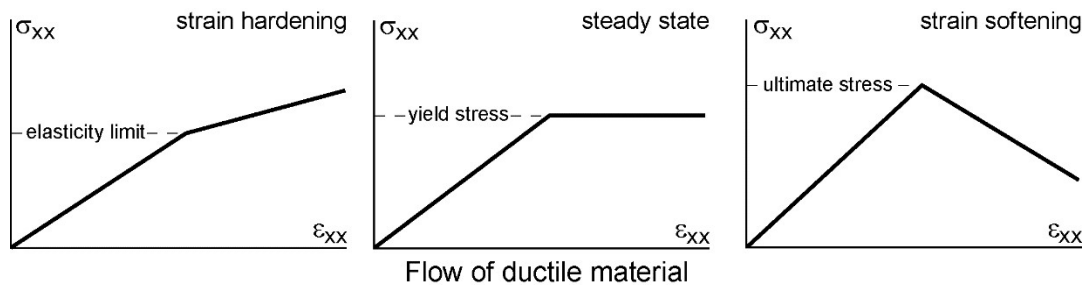
Provided the temperatures are sufficient, rocks begin to creep at low stresses. It means that ductile parts of the lithosphere are very weak in comparison to the elastic-rigid lithospheric parts and may be treated as viscous at long time scales.
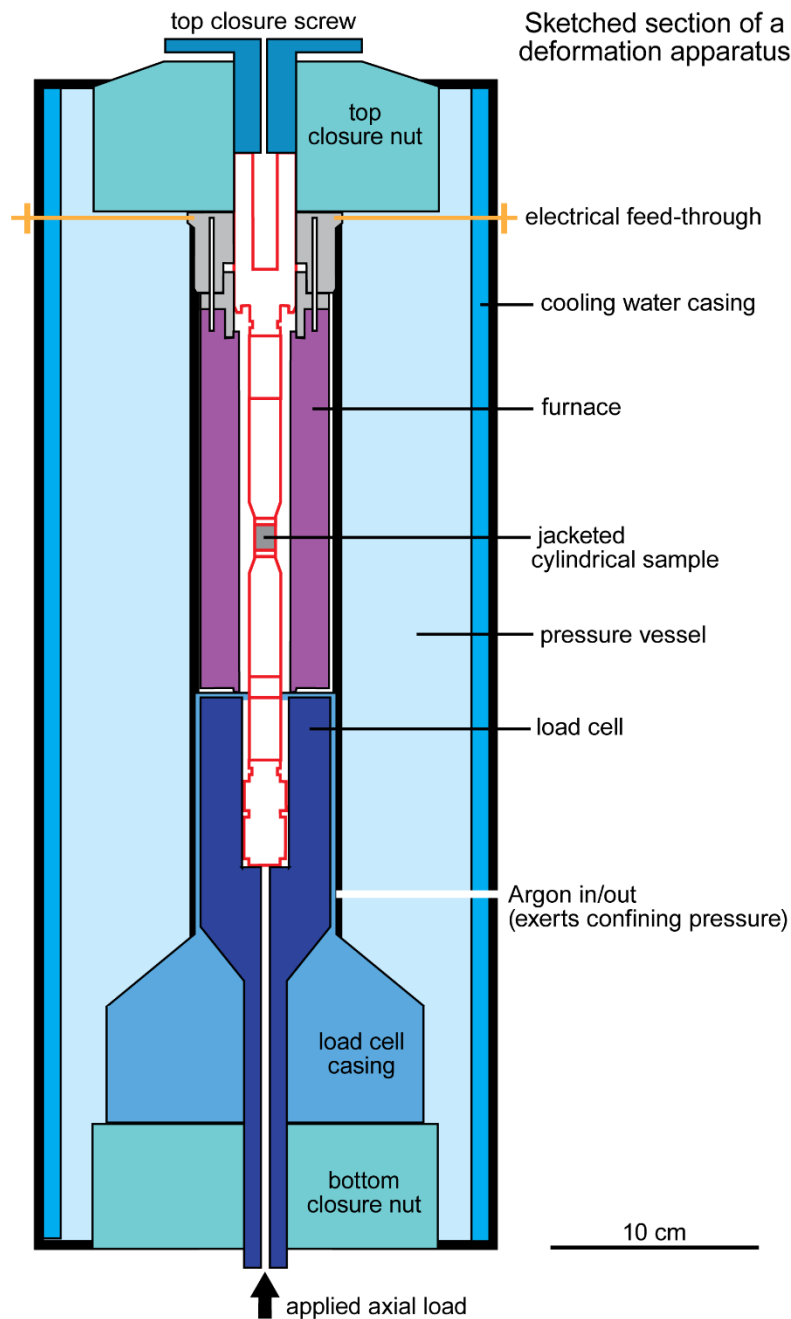
Brittle-ductile transition

The effect of increased confining pressure is to inhibit fracturing and cracking. As the confining pressure (i.e. the lithostatic pressure in the Earth) is increased the behavior of rocks passes through a transition from brittle to ductile behavior for each particular type of rock. This **brittle-ductile transition** is generally placed at the lower limit of most crustal seismicity. This transition is not sharp, nor is it a consistent depth or temperature. It is a function of hydrostatic pressure, temperature, and strain-rate as well. In general, the lower the temperature and hydrostatic pressure, and the higher the strain-rate, the more likely is a rock to behave in a brittle manner. Conversely, the higher the temperature and hydrostatic pressure and the lower the strain-rate (or the longer time the stress is applied), the more likely is a rock to behave in a ductile manner. In reality, there is a broad transition between brittle and ductile behaviors, where "**semi-brittle**" or "**semi-ductile**" deformation involves a mixture of frictional sliding and ductile flow on the microscale.

STRESS AND STRAIN IN ROCKS

Rocks and minerals are natural solid materials. To investigate the behavior of rocks and relate strain and strain-rates with stress, one needs to experimentally deform rocks under varied and controlled conditions of temperature, pressure, fluids and time. Such experiments are the foundation of **rock mechanics**.

Most mechanical tests consist of compressing a small cylindrical rock sample along its axis with a piston while exerting a confining pressure on all sides of the sample with a pressurized fluid. The fluid, contained in a pressure chamber, transmits a uniform confining pressure to the rock cylinder through an impermeable, flexible sleeve called a **jacket**, made of a material (usually copper) sufficiently weak at experimental conditions to not distort measurements. The confining pressure simulates the pressure to which rocks deep in the Earth are subjected, i.e. the weight of the overlying rock. The axial compression replicates a geologically realistic tectonic force. This experimental setting is known as a **triaxial test** because it allows predetermining stress to be applied along each of the three principal axes, with the main compression oriented parallel to the cylinder long axis. However, two of the principal stresses are equal to the confining pressure.

The stress along the axis of the cylinder is calculated knowing the axial force applied by the piston and the area of the cylinder extremities. The difference between the axial stress and the confining pressure is the **differential stress** $(\sigma_1 - \sigma_3)$. Pore pressure, if necessary, is introduced through additional devices. By varying any or all the pressures (axial load, confining and pore pressures), one can obtain different stress configurations: The axial stress can be either larger (compression) or smaller (tensile) than the confining pressure. Axial loading allows only strain of a few percent. Experiments reaching high shear strains are in torsion, whereby samples are twisted between coaxially rotating plates.

top closure screw

Sketched section of a
deformation apparatus

top
closure nut

electrical feed-through

cooling water casing

furnace

jacketed
cylindrical sample

pressure vessel

load cell

Argon in/out
(exerts confining pressure)

load cell
casing

bottom
closure nut

10 cm

applied axial load

Results are graphically displayed in diagrams where the differential stress $(\sigma_1 - \sigma_3)$ is plotted against the strain ε, which is calculated from the measured displacement of the piston $\Delta \ell$ reported to the initial length of the specimen ℓ_0: $\varepsilon = \Delta \ell / \ell_0$. This displacement is timed to access the strain-rate. The detailed shape of the stress/strain curve depends on the rate of loading, and on other parameters that will be discussed after a general description.

Constant strain-rate experiments - Effects of variation in stress
In constant strain-rate experiments, the piston of the machine moves at a constant rate throughout the experiment. The specimen deforms at a constant rate. To maintain this rate, the stress is allowed to vary and this is recorded. Three main fields in terms of progressive increase in tension (elastic, viscous and final break = **ultimate failure**) can be identified on a typical strain-time diagram.

Rheological data from deformation experiments on Carrara marble
under constant shear strain rate
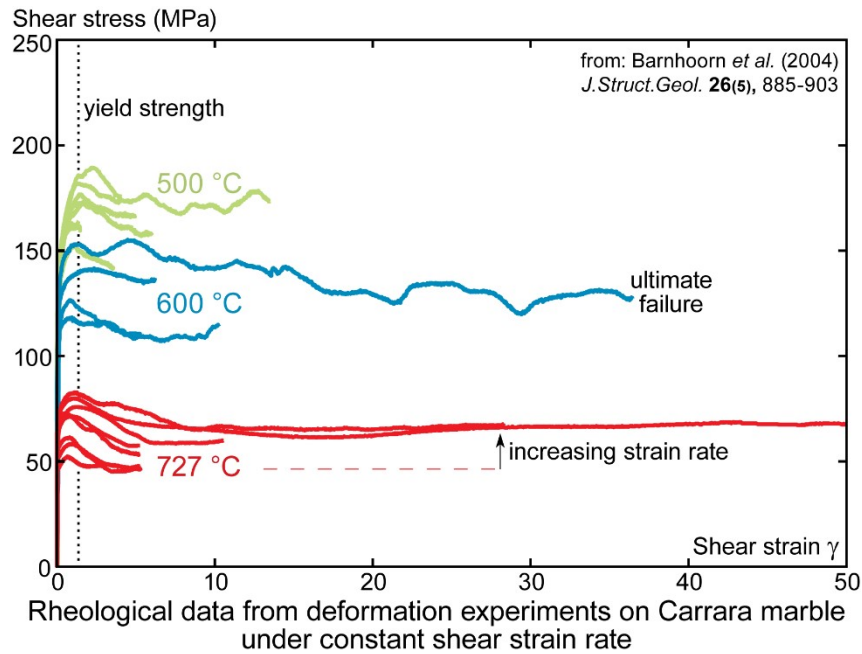
Elasticity limit

Starting from the origin, i.e. at the onset of the experiment, the stress-strain graph begins with a straight linear segment along which the strain increment is proportional to the stress increment. The sample recovers its initial size as soon as the load is removed.
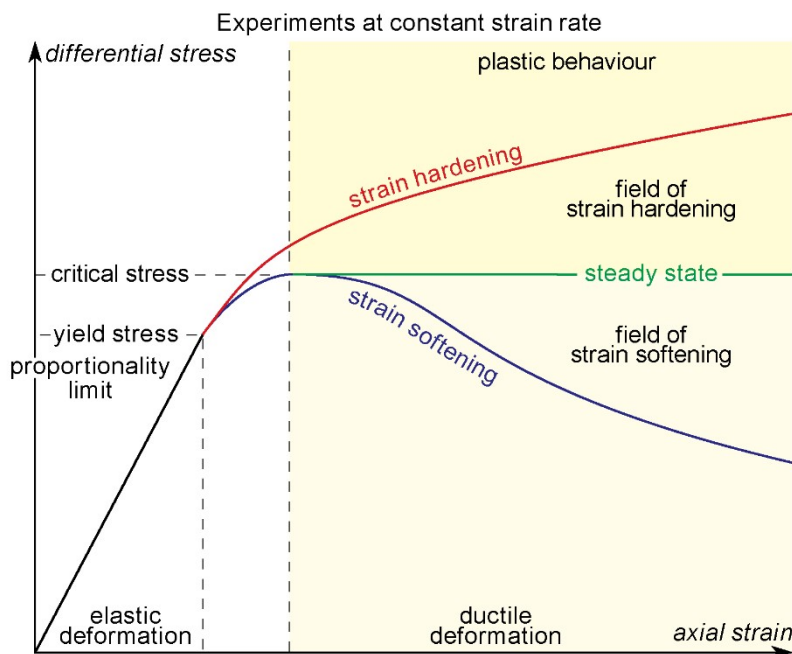
Idealised stress-strain curves obtained in experimental rock deformation
adapted from Tullis & Tullis (1986) *Geophysical Monograph* **36**, 297-324

This linear plot documents that the rock first deforms as a perfectly elastic material. The stress corresponding to the end of this linear section is the **proportionality limit**. For slightly larger values of differential stress, a change in the slope of the stress-strain graph indicates the beginning of permanent deformation: The stress is no longer proportional to strain. The point where permanent strain begins is the **yield point** (corresponding to a **yield stress)**. This critical stress is generally difficult to identify accurately because the stress-strain graph starts to curve rather gradually. In most

materials, the proportionality and the yield points are practically the same. For other materials, such as rubber, the segment of the stress-strain graph between the proportionality limit and the higher yield point documents inelasticity (i.e. non-linear elasticity).

Strain hardening

Continuing an experiment at low temperatures, the slope of the stress-strain graph of many materials diminishes but remains positive beyond the yield point. An ever increasing (but slower than in the elastic domain) stress is required for deformation to increase from the yield point onward. The material exhibits essentially ductile behavior while undergoing irrecoverable deformation. This effect is **strain hardening**.

Strain hardening reflects intracrystalline deformation. Deformation displaces atoms of the crystal lattice with the introduction of dislocations, which in turn create local elastic strain. The movement of these dislocations (**dislocation glide**) results in permanent deformation. The dislocation density increases, hence the elastic energy in the crystal lattice increases during hardening, which explains why the material becomes stronger. Inversely, strain hardening can be suppressed by prolonged but moderate heating (**recovery**) or by intense heating that induces total recrystallization of the material (**annealing**).

Resilience

Strain hardening is responsible for another phenomenon. If the load is removed during an experiment, the unloading stress-strain curve is linear and has the same slope as its initial, elastic part, down to the strain axis. Elastic strain is recovered but strain due to ductile-plastic deformation remains. Unloading does not interrupt the stress-strain curve. If a load is reapplied to the same sample, it follows the same line as the unloading path up to the point where unloading was prompted. Reinitiated deformation is again elastic, but the new elasticity limit is higher than the first one (note that the slope, the elasticity modulus, remains the same, though). Then the strain hardening curve resumes.

The yield strength of the specimen has increased (**work hardening**) because the original fabric, thus properties of the sample has been modified by permanent plastic deformation. The rock has acquired more **resilience**.

Idealised stress-strain curve of a loading-unloading test
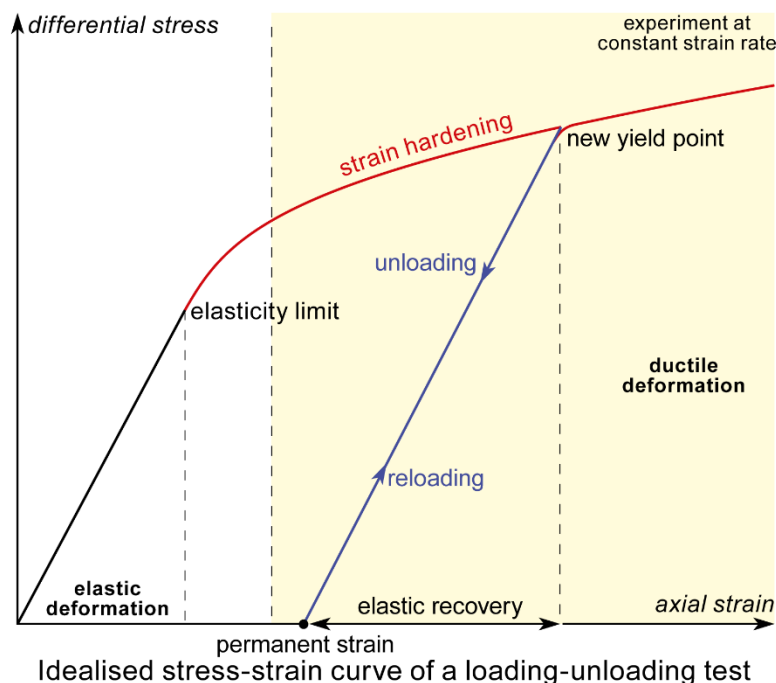
Resilience is the strain energy stored per unit volume of material before failure. Elastic materials give back the strain energy they absorbed when the load is removed before failure.

Stress relaxation

If the experiment is interrupted and the specimen held at constant strain, the stress will relax slowly. If loading is resumed, the specimen will behave as if it had been unloaded and reloaded elastically up to the point where straining was interrupted. Stress relaxation does not interrupt the stress-strain curve.
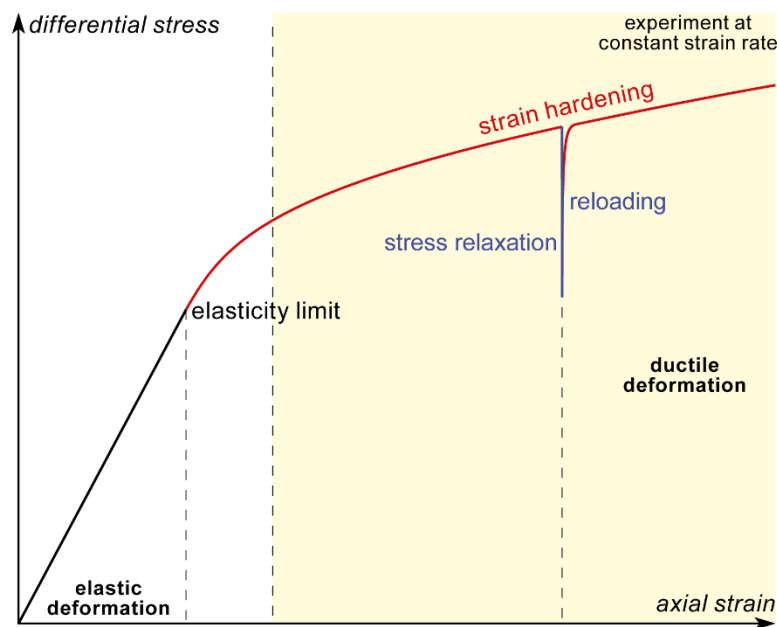
Idealised stress-strain curve of a specimen hold at constant strain

Steady state

The **yield strength** is a variable that depends on the past plastic deformation of the sample. At high temperatures or slow strain-rates, the curve becomes horizontal from the yield point onward: no strain hardening accompanies permanent deformation, and conditions approach **steady state**. The strain increases indefinitely with no increase in stress. This slow deformation, known as **creep**, is the essential characteristic of plastic strain. A perfectly plastic material is in steady state at the yield stress, and from that point on the stress-strain graph.

Steady state follows minor strain softening in most rocks.

Strain-softening

Above a second critical value of highest stress known as the **ultimate strength**, the stress-strain curve may descend either to a plateau value or (ultimately and) to a failure point. The material needs progressively lower stresses to deform further. This behavior is called **strain softening**.

Failure

The stress-strain curve falls to finish the experimental strain softening or steady state. The material exhibits accelerated viscous flow (necking for tension experiments) leading to rupture at the **failure stress**. Breakage of the sample occurring before the yield point is a **brittle failure**.

Shear fracture criteria

At rupture, the normal stress σ_N and the shear stress σ_S acting on a plane are related by an equation of the form:

$$\sigma_S = f(\sigma_N) \tag{6}$$

and experiments have shown that, for materials with no cohesion strength such as soils, the relationship is

$$\sigma_S = \sigma_N \tan \phi$$

where ϕ is known as the **angle of internal friction** (it is in this linear equation the slope of a line). The term internal friction describes a material property of slip resistance along the fracture.

Coulomb criterion

Coulomb proposed in 1773 that shear fracture occurs when the shear stress on a potential fault plane reaches and overcomes a critical value. The relationship (6) becomes the **Coulomb failure criterion:**

$$\sigma_S = c + \mu \, \sigma_N \tag{7}$$

where c is a material constant known as the **cohesion** or the **shear strength**;

    μ is another material constant known as the **coefficient of internal friction** equivalent to the term $\tan \phi$ seen for soils.

Equation (7) assumes that shear fracture in solids involves two factors: breaking cohesive bonds between particles of intact rock (the c term), together with frictional sliding (the term μ, proportional to the normal compressive stress σ_N acting across the potential fracture plane). This physical interpretation predicts a linear increase of rock strength with normal stresses acting on the rock and fits reasonably well many experimental data. Experiments provide cohesive strength of the order of 10-20 MPa for most sedimentary rocks and 50 MPa for crystalline rocks. The average angle of internal friction is 30°.

The Coulomb criterion predicts that shear fractures form at less than 45° to σ_1 because of the positive slope of the shearing resistance curve and the symmetrical shape of the shear stress curve. This criterion also termed Mohr-Coulomb and Navier-Coulomb yield criterion governs the creation of a new fracture.

Byerlee's law

The frictional strength on fault planes is generally constant. The coefficient of internal friction μ on existing fractures in consolidated rocks determines which shear stress is required to cause further movement on the fault planes:

$$\sigma_S = \mu \, \sigma_N \tag{8}$$

This equation, usually valid when two rough surfaces are in contact, is known as Amonton's law. A direct consequence of this law (and the Coulomb criterion) is that the shear stress required for sliding is independent of the surface contact area and increases with normal stress, therefore with confining pressure. The parameter μ, in this general sense, is also referred to as the **coefficient of static friction**. James Byerlee, an American geophysicist, compiled experimentally determined values of the shear stress required for frictional sliding on pre-cut fault surfaces in a wide range of rock types. He found two best-fit lines that depend on the confining pressure. For confining pressures corresponding to shallow crustal depths (up to $200\,\text{MPa} \approx 8\,\text{km}$), $\mu = 0.85$ and equation (8) becomes:

$$\sigma_S = 0.85 \, \sigma_N \tag{9}$$

Two diagrams illustrate this linear function.
The first one refers to very low (<5 MPa) normal stress conditions.
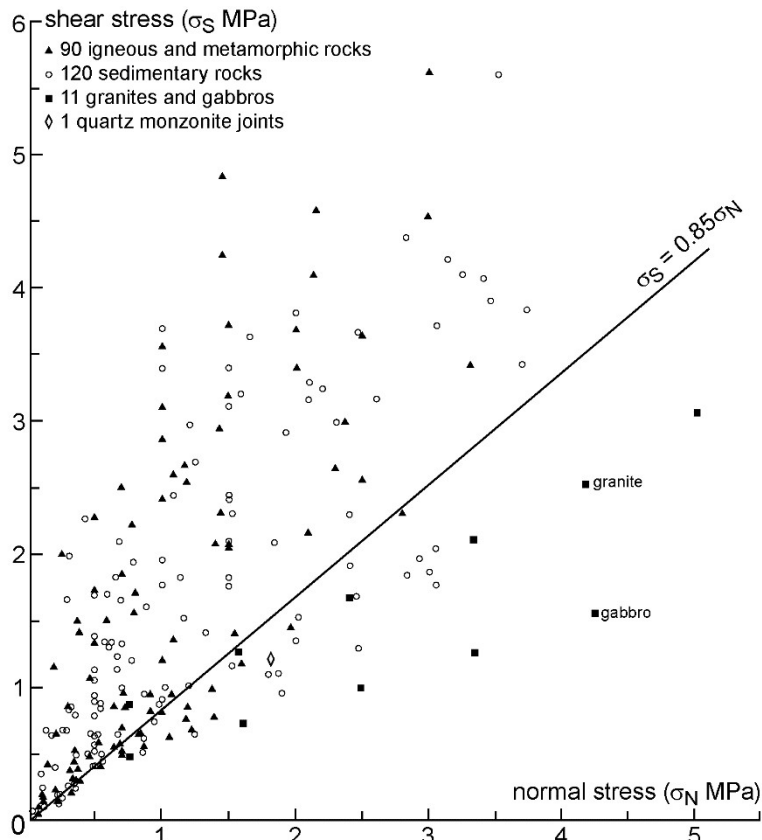
Shear stress as a function of normal stress at the maximum friction
for a variety of rock types at confining pressure up to 50 bars (5 MPa)
redrawn after Byerlee 1978 *Pure and Applied Geophysics* **116 (4/5)**, 615-626

The second one fits laboratory results generated under higher (up to 100 MPa) normal stress conditions.
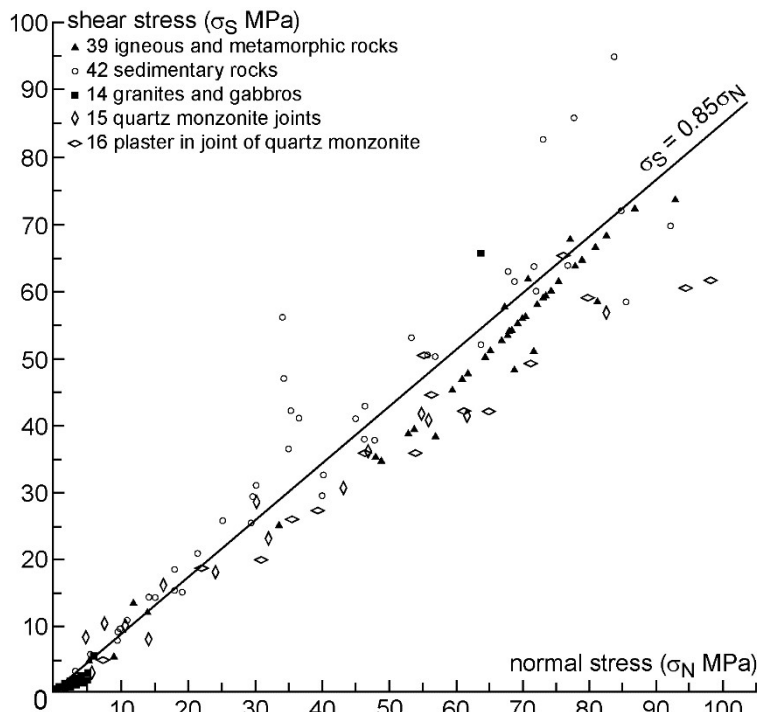

Shear stress as a function of normal stress at the maximum friction
for a variety of rock types at confining pressure up to 1000 bars (100 MPa)
redrawn after Byerlee 1978 *Pure and Applied Geophysics* **116 (4/5)**, 615-626

44

The large scatter of data point under very low normal stress reflects surface roughness, the area of contact of the asperities, less influent under higher confining pressure because the latter prevents dilatancy of the shear fracture, hence unlocking of interwoven surface irregularities. Instead, shearing and smearing of asperities tends to stabilize frictional properties

For confining pressures between 200 and 2000 MPa, the frictional strength of pre-cut rocks is better described by including a "cohesion-like" parameter:

$$\sigma_S = 50\text{MPa} + 0.6\,\sigma_N \qquad\qquad (10)$$

Shear stress as a function of normal stress at the maximum friction
for a variety of rock types at confining pressure up to 20 kbars (2 GPa)
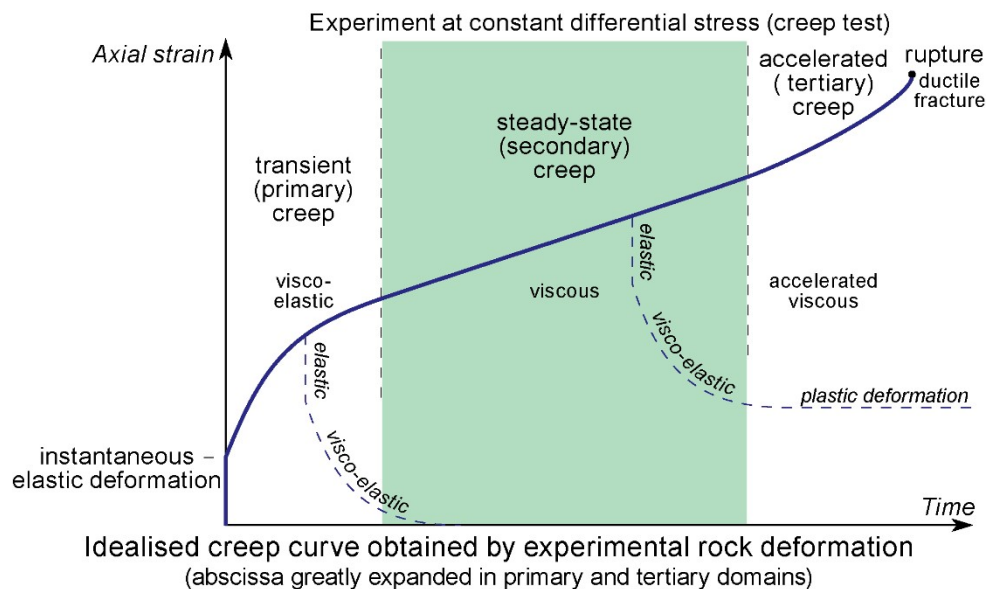redrawn after Byerlee 1978 *Pure and Applied Geophysics* **116 (4/5)**, 615-626
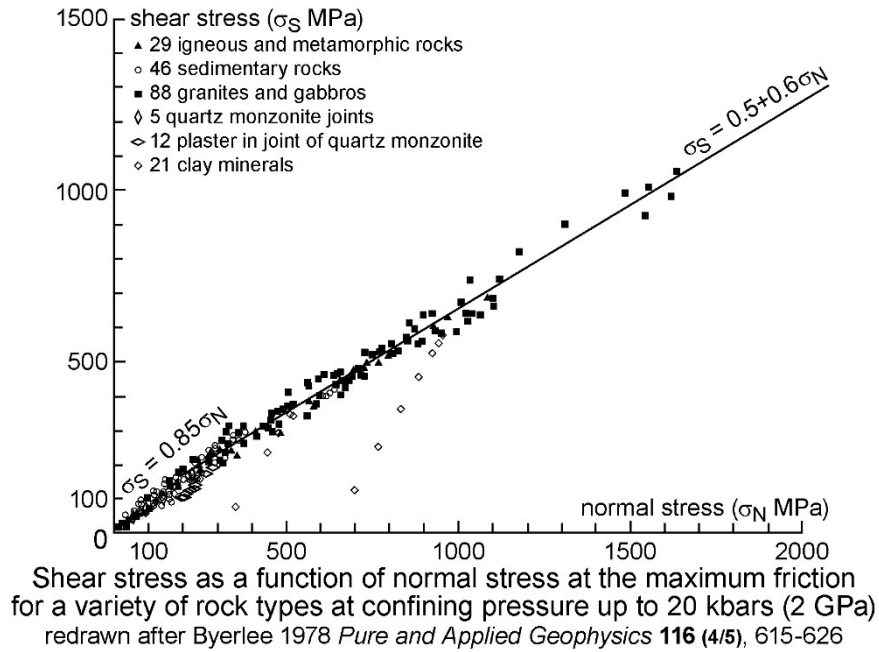
The Byerlee's law refers to equations (9) and (10), together. They are empirical and indicate that the shear stress required to activate frictional slip along a pre-existing fracture surface is largely insensitive to the composition of the rock. These laws seem to be valid for normal stresses up to 1500 MPa and temperatures < 400°C, which allows defining a lower boundary to stresses acting in the brittle lithosphere.

Byerlee's law for pre-existing fault surfaces
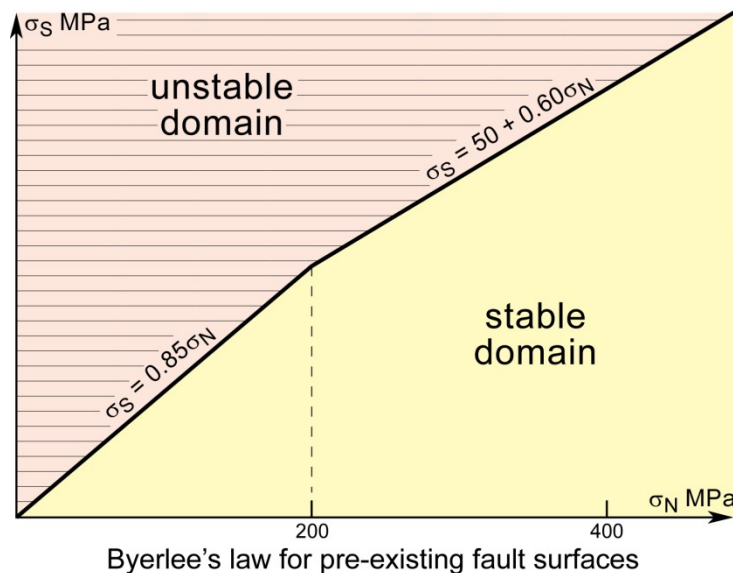
Constant stress experiments - Ductile flow

Deformation experiments lasting for days at constant differential stress and temperature produce very slow, continuous and plastic **creep** (time-dependent flow) of the sample. The resulting strain/time curve displays, after the initial elastic response, three regions representing decelerating, steady (constant rate) and accelerating creep:

In the first region, the slope of the curve (i.e. the strain-rate) decreases progressively; this decelerating behavior is called **primary** or **transient creep**; it is logarithmic because the total strain increases with the logarithm of time. The phenomenon of decreasing creep rate at constant stress is **cold working,** a strain hardening because the material becomes less ductile with increasing strain. The primary creep is reversible if the experimental load is ended, the visco-elastic strain being slowly removed after instantaneous retrieval of the elastic strain. The primary creep strain is usually < 1% of the sum of the three creep regions.

Idealised creep curve obtained by experimental rock deformation
(abscissa greatly expanded in primary and tertiary domains)

- In the second, usually, the largest regions of creep curves, the slope (strain-rate) is constant and residual, the plastic strain is irrecoverable; this linear flow behavior is called **secondary** or **steady-state creep**. Even with constant stress steady-state creep could continue indefinitely. It may represent the long-term deformation processes that occur within the Earth over geological times without failure. Therefore, it is the part of the experiment that interests geologists. The material behaves like a viscous, yet non-Newtonian fluid. As a first-order but approximate explanation, steady-state creep results from recovery processes (mostly thermal softening) balancing strain hardening (due to dislocations) as it appears.

- In the third region, not always observed or absorbing a very small amount of strain, the strain-rate increases exponentially until the **rupture** of the specimen. Accelerating flow is mainly caused by the spread of microfractures or slip surfaces through the rock in such a way that they link up (accumulating damage) to form continuous pervasive cracks causing loss of cohesion, and failure; this is called **tertiary** or **accelerating creep**.
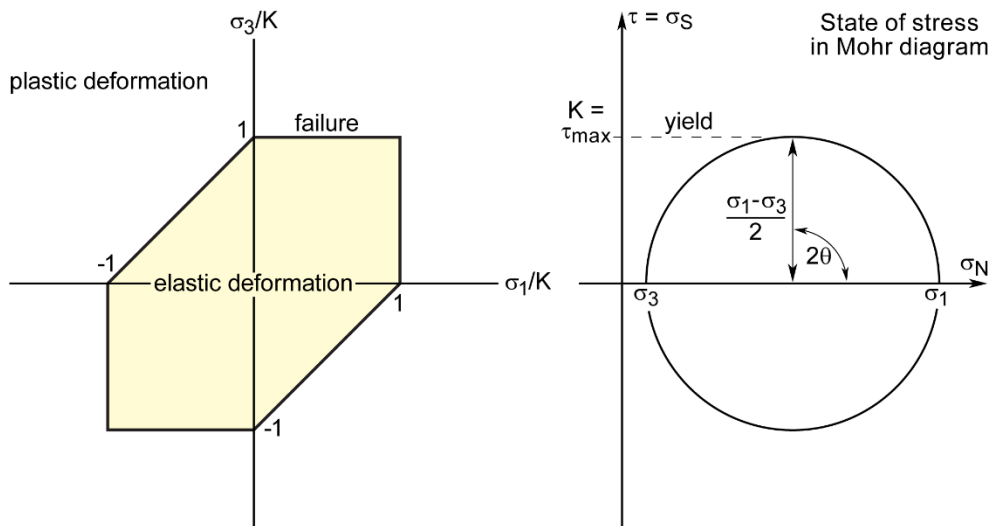
Failure criteria

Ductile isotropic materials have equal or nearly equal strength values in uniaxial tension and uniaxial compression. Yield is generally caused by the slip of crystal planes along the plane of maximum shear stress. Therefore, and independent from the state of stress, a given point in an isotropic body does not deform plastically as long as the maximum shear stress at that point is under the yield shear stress, which defines plasticity.

Tresca criterion

The maximum shear stress criterion, known as Tresca criterion, assumes that the differential stress and the maximal principal stress are smaller than the plastic, yield shear strength K:

In uniaxial compression: $\sigma_1 \leq K$

In uniaxial tension: $\sigma_3 \leq K$

In general: $(\sigma_1 - \sigma_3) \leq K$

Graphically, this criterion requires that the two principal stresses plot within an irregular hexagon of a two-dimensional space.

Tresca failure criterion of maximal (critical) shear stress

The six-sided prism in the two-dimensional stress space is a hexagon of infinite length in the along the hydrostatic axis, inclined at 45° to all principal stress axes (planes of maximum resolved shear stress are at 45° to stress axes).
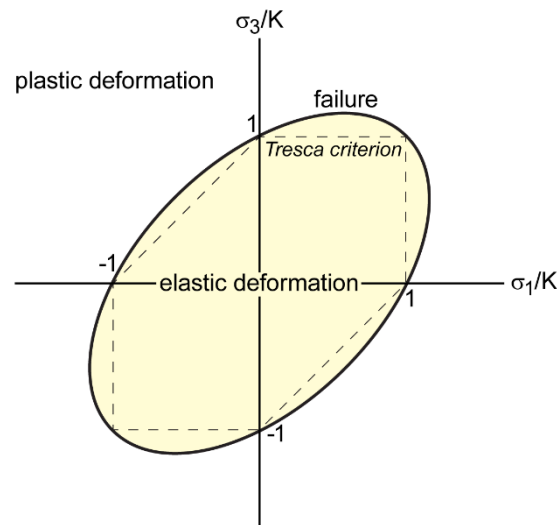
Von Mises criterion

The maximum distortion energy (a.k. von Mises) criterion, states that ductile failure occurs when the energy of distortion per unit volume of isotropic material reaches the same energy for yield/failure. Mathematically, this is expressed as

$$\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] \leq K^2$$

The left side of this equation is known as equivalent stress, and the equation holds regardless of the relative magnitudes of σ_1, σ_2 and σ_3. In plane stress, $\sigma_2 = 0$ and the criterion reduces to:

$$\sigma_1^2 - \sigma_1\sigma_3 + \sigma_3^2 \leq K^2$$

This equation represents a principal stress ellipse in which the Tresca hexagon is inscribed. This geometrical relationship shows that the maximum shear stress criterion is more conservative than the von Mises criterion since it lies inside the von Mises ellipse. Like for the Tresca criterion, the ellipse has an infinite length in the $\sigma_1 = \sigma_2 = \sigma_3$, i.e. the hydrostatic axis direction (the ellipse is an oblique section of an infinite cylinder).

von Mises failure criterion (ellipse) of maximal distortion energy

Deformation tests yield results better reproduced by the von Mises stress than by the Tresca criterion.

Steady-state creep function

Creep mechanisms depend strongly on temperature. Dislocation glide is dominant in the lower temperature range, giving way to dislocation and lattice diffusion (Nabarro-Herring) creep at increasing temperature, followed by grain boundary diffusional (Coble) creep at very high temperature. These mechanisms are not described here. In general, the differential stress $(\sigma_1 - \sigma_3)$ is related to the rate of steady-state creep $\dot{\varepsilon}$ by the non-linear, empirical (phenomenological) equation of the form:

$$\dot{\varepsilon} = A\sigma^n d^{-m} f_{fluid}^r \exp\frac{-Q+pV}{RT}$$

where A, the **frequency factor,** is a material constant defined for the particular creep mechanism in $Pa^{-n} \cdot s^{-1}$.

σ the differential stress

n is also an experimentally determined constant depending on the material and the creep mechanism; it commonly varies between 3 and 5 but can be higher for dislocation glide.

d is the grain size with exponent $-m$

f_{fluid} is the fluid fugacity (generally water) with exponent r

Q is a creep **activation energy,** a constant that must be determined experimentally. Q has a unit of kcal (or Joules) / mole.

p is pressure

V the activation volume

R is the gas constant (in $J \cdot mol^{-1} \cdot K^{-1}$).

T is the absolute temperature in Kelvin

Grain size and fluid fugacity have nearly no effect on dislocation creep because it results from practically rigid displacement of matter by the motion of within crystal lattice defects (the dislocations). The generalized equation is thus reduced to:
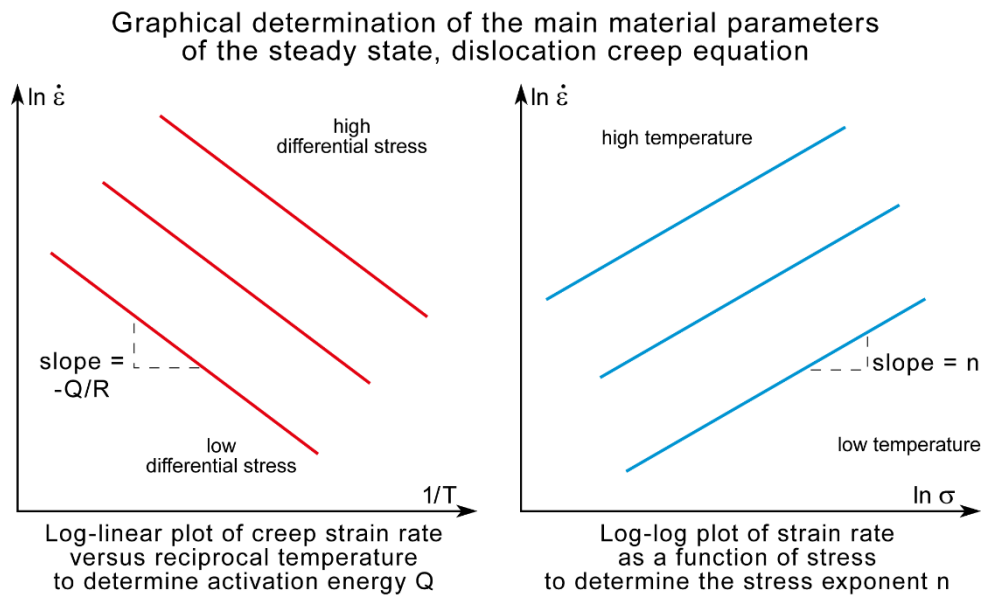
$$\dot{\varepsilon} = \frac{d\varepsilon}{dt} = A(\sigma_1 - \sigma_3)^n \exp\frac{-Q}{RT} \tag{11}$$

The right part of equation (11), referred to as Weertman equation, implies that viscosity decreases exponentially with temperature. High-temperature diffusion creep involves the additional grain size, power-law dependency and A includes a diffusion coefficient. Since diffusion implies material

transport on the scale of the grain, coarser grains lead to slower creep rates. Equation (11) can be rearranged into logarithm form:

$$\ln \dot{\varepsilon} = \ln A + n\left(\ln \sigma\right) - Q/RT$$

The thermal activation energy Q can be obtained from the gradient $-Q/R$ of the natural log-linear of the creep rate plotted against the reciprocal of temperature.

Graphical determination of the main material parameters of the steady state, dislocation creep equation

Log-linear plot of creep strain rate versus reciprocal temperature to determine activation energy Q

Log-log plot of strain rate as a function of stress to determine the stress exponent n

The stress exponent n can be determined by plotting the strain-rate as a function of stress in a log/log diagram.

The steady-state rheology of rocks is of great importance to establish constitutive relationships involving stress, strain-rate, and intrinsic flow parameters. One particular strain-rate yields only one particular deviatoric stress in steady state. A power law with $n = 3$ is often used to represent mantle rocks.

Strength of materials - Interrelationship of stress, strain and time

One important variable is the **strength** of a material, which is the stress at which failure occurs. Many materials possess both a **yield strength** defined as the stress above which permanent deformation occurs and a **failure strength** at which fracturing occurs. Laboratory measurements provide limiting values of lithospheric stress, provided that one effective principal stress is known.

The relationship between stress and strain for real materials that exhibit a combination of elastic, viscous and plastic properties depends critically on the length of time for which the differential stress is applied. In laboratory experiments with a duration of up to a few days, the behavior of the material is effectively "instantaneous" in geological terms and differs significantly from that of the same material under stresses with a more geologically realistic duration of months or years. The long-term strain behavior of materials is called **creep**. The important characteristic of creep behavior is that values of strength measured over short periods are much larger than those measured (or extrapolated) for long periods. The long-term or **creep strength** of most rocks is only in the range 20 60 % of their instantaneous strength.

Strain-rate is expressed as a change in dimension per unit time, in the form 10^n s^{-1}. Extrapolation of the experimental data to average geological strain-rates of approximately 10^{-14} s^{-1} indicates that rocks should have strengths of a few hundred bars at 200-300°C and maybe less than 100 bars at 800-900°C. Also, all the relevant brittle and ductile processes are thermally activated, i.e. the rates of deformation at fixed differential stress increase with increasing temperature and time necessary to achieve a given deformation increases as temperature decreases.

Effects of material components and environmental parameters

All effects reported in this paragraph differ from rock to rock, shales showing some particularity. However, behaviors are generally comparable in shape if not in values, and descriptions are summaries that do not document detailed material properties and responses to varying parameters.

Time factor

Time plays a prominent role in deformation. Rapid loading of **rate-sensitive** materials exhibits different stress-strain curves than those obtained under slow loading. Experimentally, the effect of a slow strain-rate is analogous to increasing temperature: Decreasing strain-rate decreases the rock strength and increases ductility. The values of the yield and ultimate strengths of rocks are much higher if measured over short periods than over geologically significant periods. Ductile flow has been observed to take place under a constant long-term load whose value is considerably lower than the elastic and plastic limits (e.g. bending of graves and candles under their weight). Accordingly, rocks under geological load flow much below the yield strength measured under experimental conditions of temperature and confining pressure comparable to those in the earth.

Effects of strain-rate

If n = 1 in equation (11), then the stress is proportional to the strain-rate and the material behaves like a Newtonian, perfectly viscous fluid (equation 3). The viscosity of rocks with $n > 1$ is characteristically dependent on the strain-rate. The strain-rate $\dot{\varepsilon}$ is related to the applied stress by the power-law expressed in equation (4):
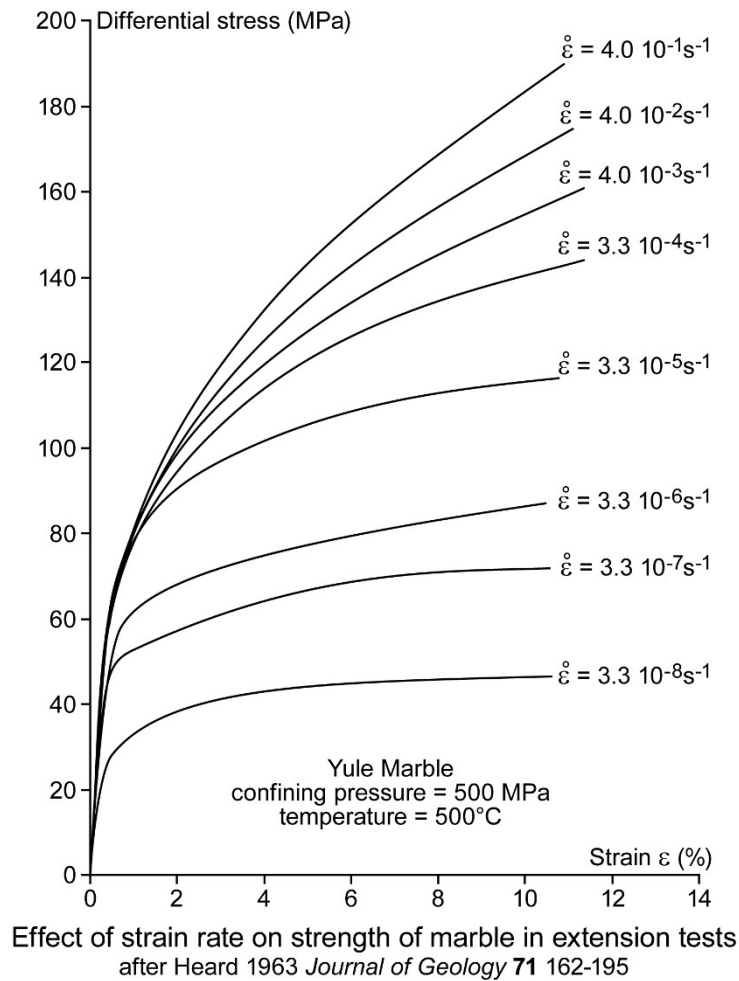
$$\sigma^n = \mathbf{A}\dot{\varepsilon} \tag{12}$$

with $\mathbf{A} = A \exp\dfrac{-Q}{RT}$ of equation (11), a function of material properties, pressure, and temperature.

Since the proportionality between strain-rate and stress is non-linear, only the effective viscosity can be defined according to:

$$\eta_{eff} = \sigma/\dot{\varepsilon} = \mathbf{A}^{1/n}\dot{\varepsilon}^{(1/n)-1} \tag{13}$$
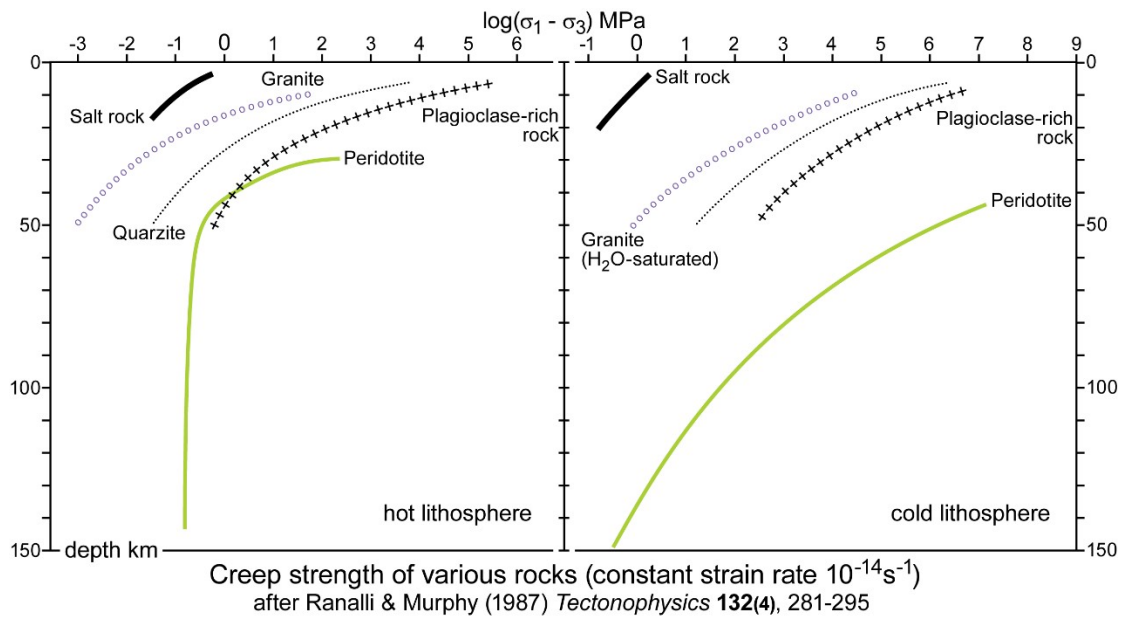
Equation (13) shows that a small increase in the strain-rate results in a rapid decrease of the effective viscosity, provided $n > 1$. For this reason, the effective viscosity is also called **stress-dependent** or **strain-rate-dependent** viscosity.

Effect of strain rate on strength of marble in extension tests
after Heard 1963 *Journal of Geology* **71** 162-195

The constant **A** also depends on the deformation mechanisms (intra-/inter-crystalline, diffusion, etc.) which, in turn, depend on the rock type and the conditions of deformation. For any of these nonlinear mechanisms, changes in strain-rate alter significantly the strength of rock. This is opposed to cataclastic flow, which is time, hence strain-rate independent.

Effect of temperature

Increasing the temperature under constant loading conditions reduces the yield stress without changing the shape of the stress-strain curve. For example, the yield stress of marble at 800°C is about one-sixth of its value at room temperature. Lowering the yield stress has for effect to enlarge the field of ductile, permanent deformation at the expense of the elastic and failure fields. Moreover, the rate at which the material deforms under given applied stress is increased while strain hardening is reduced or even eliminated. Consequently, the material shows an increase in ductility (i.e. the % of strain a rock can take without fracturing at a macroscopic scale). A simplified explanation is that heat weakens the bonds between the atoms without breaking them. The temperature effect is well known to blacksmiths who heat metals to fashion ironwork out of a ductile material that does not break. Similarly, the Earth's internal heat renders brittle rocks more ductile.

log($\sigma_1 - \sigma_3$) MPa

Creep strength of various rocks (constant strain rate $10^{-14}s^{-1}$)
after Ranalli & Murphy (1987) *Tectonophysics* **132(4)**, 281-295

These explanations are consistent with the geological observation of metamorphic rocks deformed at elevated temperature and pressure. These exhibit much more ductile structures than do the equivalent rocks at the surface. Temperature affects all properties of rocks (importantly density) and therefore the mechanical properties of the lithosphere.

The geothermal gradient, i.e. the distribution of temperature with depth, varies between 15°C/km in stable cratons and 35°C/km in the much-thinned crust. Theoretical considerations refer to two types of geothermal gradients:

- Steady-state geothermal gradients are those for which the temperature at any given depth does not change through time.
- Transient geothermal gradients are those for which the temperature at any given depth varies through time.

Steady-state geothermal gradients are valid for instantaneous or short-lived processes; it is obvious that all parameters, including temperature and depth, vary over long-lasting geological processes. This variation depends on processes producing, consuming, or transporting heat at the same time. In the lithosphere, the main mechanisms are heat conduction, heat advection and the heat production from the decay of radioactive elements.

Geothermal gradients are not linear, in particular for the continental crust whose upper parts are enriched in incompatible elements (large radius elements such as U, Th, and K) which tend to be transported in melts. Due to repeated events of melting/magmatism, the continental radiogenic heat production decreases with depth.

Effect of hydrostatic (confining) pressure

Rocks deeply buried in the crust are subjected to the **lithostatic pressure** (the vertical normal stress σ_{ZZ}) which can be assumed to be effectively hydrostatic (i.e. equal in all directions), and related to the thickness and mean density of the overlying column of rock. Accordingly, the lithostatic pressure is generally calculated by a simple integration:

$$P_{lith} = g\int_0^z \rho(z).dz \tag{13}$$

with ρ rock densities, g the gravitational acceleration and z the depth.

Variations in **confining pressure** are experimentally introduced to reproduce pressures deep in the crust and mantle. The hydrostatic stress (the experimental confining pressure) causes elastic volume changes which depend on the compressibility of the material. The size of these volume changes is negligible except at great depth.
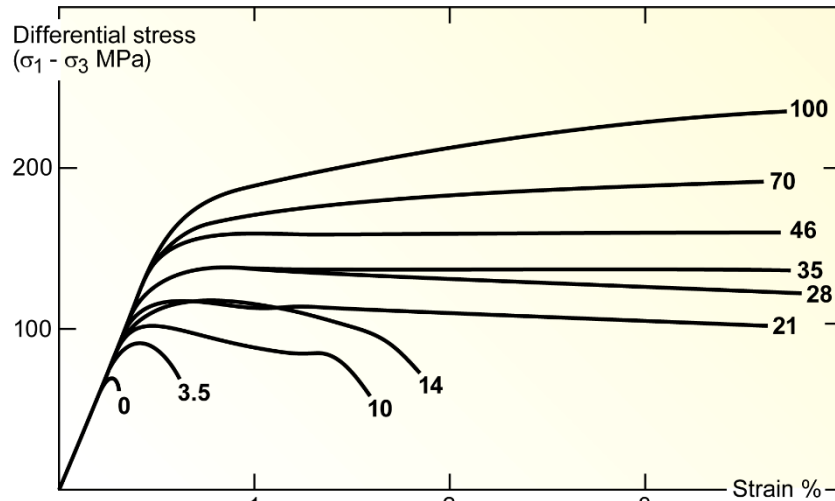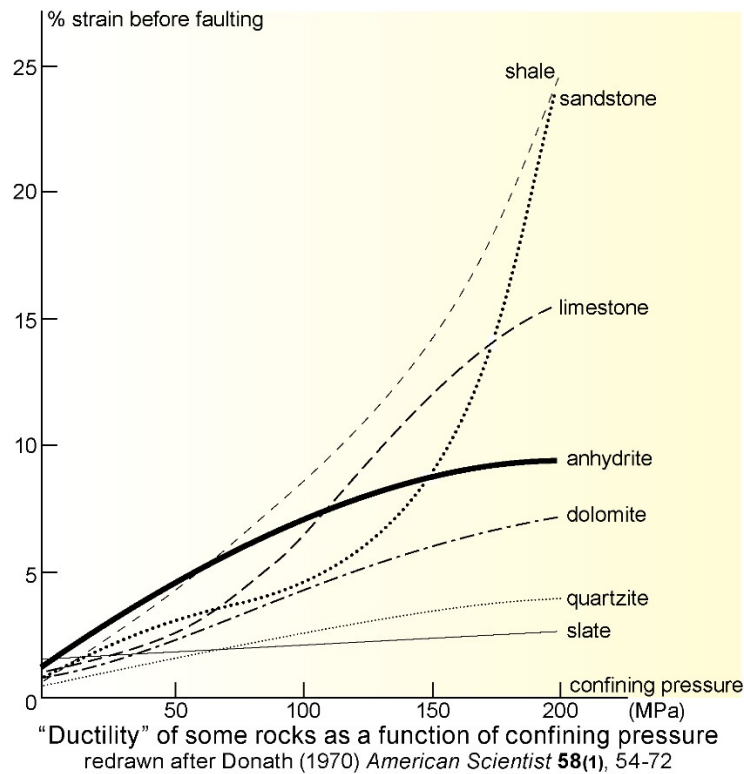
Stress strain curves in triaxial compression of Wombeyan marble
under various confining pressures (bold numbers, MPa)
after Paterson (1978) *Experimental rock deformation - the brittle field.* Springer Verlag, fig. 47 p163.

Realistic confining pressures for most naturally deformed crustal rocks increase approximately by 26 MPa/km in the crust and by 35 MPa/km in the mantle. The pressure at the base of a 35 km thick crust is about 10 GPa.
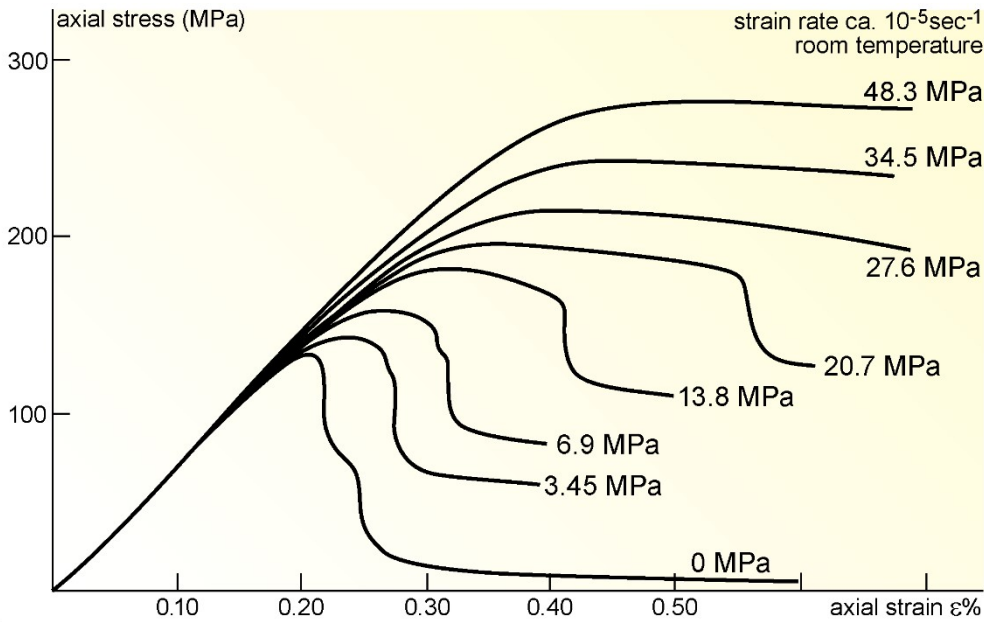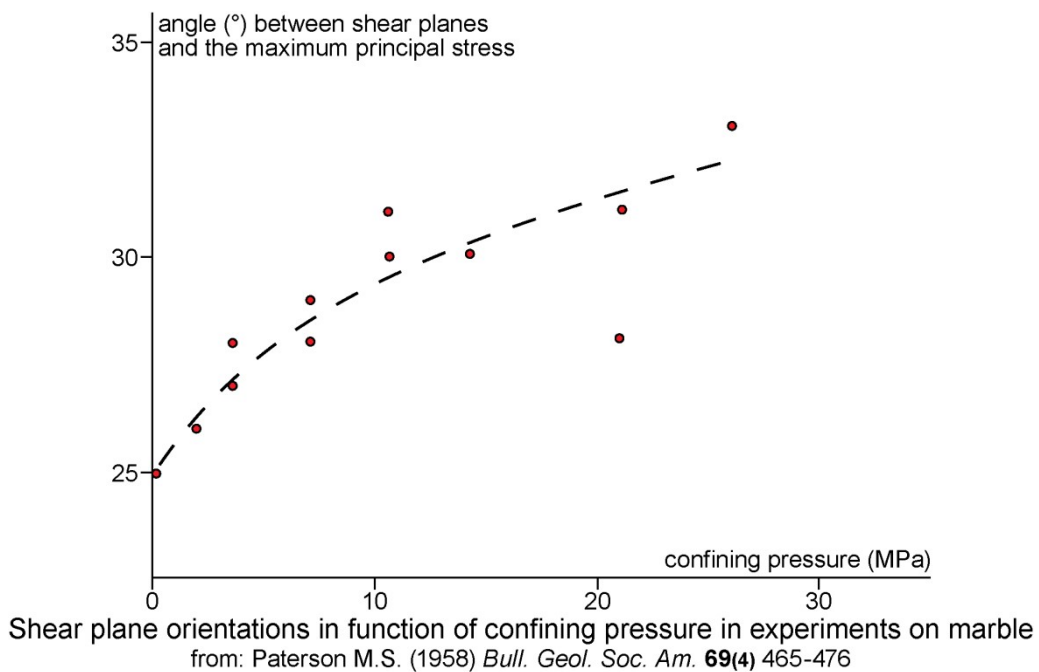

Stress-strain curves for Tennessee Marble at various confining pressure conditions
from Wawersik & Fairhurst 1970 *Int. J. Rock Mech. Min. Sci.* **7(5)**, 561-575

A more important aspect of increasing confining pressure is to cause a significant, in general almost linear increase in both the yield stress and the failure stress, giving the material a higher effective strength whether in compression or extension. In other words, increasing the confining pressure allows a greater amount of strain to accumulate before failure; it increases the rock's ability to flow ductilely.

"Ductility" of some rocks as a function of confining pressure
redrawn after Donath (1970) *American Scientist* **58(1)**, 54-72

To give a simplified explanation, moderate lithostatic pressures near the Earth's surface allow the atoms of stressed rocks to move freely and easily break their bonds; as a result, near-surface rocks under sufficient stress can undergo brittle failure. The same rock at several kilometers depth would deform ductile instead because the greater lithostatic pressure impedes the breakage of chemical bonds.

Shear plane orientations in function of confining pressure in experiments on marble
from: Paterson M.S. (1958) *Bull. Geol. Soc. Am.* **69(4)** 465-476

The dihedral angle between conjugate fractures widens with increasing confining pressure. The experimental observation is consistent with the Mohr criterion of failure.

Effect of pore-fluid pressure and impurities

Since the fluid pressure is hydrostatic and opposed to the lithostatic pressure, increasing pore pressure is equivalent to decreasing confining pressure. Fluids in rocks reduce both the rock strength (failure occurs at lower differential stress) and its ductility (failure occurs at lower strain). The combined reduced ductility and strength promotes flow under high pore pressure.

The presence of a fluid phase in rocks undergoing deformation is important in two ways.

- It can promote mineralogical reactions, particularly at elevated temperatures, which affect the mechanical properties of the rock.

- It can reduce the effect of the lithostatic (confining) pressure (i.e. the effective normal stresses), the intensity of the pore-fluid pressure countering the direct pressure between adjoining grains.

The chemical effect is often small compared to the mechanical influence of the pore-fluid pressure, which is expressed by:

$$P_{eff} = P_{lith} - P_{fluid} \qquad (14)$$

where P_{eff} is the effective pressure on the solid material, P_{lith} is the confining pressure, and P_{fluid} the fluid pressure.

A **coefficient of fluid pressure** λ is often used to denote the ratio of fluid pressure and lithostatic load:

$$\lambda = P_{fluid} / P_{lith} \qquad (15)$$


Effect of pore-fluid pressure on quartz rheology
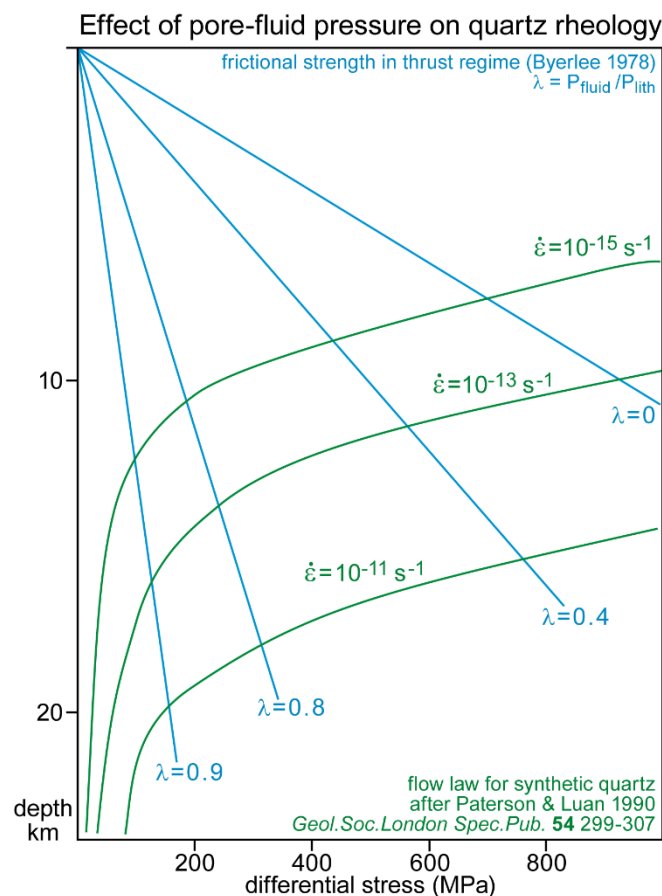
Exercise

Calculate the coefficient of fluid pressure in the upper part of the crust, where fractures reach the surface. Bulk rock density is 2400 kg.m^{-3}.

Water density = 1000 kg.m^{-3}, builds columns in open fractures, λ = 0.42

For saturated rocks, in which the pore-fluid pressure may be very high, the effect of the confining pressure is canceled out and the rock strength is reduced to near-surface conditions. In boreholes, high P_{fluid} may cause fracturing of the casing or wall rocks, whereas low P_{fluid} are responsible for blowout and borehole closure.

The stress/strain curves for wet and dry quartz crystals illustrate the effect of high fluid pressure on rocks at elevated temperature. The yield stress at 950°C in wet quartz is only about one-tenth of that required for the dry quartz at the same temperature. The ductility, in this case, is increased by the presence of water, which explains why certain materials, normally strong even at high temperatures, can flow under metamorphic conditions in the presence of aqueous fluids.
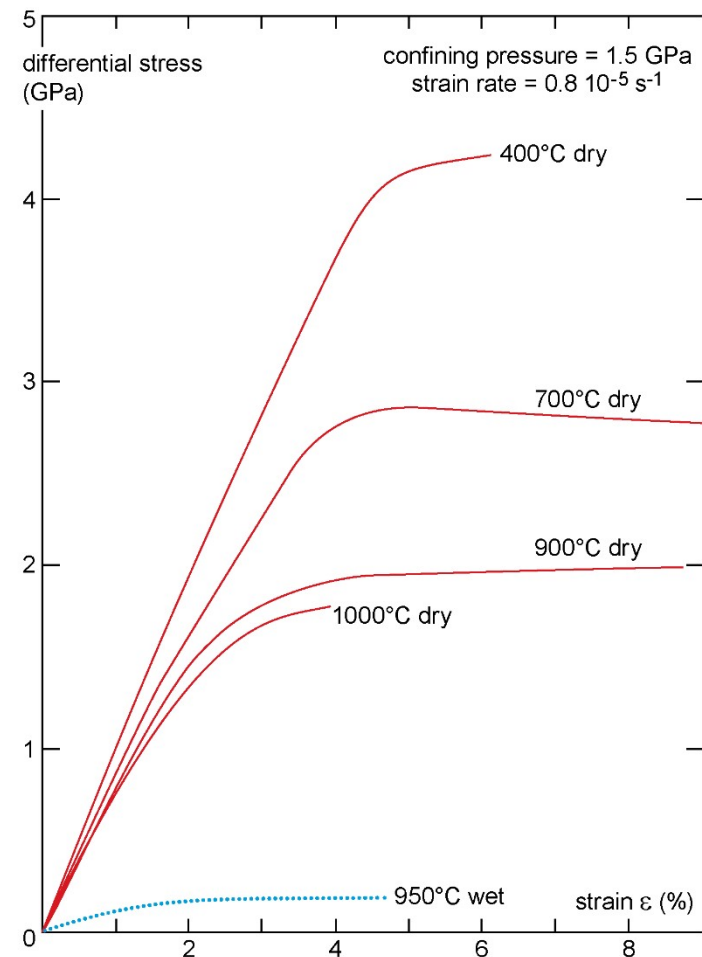
Strain-strain curves of wet and dry natural quartz crystals
after Griggs 1967 *Geophysical Journal International* **14 (1-4)** 19-31

By comparison, the activation energy in equations (10) and (12) depends on any fluid phase or impurity, for example, such as depending on the diffusion of CO_2 through the calcite structure, or amounts of (OH) in the quartz structure or H in olivine. Impurities usually lower the yield stress values and increase the field of viscosity in the stress-strain curves.

Plastic properties of rocks

The rheology of polymineralic rocks is controlled by the behavior of the weakest mineral phase, on condition that it constitutes an interconnecting network in a rock, commonly above 30 volume %. Since natural rocks are mostly polycrystalline materials, some mathematical techniques, like those of Reuss and Voigt, have been designed to "average" material properties of each constituting phase and by such means approximate perhaps better the bulk rock behavior. These techniques consider the volume fractions of the various components, their physical properties, and their relative arrangement.

Simplified calculations consider mixtures of two laminated or linear (fibers), isotropic, purely elastic or purely viscous phases only. The Voigt model (axial loading of the laminae or fibers, the strain is everywhere uniform) yields upper "bounds" of the possible effective parameter under investigation. The Reuss model (perpendicular loading, stress is everywhere uniform) yields lower "bounds". The real answer might lie between these two solutions. More complex solutions (e.g. Hashin-Shtrikman bounds) have been developed in material sciences.

STRENGTH PROFILES

Knowing the amounts and rates of displacements involved in plate tectonics, the rheology of the lithosphere has been assumed to be viscous. However, several types of observations show that the rheology changes with depth. Geologists observed that rocks from shallow crustal levels have deformed in a brittle fashion, while rocks from lower crustal levels deformed in a ductile manner. Supportively seismicity occurs within the upper crust in both strong and thin oceanic lithospheres and comparatively weaker and thicker continental lithospheres. Rheological models of the lithosphere should account for these variations. They are graphically summarized as strength versus depth profiles called **yield-strength envelopes**.

Strength profiles use intuitive relationships on the constitutive relationships that govern the rheology of rocks. The first intuitive principle is that of minimum effort: a rock will break if it requires less stress than to flow and vice-versa, according to physical conditions considered. At least two contrasting and superposed lithospheric regions are distinguished: (1) pressure-controlled regions of elastic-rigid behavior where it is easier to obey laws for the fracture strength, and (2) temperature-controlled parts with ductile (viscous or plastic), non-linear flow laws for the yield strength.

Construction

The construction of strength profiles is based on considerations of two experimentalists whose names have been attached to the resulting model: "Brace-Goetze lithosphere".

Assuming that the rheological data obtained from laboratory tests on the mechanics of cold (brittle) and hot (ductile) rocks and minerals remain valid when extrapolated to macroscopic scales and geological times, the rheological data can be assembled to produce a first-order model of maximum rock strength in function of depth in the considered lithosphere. The **integrated strength** of this lithosphere (in Nm^{-1}) is defined by the integration over depth of the assembled rheological profile.

Brittle levels

The upper part of the lithosphere can be considered elastic on a wide range of time scales. This hypothesis explains that heavy loads such as seamounts and mountains are supported by the lithosphere for millions of years. On shorter time scales, the Earth behaves elastically in the cases of seismic events and seismic wave propagation. The main importance of the elastic behavior of rocks for geodynamic processes lies in the fact that the lithosphere is strong and capable of supporting and transmitting large tectonic stresses for long periods. Elasticity controls the stress field in rigid domains, which in turn controls the position and timing of deformation in non-rigid domains. This means that it is fundamentally impossible to assess the stress-state of the lithosphere without considering the effects of elasticity.

Stresses in the Earth cannot exceed the strength of rocks. The upper part of the lithospheric strength profile uses Byerlee's friction law for the limiting strength and assumes a hydrostatic pore-pressure gradient. This law involves zero strength at the surface (the failure envelope goes through the origin of the diagram). This is accepted because the upper crust is crossed by many fractures of every shape and size. The relevant deformation process is, therefore, sliding along such pre-existing surfaces. An Andersonian stress state and fault orientation (i.e. one of the principal stresses is vertical and that conjugate faults make an acute angle bisected by the maximum principal stress) is assumed.

According to Byerlee's law, the brittle strength of rocks increases linearly with depth and is independent of material. Thus, the brittle shear strength of the lithosphere will increase roughly linearly throughout its crust and mantle layers.
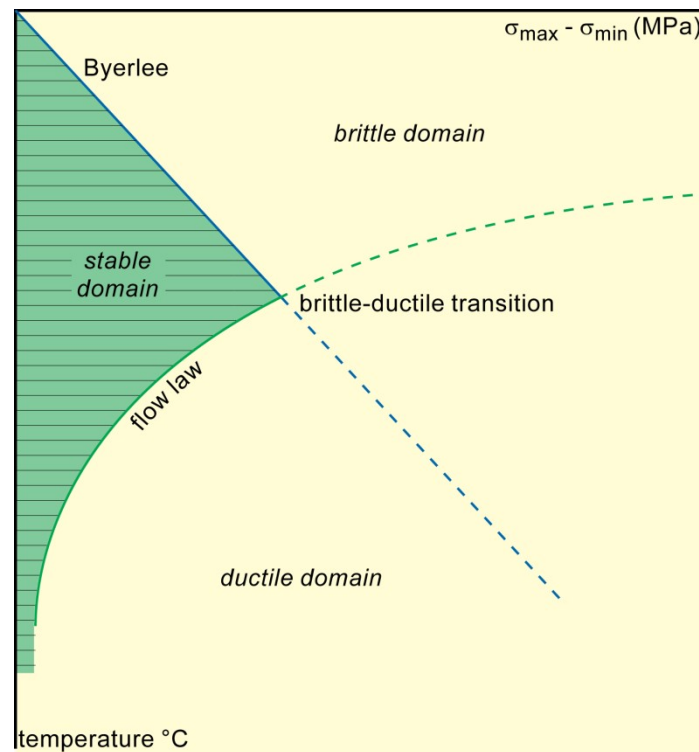
Elementary construction of a rheological profile

Ductile levels

The lower part of the lithospheric strength profile is based on the extrapolation to relevant temperatures of the steady-state power-law (equation 11). It is generally agreed that the bulk viscous behavior of the lithosphere may be described by such a non-linear, temperature and stress-dependent power-law rheology. The curved strength envelope of rocks in ductile deformation is largely insensitive to pressure variations (hence to depth) but decreases exponentially downwards due to thermal softening. Accordingly, the viscous shear strength is likely to decrease with depth. Depending on the state of stress, grain size, and composition, rocks with a non-linear viscosity at moderate temperature/depth may follow a linear, Newtonian constitutive law at very high temperatures.

To calculate the strength of the lithosphere, a strain-rate and a geothermal gradient must be applied to appropriate rocks (e.g. wet quartzite in the crust, olivine in the mantle).

Effect of the stress regime

The Mohr construction demonstrates that the normal stress σ_N and shear stress σ_S are related to the maximum and minimum principal stresses:

$$\sigma_N = \sigma_1 \cos^2 \theta + \sigma_3 \sin^2 \theta$$

and $\hspace{20em}$ (16)

$$\sigma_S = \frac{1}{2} \sin 2\theta (\sigma_1 - \sigma_3)$$

with θ the angle between σ_1 and the normal to the fault plane, which is the same angle as between the fault plane and σ_3 (script on stresses).

The application of the Byerlee's law to the lithosphere requires that it is reformulated in terms of the principal stresses, instead of using normal and shear stress. In effect, we need to know the total

58

differential stress $(\sigma_1 - \sigma_3)$ sufficient to activate fault slip. For this purpose, one substitutes equations (16) into equations (8) and (9).

For $\sigma_N \leq 200\,\text{MPa}$:
$$(1/2)\sin 2\theta(\sigma_1 - \sigma_3) = 0.85\left(\sigma_1 \cos^2\theta + \sigma_3 \sin^2\theta\right)$$

For $200 \leq \sigma_N \leq 2000\,\text{MPa}$
$$(1/2)\sin 2\theta(\sigma_1 - \sigma_3) = 50 + 0.6\left(\sigma_1 \cos^2\theta + \sigma_3 \sin^2\theta\right)$$

Remembering the trigonometric identity $\sin 2\theta = 2\sin\theta\cos\theta$:

For $\sigma_N \leq 200\,\text{MPa}$:
$$(\sin\theta\cos\theta)(\sigma_1 - \sigma_3) = 0.85\left(\sigma_1 \cos^2\theta + \sigma_3 \sin^2\theta\right)$$

For $200 \leq \sigma_N \leq 2000\,\text{MPa}$
$$(\cos\theta\sin\theta)(\sigma_1 - \sigma_3) = 50 + 0.6\left(\sigma_1 \cos^2\theta + \sigma_3 \sin^2\theta\right)$$

Developing these equations to group σ_1 and σ_3 one obtains:

For $\sigma_N \leq 200\,\text{MPa}$:
$$\sigma_1 = \frac{\sin\theta(\cos\theta + 0.85\sin\theta)}{\cos\theta(\sin\theta - 0.85\cos\theta)}\sigma_3 = \tan\theta\frac{(\cos\theta + 0.85\sin\theta)}{(\sin\theta - 0.85\cos\theta)}\sigma_3$$

For $200 \leq \sigma_N \leq 2000\,\text{MPa}$
$$\sigma_1 = \frac{\sigma_3 \sin\theta(\cos\theta + 0.6\sin\theta)}{\cos\theta(\sin\theta - 0.6\cos\theta)} + \frac{50}{\cos\theta(\sin\theta - 0.6\cos\theta)}$$

Taking $\theta = \pi/3$ as the standard angle for numerical application, Byerlee's law in terms of the principal stresses become:

$$\sigma_1 = 4.85\,\sigma_3 \approx 5\,\sigma_3 \qquad \text{for} \qquad \sigma_3 < 101\,\text{MPa}$$
$$\sigma_1 = 3.12\,\sigma_3 + 176 \qquad \text{for} \qquad \sigma_3 > 101\,\text{MPa}$$

Exercise
Draw a Mohr diagram that refers to the last set of equations, and discuss how it verifies the Byerlee's Law.

In an Andersonian stress state and fault orientation, the condition for failure is given by the maximum value of :
$$(\sigma_v - \sigma_h)$$
where σ_h and σ_v are the vertical and horizontal stresses, respectively. In the Earth, σ_v is usually due to the weight of the overburden (equation 13). In Extension $\sigma_v = \sigma_1$ and compression $\sigma_v = \sigma_3$.
Under low stress, in extension:
$$\sigma_3 = \sigma_h \approx \sigma_1/5 = \rho.g.z/5$$
Therefore, the condition for failure in extension is given by:
$$(\sigma_v - \sigma_h) = 4\rho.g.z/5$$

Substituting $g = 9.81\,\text{m.s}^{-2}$ and $\rho = 2800\,\text{kg.m}^{-3}$ gives a slope of $21.97\,\text{MPa.km}^{-1}$ for extension. For high stresses the slope is $18.6\,\text{MPa.km}^{-1} + 67.7\,\text{MPa}$.
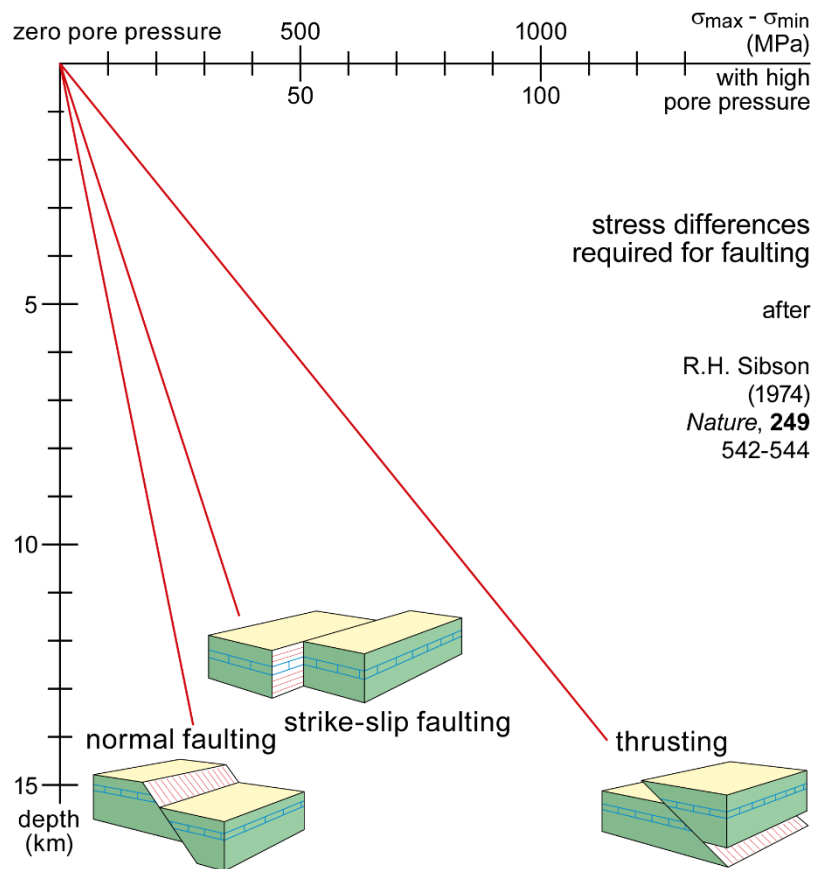
In compression, failure occurs when the horizontal compressive stress has built up to a sufficient value that it has effectively balanced the horizontal stress. This occurs when:
$$\sigma_h = 5\rho.g.z \qquad\qquad (17)$$
the condition of failure is therefore given by:
$$(\sigma_v - \sigma_h) = -4\rho.g.z \qquad\qquad (18)$$

Substituting g and ρ gives a slope of $-109.87\,\text{MPa.km}^{-1}$ for compression. For high stresses, the slope is $-57.7\,\text{MPa.km}^{-1} - 210\,\text{MPa}$.

To summarize: Effects of traction and compression are not symmetric.

The Byerlee's law tends to give a lower bound on the yield stress because it assumes that the lithosphere is already fractured. Seismic waves propagate through the lithosphere, thus suggesting that rocks are dominated by elastic behavior and that the background stress state of the lithosphere is lithostatic. The condition of failure should be considered for a perfectly elastic material. In a perfectly elastic body, the principal stresses are given by:

$$\sigma_2 = \sigma_3 = \sigma_1 \left[\nu / (1 - \nu) \right]$$

where ν is the Poisson's ratio (Equation 2), about 0.25 for rocks. Therefore, in the elastic lithosphere:

$$\sigma_1 = 3\sigma_3$$

Substituting the weight of overburden for these principal strains we see that failure will occur if:

In extension
$$(\sigma_v - \sigma_h) = \rho.g.z - (\rho.g.z/3) = 2\,\rho.g.z/3 \tag{19}$$

In compression
$$(\sigma_v - \sigma_h) = \rho.g.z - 3\,\rho.g.z = -2\,\rho.g.z \tag{20}$$

The same parameters as above yield a slope of 18.3 MPa.km^{-1} in extension and –54.9 MPa.km^{-1} in compression.

The increase of the frictional strength with depth for normal, reverse and strike-slip faults all will be different because of the difference in orientation with the vertical stress. Minimum stress levels for movement on normal faults are only a quarter of those required for thrust faulting and about half that for strike-slip faulting.

Effect of pore pressure

Water may weaken a rock through hydrolytic weakening and diffusive processes enhancing ductile flow. In addition, pore pressure modifies the Coulomb frictional failure criterion (equation 7) that adequately describes the brittle behavior of most rocks. In the case of pre-existing faults of favorable

orientation and negligible cohesion (e.g. $c = 0$), the normal pressure is the lithostatic overburden counteracted by the fluid pressure. The failure criterion can be written as:

$$\sigma_1 - \sigma_3 \geq \beta \rho g z (1 - \lambda) \tag{21}$$

where the same abbreviations as in previous equations represent the same terms, ρ is the average density of rocks above the depth z, and λ the ratio of pore fluid pressure to lithostatic pressure (equation 15); the typical hydrostatic value is $\lambda = 0.4$. β is a numerical parameter comparable to the friction coefficient μ of equation (8) and depending on the type of faulting with values 3, 1.2 and 0.75 for thrust, strike-slip, and normal faulting, respectively. These variables invite reformulating faulting conditions for each tectonic regime. This is done combining the classical relationship between normal and shear stress (equations 16) with equations involving pore pressure.
The governing equations are:

in compressional faulting
$$(\sigma_1 - \sigma_3) = \frac{2\left[c + \mu \rho g z (1 - \lambda)\right]}{\left(\mu^2 + 1\right)^{1/2} - \mu} \tag{22}$$

in tensional faulting
$$(\sigma_1 - \sigma_3) = \frac{-2\left[c + \mu \rho g z (1 - \lambda)\right]}{\left(\mu^2 + 1\right)^{1/2} + \mu} \tag{23}$$

in strike-slip faulting
$$(\sigma_1 - \sigma_3) = \frac{2\left[c + \mu \rho g z (1 - \lambda)\right]}{\left(\mu^2 + 1\right)^{1/2}} \tag{24}$$
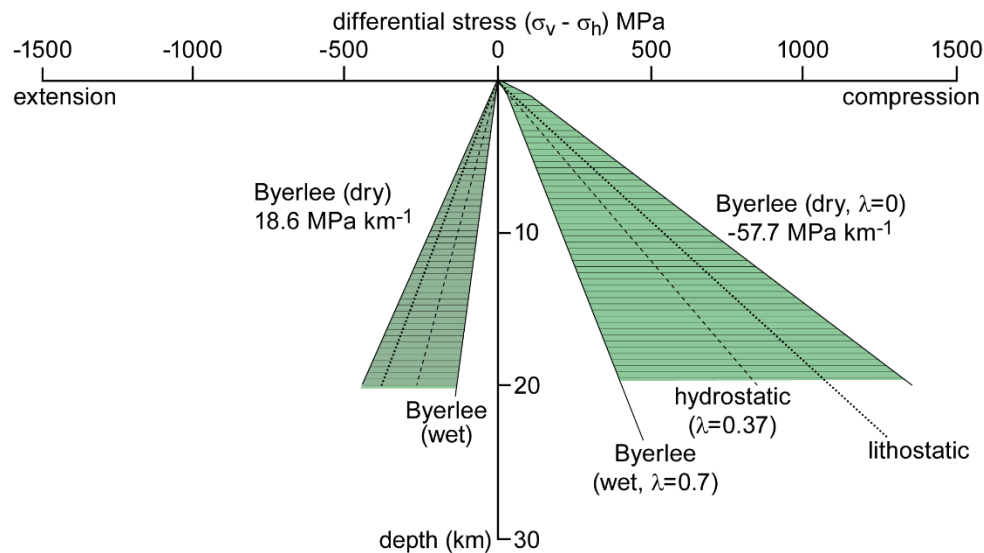

Strength profiles for dry and wet conditions in the crust

These equations (with μ the coefficient of friction) show that the stress required for slip will be lower if the confining effect of lithostatic pressure is reduced by higher fluid pressure, which increases λ.

Effect of strain-rate
Equation 12 shows that the effective viscosity is strain-rate-dependent. Strain-rate has a comparable effect as the temperature in shifting the exponential flow law, mostly upward for slower strain-rate or higher temperature, downward for faster strain-rate or lower temperature.
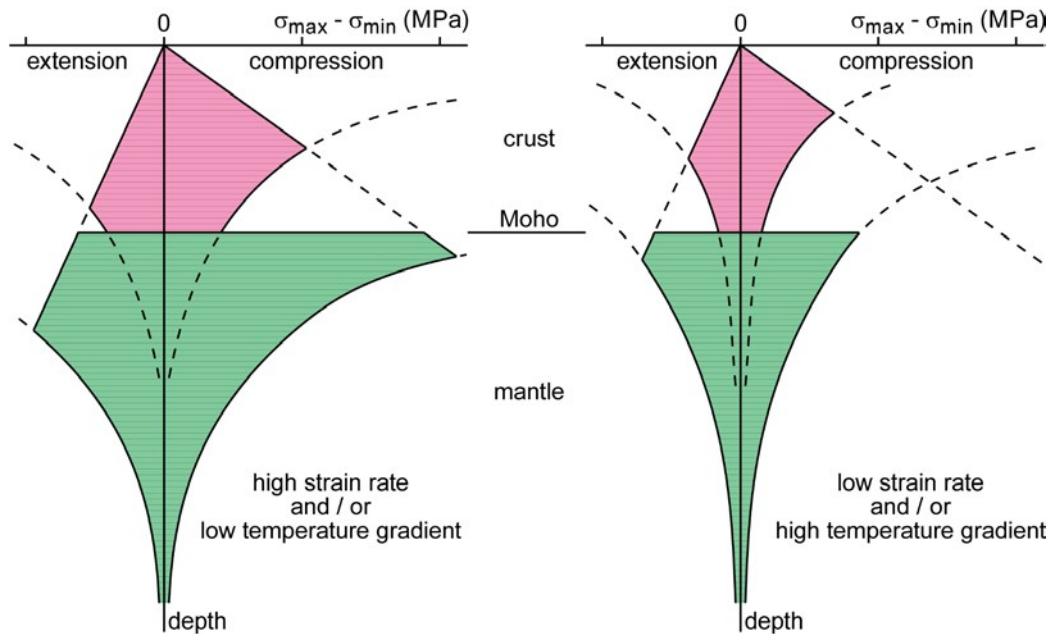
Qualitative strength profiles of a two-layer lithosphere
under different bulk strain rates

This shift moves the intersection of the creep strength curve with the Byerlee yield line. Therefore, it changes the depth and stress level at which, for a given model lithosphere with layers of constant composition, brittle failure switches to ductile flow. Importantly, the upper mantle, just below the Moho, becomes brittle when deformation is fast.

Description

A yield strength envelope shows that the relative importance of temperature and pressure changes in the brittle and ductile regimes of rock deformation. The result is asymmetric and emphasizes the strength of the brittle-ductile transition significantly bigger and shallower in compression than in extension. The area under the yield strength envelope leads to the integrated strength, which is a measure of the total lithospheric strength. This implies that (1) the integrated yield strength transmits the global plate tectonic stress field and (2) the driving forces of plate tectonics cannot exceed the integrated lithospheric strength. This provides an important constraint on the geodynamics of oceans and continents.

The strength predicted in the brittle, frictional part of the crust depends only on the assumed pore pressure. As the slope of the brittle yield strength envelope depends on the mode of faulting, two profiles are plotted on both sides of the vertical temperature axis. One on the positive side of stresses refers to the profile in compression (equations 20 and 22), the one on the negative side corresponds to the profile in tension (equations 19 and 23).

The strength predicted in the ductile, deeper part depends strongly on the assumed rock type, temperature, and strain-rate. Usually, a quartz rheology is applied for the continental crust. Rocks change with depth and the same sort of predictions can be made for any mineral phase. Different viscous envelopes become relevant and the lithosphere may appear rheologically stratified. In particular, at the Moho, mantle (olivine) rocks are more creep resistant and stronger than other silicates at the same temperature.
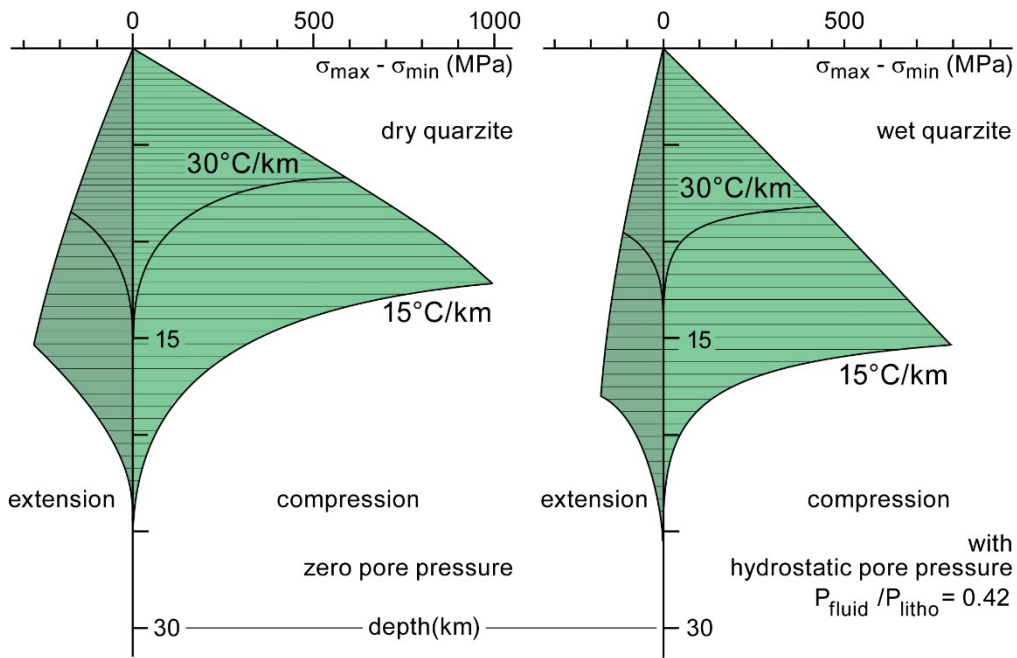
Crustal strength profiles for constant strain rate of 10^{-14} s^{-1}
Effects of pore pressure and temperature gradient

Byerlee's law combined with the quartz and olivine flow law provides a maximum stress profile to about 25 or 50 km, respectively. For a temperature gradient of 15°C/km, stress will be close to zero at the surface and reaches a maximum of 600 MPa (quartz) or 1100 MPa (olivine) for hydrostatic pore pressure.
The rheological stratification of oceanic lithospheres differs from that of continental lithospheres.
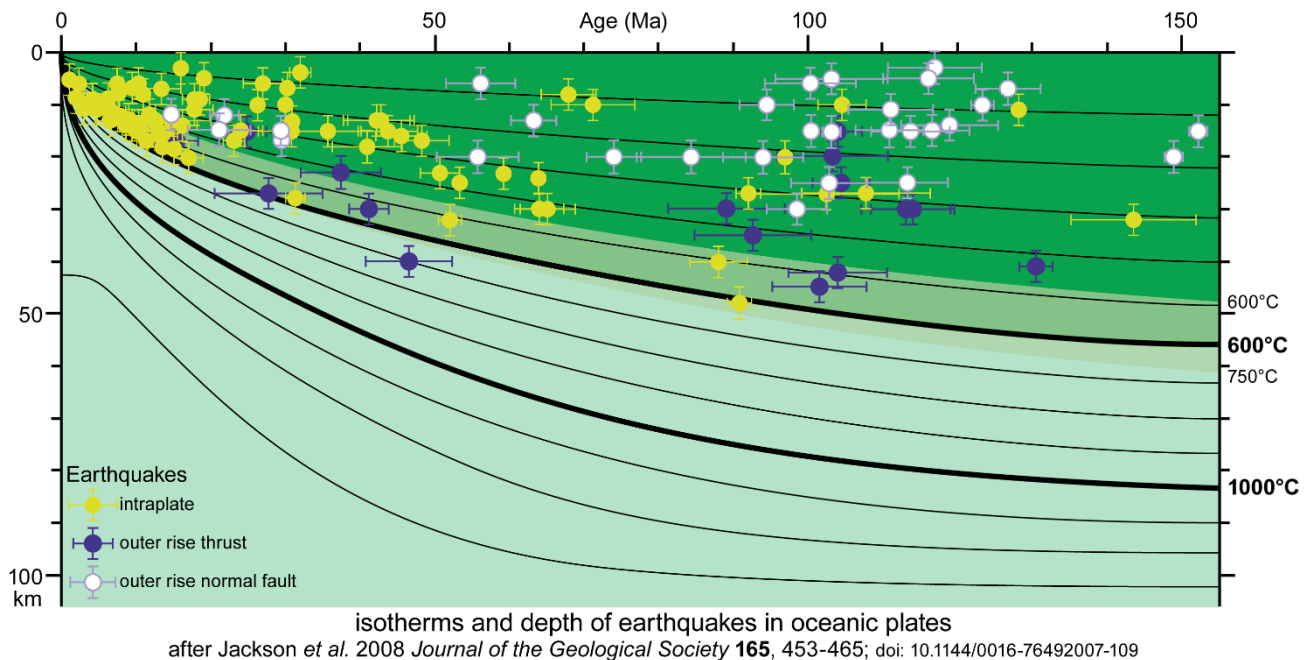
Oceanic lithosphere

The strength profiles of the oceanic lithosphere have simple shapes. The basaltic crust has a near 0 km thickness at the mid-oceanic ridge and the thin sedimentary cover is too negligible to influence the overall rheology of the whole lithosphere. The oceanic crust thickens to about 5-7 km by cooling the sub-crustal mantle lithosphere. Thus, the bulk composition of the oceanic lithosphere is rather uniform and the rheology of olivine should govern its bulk behavior. Strength profiles first increase linearly with depth according to the Byerlee law, then decrease exponentially according to the olivine viscous power law to grade into the nearly no strength of the asthenosphere (etymology: strength-less sphere). Since Byerlee laws apply whatever the type of rocks, the presence of plagioclase and pyroxene in the cold crust of basalts and gabbros can be omitted. The brittle and viscous yield envelopes intersect at the brittle-ductile transition.
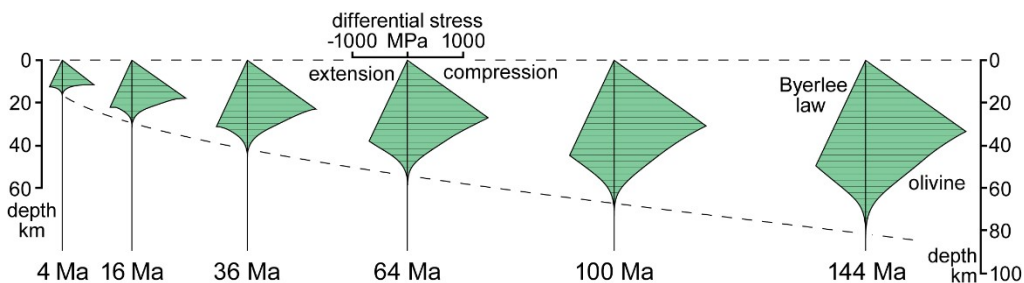
Strength profile of the oceanic lithosphere in function of age

During cooling, the depth of the brittle-ductile transition increase and the size of the integrated region below the strength profile varies. In other words, the bulk strength of the oceanic lithosphere increases with age until the lithosphere reaches a steady-state thickness of about 90-100 km at the 60-80 Ma age. Whatever this age, the profiles are characterized by a single strength maximum.

isotherms and depth of earthquakes in oceanic plates
after Jackson *et al.* 2008 *Journal of the Geological Society* **165**, 453-465; doi: 10.1144/0016-76492007-109

Exercise

Construct the thickness evolution of the oceanic lithosphere from the MOR outward. Assume that the 5km thick oceanic crust at the MOR does not change with time. Conversely, the thickness (H) of the oceanic mantle lithosphere varies according to:

$$H(km) = 10 * \sqrt{\text{Age in Million Years}}$$

Calculate the thickness and the average density of the lithosphere when it is 2.5, 5, 10 and 20 Ma old. Use 2900 kg.m^{-3} as the density of the oceanic crust and 3300 kg.m^{-3} for the mantle. At which age and thickness the lithosphere reaches the same average density as the asthenosphere (3250 kg.m^{-3})? Comment this result.

Continental lithosphere

The concept is more complex to apply to continents where the crust is compositionally heterogeneous and thicker than in oceans.
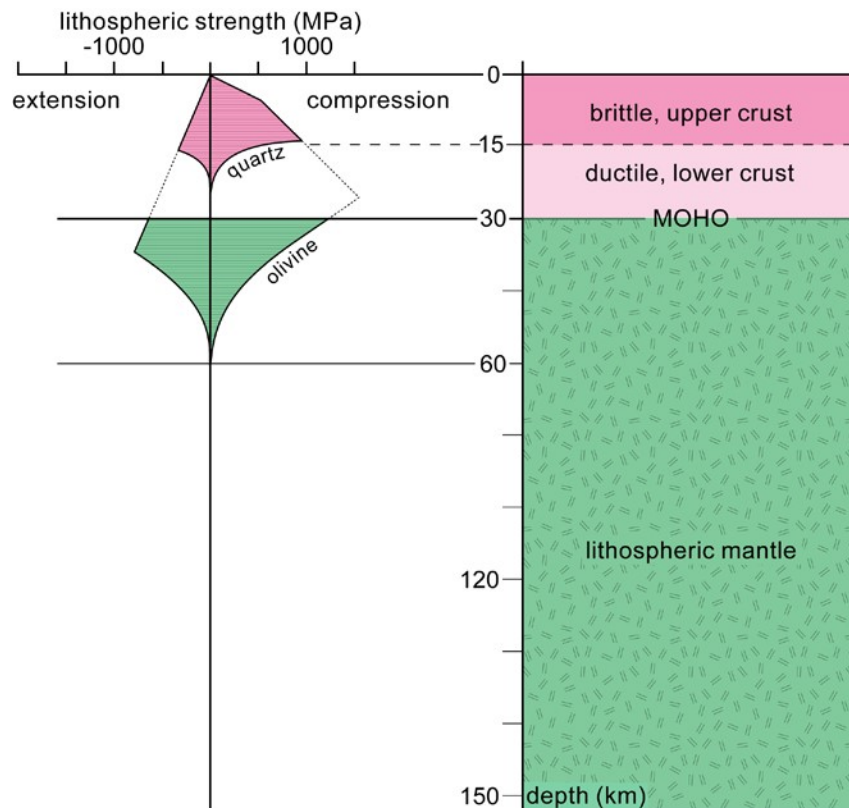
Continental crust

The rheology of the crust is generally approximated as that of the most common mineral, wet quartz, which is brittle at shallow depth but typically ductile at temperatures >300-350°C, well above the Moho (about 500°C). If the crust deforms at a constant strain-rate, then its brittle strength increases with depth and crosses the viscous strength envelope at the "brittle-ductile" transition, at about 15 km depth for a typical geothermal gradient of 20° C km^{-1}. The crust above this depth deforms in a brittle fashion. Below this depth, the crust is viscous and the viscous strength decreases with depth. The brittle-ductile transition supports the highest shear stresses anywhere in the crust and it is within this depth range that the highest moment release might be anticipated from intraplate earthquakes. At an increased strain-rate, the viscous stresses will become larger so that the brittle-ductile transition moves downwards and vice versa.

Mantle

Below the Moho, the viscous curve for wet quartz is replaced by that for olivine. Olivine supports substantially higher shear stresses than quartz and the Moho is, therefore, the region of highest strength in the lithosphere. As for the oceanic lithosphere, the thickness of the continental mantle lithosphere is age-dependent, being the thickest under the oldest cratons.

Total lithosphere

Known experimental data seem to indicate that the lower crust is markedly weaker than the peridotite mantle immediately above the Moho. Consequently, the continental lithosphere has one (at the bottom of the crust) or two (the previous one plus another at mid-crustal levels) soft ductile layers sandwiched between brittle layers.



idealised strength profile of a continental lithosphere

The prevailing model (commonly referred to as the "jelly sandwich" model) is that of a strong upper crust, down to about 15 km, overlying a soft middle to lower crust (down to 20-30 km) and a strong lower crust and uppermost mantle down to the depth at which the olivine viscous law applies. On a rheological profile, the strong brittle levels reaching the Byerlee line are superposed and alternate with strength indentations corresponding to the various ductile levels. The continental lithosphere is thus rheologically layered. Owing to their shape, these diagrams are popularly called "Christmas trees". A rheologically stratified lithosphere has resulted in generally good agreement with the depth distribution of earthquakes in oceanic and continental lithospheres.

In this model, a significant part of the total strength resides in the lithospheric mantle. Where the geothermal gradient is very high, the thickness of the viscous levels increases which causes both lower crust and mantle to be softer. Following the same reasoning, the rigid levels will be much thicker in old, cold cratonic areas than in areas of the thinned lithosphere with high heat flow, or thickened orogenic zones. This may account for a long-termed focusing of lithospheric deformation into orogenic zones. Once a part of the continental lithosphere has become a locus of internal deformation and crustal thickening, it may concentrate the deformation for a long time, until it cools below the plastic limit of crustal rocks or the tectonic forces cease.

The rheological layering may be complicated if the lower crust is mafic or granulitic. In that case, the rheology of plagioclase or dry quartz, respectively, is applied. Molten rocks may be significantly weaker, resulting in decoupling horizons where contrasting rheologies occur. In that perspective, the continental mantle can be attenuated by positive thermal anomalies due to, for instance, plume heads. In that manner, one may include many layers and quite a variety of strength profiles for the continental lithosphere. In short, the many parameters involved prevent defining a standard strength for the continental lithosphere. Yet, the integrated strength for the continent will usually be less than that of the ocean of the same age.

Strength of the brittle-ductile transition

Different geotherms, strain-rates, and thicknesses are used to generate rheological models covering a variety of geodynamic settings. On any model, at some depth, the flow law curve intersects the line representing Byerlee's law. The intersection of the brittle and ductile laws is taken to mark the brittle-ductile transition in the Earth. It is the strongest part of the strength profile.
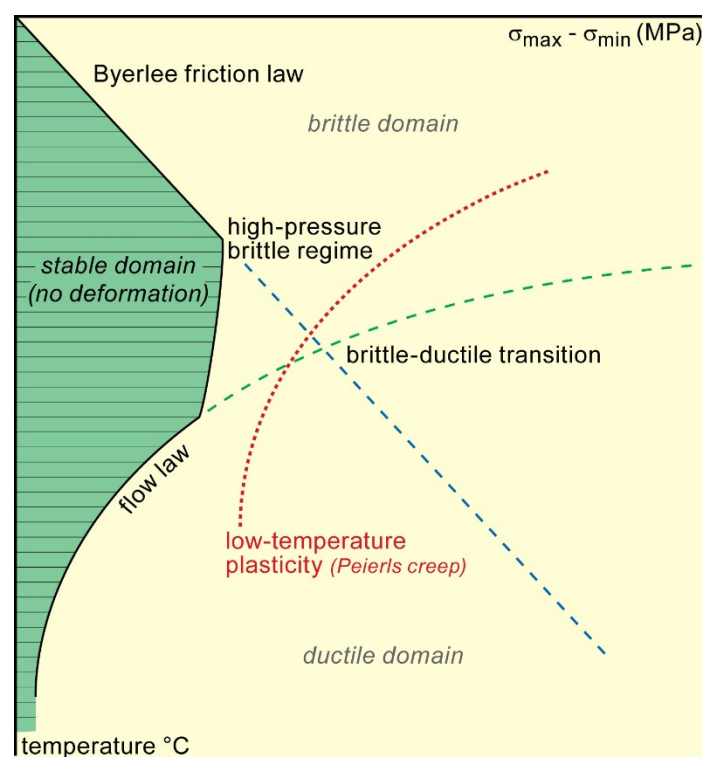
Unscaled effects of high pressure brittle regime and
low-temperature plasticity on strength profiles

High-pressure brittle regime

Friction (Byerlee's) laws are experimentally verified for confining pressures up to a few hundred MPa, corresponding to the upper crust. In strength profiles, these linear laws are customarily extrapolated to lower crustal and even upper mantle depths. This extrapolation is valid only if shear fracture mechanisms are pressure independent. Growing experimental evidence for granite suggests that a change in fracture mechanism takes place at 250-300°C and ca. 1500 MPa. Beyond these values, the fracture strength slightly decreases with the increasing temperature down to the creep strength curve. The brittle field is accordingly divided into two parts: a shallow part where frictional strength increases linearly and significantly with depth and a deeper part where frictional strength is nearly constant with depth. The result is to cut down the strength of the brittle-ductile intersection.

Low-temperature plasticity (Peierls creep)

Besides, ductile creep is related to the intracrystalline dislocation mobility, which depends on a complex interplay between stress and thermal activation. In particular, measurements and calculations of dislocation dynamics pointed out that experimentally-derived creep curves overestimate the strength of rock-forming minerals at low temperatures. The resulting stress-strain curve is expressed by a non-linear constitutive law similar in shape as equation (11) but shifted mostly towards lower temperatures for similar deviatoric stresses. In other words, the creep curve intersects the brittle line at lower stress values than those inferred from high-pressure experiments. Extrapolated to natural conditions, the **low-temperature plasticity** (a.k.a. Peierls mechanism) may limit the deviatoric stresses that can be sustained at the brittle-ductile intersection on the simple plot.

Application

The rheological knowledge is fundamental to the laboratory, scaled experiments on analog models. These tectonic models are attempts, in miniature, to simulate conceptually and mechanically the deformation of the Earth's lithosphere (in particular large-scale features such as sedimentary basin formation or orogen development) under defined loading conditions. Analog models supplement numerical modeling because the inherent limitations of each method are different.
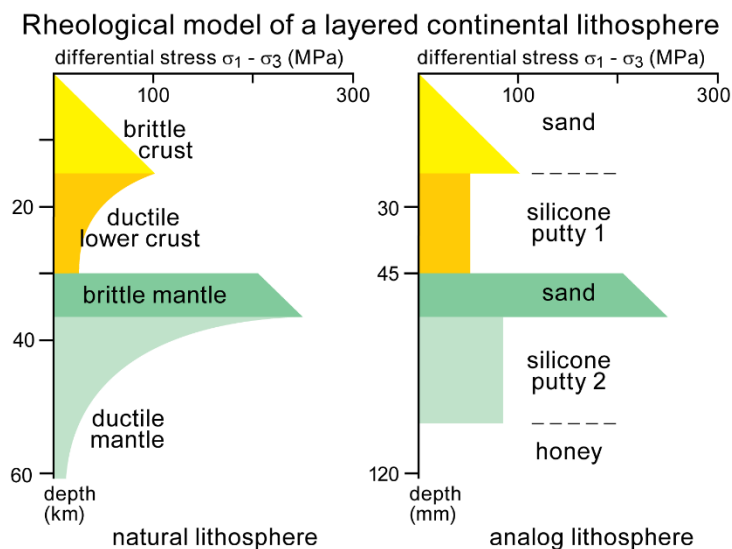
In theory, the lithospheric rheological profiles give a lower limit of the strength of the lithosphere, because they describe the stress levels required to drive suitably oriented faults. In practice, they only provide a qualitative guide to the actual strength, i.e. the greatest possible stress as a function of depth. These profiles show that most of the total strength of the lithosphere resides in the seismogenic crust. It is clear that the thermal structure of deforming plates, and therefore their age, is very important to the mechanics of the phenomenon. Such models are used to analyze observed variations in the crustal style of deformed continental and oceanic lithospheres.

Limitations

First-order uncertainties limit a direct assessment of lithosphere rheology.

Composition

Exact knowledge of the composition of rocks belonging to the lithosphere under consideration is impossible. However, rocks are heterogeneous mineral aggregates constituted of a few main components. The first assumption is that the main constituents govern the rheology of aggregates. Rocks are therefore simplified to their major constituents such as feldspar and quartz in continental crustal rocks, and olivine in mantle rocks.

Oceanic lithosphere

The oceanic crust is basaltic and wet. Wet basalts are therefore the reference rheology.

Continental lithosphere

The composition of the continental lithosphere is envisioned from three information sources: (a) seismic data (b) petrology of surface exposures, and (c) xenolith data. All lead to very broad rheological approximations.

Granitoids, quartz-rich sediments, and quartz-bearing schists usually dominate the bulk composition of the continental upper crust. Quartz is therefore considered to control the bulk rheology of the upper continental crust. Since the upper part of the crust is hydrated, the rheological parameters of wet quartzite are taken as the best approximation.

The bulk of the continental lower crust generally has seismic velocities compatible with mafic composition. However, more felsic compositions may predominate in particular regions. Accordingly, the mechanical behavior of the continental lower crust is modeled with the creep parameters of both mafic and felsic granulites, and of wet and dry diabase.

This crustal stratification rests on an ultramafic upper mantle. Thus, the flow properties of olivine and peridotites should be representative of the response of the upper mantle. The wet rheology may be appropriate for the continental lithospheric mantle in zones recently affected by the subduction of oceanic lithosphere and post-Paleozoic tectonothermal events, while the dry rheology may be more relevant for older regions.

State variables

The second type of uncertainty stems from errors in the state variables such as pressure, temperature, stress level, grain size, and fluid pressure. Temperature and grain size assert direct control on the creep of rocks, whereas the effective pressure controls the frictional strength of fault.

Thermal conditions in the continental lithosphere are modeled using assumptions on the depth distribution of heat-producing elements and either a fixed asthenosphere temperature or heat flux boundary conditions at the lithosphere/asthenosphere boundary. Cooling of the lithosphere introduces an age-dependence in the rheology of the lithosphere.

The lithostatic pressure can be estimated as the overburden load with reasonable boundaries. The effective pressure (equation 14) is much less certain. However, typical crustal permeability is such that fluid pressures above lithostatic cannot be maintained for geological times.

The shear stress level ranges from 1 to 150 MPa at depths between 1 and 100 km. Linear extrapolation of the Byerlee's law probably overpredicts the stresses at a depth greater than 5-10 km (fitting stresses in the KTB borehole at 8 km, but only 150 MPa at 11 km in the Kola well).

Conclusion

Rheology offers explicit equations, the quantitative constitutive laws that make responses of rocks to inherent physical properties and imparted forces predictable.

> Increasing pressure strengthens rocks
> Increasing temperature weakens rocks.
> Fluids weaken rocks by affecting the bonding of materials while high fluid pressure reduces effective stress.

At depth, temperature overcomes the strengthening effect of confining pressure, which generally leads to ductile behavior, by opposition to frictional processes that dominate at low temperatures. Generally, pressure and temperature are strongly dependent on depth and vary relatively less laterally. Therefore, the lithosphere tends to have horizontally stratified mechanical properties. Different factors such as composition contribute to this stratification. The horizontally stratified compositions may reflect layering in undeformed sediments or stacks of structural units, such as thrust sheets.

The large-scale characteristics of a lithospheric system show a satisfactory agreement with the inferred rheological structure: sub-horizontal decrements should be the rule rather than the exception where the lithospheric rheology is strongly stratified. The most significant attributes are the maximum in

strength at the brittle-ductile transition in the continental crust and a second maximum at the Moho, where the rock composition changes from quartzo-feldspathic rocks to olivine-dominated peridotite. Comparison of the high-temperature bulk steady-state rheologies of crustal rocks with flow laws for polycrystalline olivine indicates that the Moho should be a rheological discontinuity with the uppermost mantle being significantly stronger than the lowermost continental crust.

The strain-rate has a decisive influence on the rheology of rocks: materials deforming under low strain-rates (geologically realistic) exhibit creep, whereas high strain-rates are associated with geologically instantaneous strain. Slow deformation and creep allow diffusional crystal-plastic processes to more closely keep up with applied stress.

Lithospheric strength profiles illustrate, approximately, which levels of the lithosphere behave in brittle and ductile manners. The relative magnitude of frictional and creep strength determines the first-order rheological behavior (brittle or ductile) at any given depth. However, the actual variation of strength with depth depends strongly on the deformation mechanism as well as on the type of rock. The brittle/ductile transition is likely to be gradual, but this is an often-neglected second-order effect. The strength of the lithosphere (i.e. the total force per unit width necessary to deform a lithospheric section at a given strain-rate) is a function of composition, crustal thickness, and geothermal gradients.

Recommended literature

Ashby M. F. & Verall R. A. - 1977. Micromechanisms of flow and fracture, and their relevance to the rheology of the upper mantle. *Philosophical Transactions of the Royal Society of London.* **A288 (1350)**, 59-95, 10.1098/rsta.1978.0006

Brace W. F. & Kohlstedt D. T. - 1980. Limits on lithospheric stress imposed by laboratory experiments. *Journal of Geophysical Research.* **85 (B11)**, 6248-6252, 10.1029/JB085iB11p06248

Burov E. B. - 2011. Rheology and strength of the lithosphere. *Marine and Petroleum Geology.* **28 (8)**, 1402-1443, 10.1016/j.marpetgeo.2011.05.008

Byerlee J. D. - 1978. Friction of rocks. *Pure and Applied Geophysics.* **116 (4-5)**, 615-626,

Jaeger J. C., Cook N. G. W. & Zimmerman R. W. - 2007. *Fundamentals of rock mechanics. Fourth edition.* Blackwell Publishing, Oxford. 475 p.

Kirby S. H. - 1983. Rheology of the lithosphere. *Review of Geophysics and Space Physics.* **21 (6)**, 1458-1487, 10.1029/RG021i006p01458

Kirby S. H. - 1985. Rock mechanics observations pertinent to the rheology of the continental lithosphere and the localization of strain along shear zones. *Tectonophysics.* **119 (1-4)**, 1-27, 10.1016/0040-1951(85)90030-7

Kohlstedt D. L., Evans B. & Mackwell S. J. - 1995. Strength of the lithosphere : constraints imposed by laboratory experiments. *Journal of Geophysical Research.* **10 (B9)**, 17587-17602, 10.1029/95JB01460

Ranalli G. - 1995. *Rheology of the Earth.* 2nd, Chapman & Hall, London. 413 p.

Ranalli G. & Murphy D. C. - 1987. Rheological stratification of the lithosphere. *Tectonophysics.* **132 (4)**, 281-295, 10.1016/0040-1951(87)90348-9

Scholz C. H. - 1992. *The mechanics of earthquakes and faulting.* 2nd, Cambridge University Press, Cambridge. 439 p.

Sibson R. H. - 1974. Frictional constraints on thrust, wrench and normal faults. *Nature.* **249 (5457)**, 542-544, 10.1038/249542a0

Tullis T. E. & Tullis J. - 1986. Experimental rock deformation techniques. <u>In</u>: *Mineral and rock deformation: Laboratory studies.* (Hobbs B. E. & Heard H. C.), *Geophysical Monograph Series*, **36**, American Geophysical Union, 297-324, 10.1029/GM036p0297

Turcotte D. L. & Schubert G. - 2002. *Geodynamics*. Second Edition, Cambridge University Press, Cambridge. 456 p.